FLUID DYNAMICS AND AERODYNAMICS

1.0 Introduction

Fluid dynamics deals with the motion of materials that can be represented as fluid and their interactions with boundary in the flow. Gas such as air in the atmosphere or liquid such as water in oceans can be represented as fluid. Fluid flows are encountered in engineering systems, metrology, aeronautics, combustion, hydrodynamics, etc. The study of fluid dynamics is required for designing, analyzing and optimization of engineering devices such as aircraft, piping systems, heat exchangers, and so on.

1.1 Classification of fluid dynamics

Hydrodynamics: Study the flow of liquids and their interactions with impermeable walls in the flow.

Gas dynamics: Study the flow of gas and its interactions with solid surfaces in the flow.

Aerodynamics: Study of air flow and its interactions with solid surfaces in the flow.

It should be noted that these three categories are not mutually exclusive but there are many similarities and identical phenomena between them.

1.2 Objectives of fluid dynamics

1. Prediction of forces and moment on body moving through fluid.
2. Prediction of heat transfer to bodies moving through fluid.
3. Determination of flow phenomena and heat transfer in duct.

1.3 Methods of fluid dynamics study

1. Experimental method.

2. Computational method (CFD).

3. Flow visualization method.

2.0 Equations of fluid motion

In fluid flow analysis, the conservation laws governing the motion of fluid apply to any particle or element of the fluid. The equation of motion governing fluid flow particles can be derived from the Reynolds transport theorem.

2.1 Reynolds Transport Theorem

Consider the rate of change of an extensive property of a system as it passes through a control volume shown in Fig. 2.1.

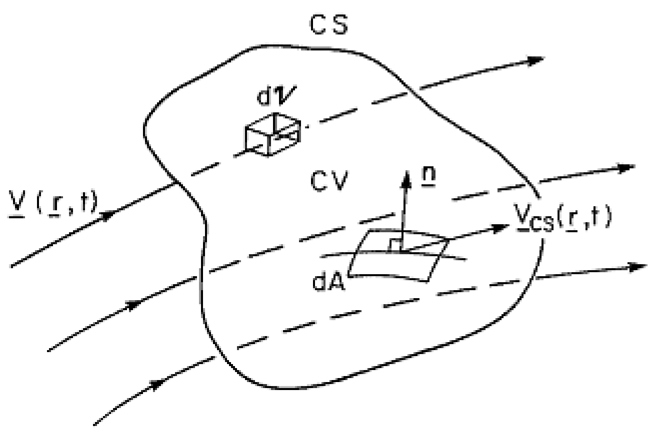


Fig 2.1: Control volume

Let B denote any *extensive* property B (e.g., mass, momentum or energy) and $b=B/m$ *denote* the corresponding intensive property. The Reynolds transport theorem for a moving and arbitrarily deforming control volume CV, with boundary CS is

$$\frac{d}{dt}\left(B_{system}\right)=\frac{d}{dt}\left(\iiint_{CV}\rho b d\upsilon+\iint_{CS}\rho b\left(V_r.n\right)dA\right) \qquad 2.1$$

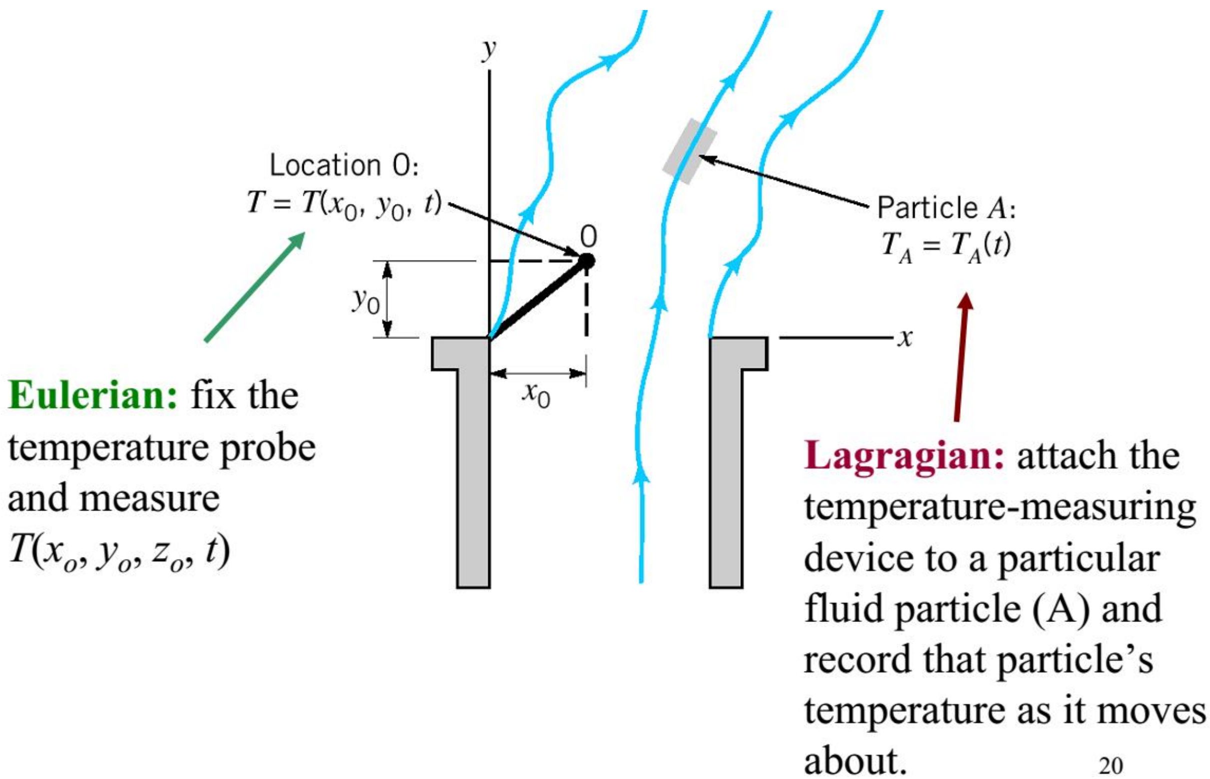
Where $\boldsymbol{n}$ is the outward normal to the CS, $\boldsymbol{V_r} = \boldsymbol{V}_{(\boldsymbol{r},\, t)} - \boldsymbol{V}_{CS(\boldsymbol{r},\, t)}$, the velocity of the fluid particle, $\boldsymbol{V}(\boldsymbol{r}, t)$, relative to that of the CS.

The theorem states that the time rate of change of the total B in the system is equal to the rate of change within the CV plus the net flux of B through the CS.

Eulerian and Lagrangian Flow Descriptions

The **LHS** of eqn (2.1) is the **Lagrangian form**; it states that the rate of change of property B evaluated while moving with the system. The RHS is the **Eulerian form**; it states that the change of property B evaluated at fixed point in space.

Example: To determine the temperature of smoke from a chimney as shown below



In order to provide the connection between the Lagrangian and Eulerian descriptions of fluid flow at the instant the system occupies the control volume, we introduce the control volume approach.

2.2 Integral Relations for a Control Volume

2.2.1 Conservation of mass

Applying eqn (2.1) to a fixed control volume, d(*B*system/*dt)* = 0, $b = B/m = 1$ and $V_r = V$.

$$\iiint_{CV} \frac{\partial \rho}{\partial t} d\upsilon + \iint_{CS} \rho (V.n) dA = 0 \qquad 2.2$$

Eqn (2.2) is the integral form of the conservation of mass law for a fixed control volume. For a steady compressible flow, eqn (2.2) becomes

$$\iint_{CS} \rho(V.n)dA = 0 \qquad 2.3$$

For incompressible flow, ρ = constant, eqn (2.3) is reduce to

$$\iint_{CS} (V.n)dA = 0 \qquad 2.4$$

2.2.2 Conservation of momentum

For the conservation of linear momentum

$$\sum F = \frac{D}{Dt}\left(\iiint_{system} \rho V d\upsilon\right) = \frac{d}{dt}\left(\iiint_{CV} \rho V d\upsilon\right) + \iint_{CS} \rho V(V_r.n)dA \qquad 2.5$$

For a steady flow fixed control volume, eqn (2.5) can be written as

$$\sum F = \iint_{CS} \rho V(V.n)dA \qquad 2.6$$

The total external forces acting on the system in eqn (2.5) are the body force $\boldsymbol{F_b}$ and surface force $\boldsymbol{F_s}$.

$$F_b = \iiint_{system} \rho g d\upsilon \qquad 2.7$$

$$F_s = \iiint_{system\_surface} \sigma.ndA \qquad 2.8$$

Eqn (2.5) can be written in integral form as

$$\frac{\partial}{\partial t}\left(\iiint_{system} \rho g d\upsilon + \iiint_{system\_surface} \sigma.ndA\right) = \frac{D}{Dt}\iiint_{system} V d\upsilon = \frac{d}{dt}\left(\iiint_{CV} \rho V d\upsilon\right) + \iint \rho V(V_r.n)dA \qquad 2.9$$

2.2.3 Conservation of Energy

The energy conservation equation can be written as

$$\dot{Q} - \dot{W} = \frac{D}{Dt}\left(\iiint_{system} \rho e d\upsilon \right) = \frac{d}{dt}\left(\iiint_{CV} \rho e d\upsilon \right) + \iint_{CS} \rho e (V_r . n) dA \qquad 2.10$$

where $\dot{Q}$ is the rate at which heat is added to the system, $\dot{W}$ the rate at which the system works on its surroundings, and *e* is the total energy per unit mass. For a particle of mass *dm* the contributions to the specific energy *e* are the internal energy *u*, the kinetic energy $V^2/2$, and the potential energy, which in the case of gravity, the only body force we shall consider, is *gz*, where *z* is the vertical displacement opposite to the direction of gravity.

For a fixed control volume it then

$$\dot{Q} - \dot{W} = \frac{D}{Dt}\left(\iiint_{system} \rho \left(u + \frac{V^2}{2} + gz \right) d\upsilon \right) = \frac{d}{dt}\left(\iiint_{CV} \rho \left(u + \frac{V^2}{2} + gz \right) d\upsilon \right) + \iint_{CS} \rho \left(u + \frac{V^2}{2} + gz \right) (V . n) dA$$

2.11

2.3 Differential relations for fluid motion

Generally, the integral relations are useful in control volume analysis of average features of flow. Such analyses usually require some assumptions about the flow. However, approaches based on integral conservation laws cannot be used to determine the point-by-point variation of the dependent variables, such as velocity, pressure, temperature, etc. Applications of differential forms of conservation laws are require to achieve this.

2.3.1 Conservation of mass – continuity equation

Applying the divergence theorem to eqn (2.2) we obtain

$$\iiint_{\substack{CV \\ (fixed)}} \left[\frac{\partial \rho}{\partial t} + \nabla . (\rho V) \right] d\upsilon = 0 \qquad 2.12$$

Since the control volume is arbitrary, eqn (2.12) can be written in differential form as

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho V) = 0 \qquad 2.13$$

2.3.2 Momentum equation

Applying the divergence theorem to eqn (2.9) and assuming arbitrary control volume, we obtain

$$\rho \frac{DV}{Dt} = \rho g + \nabla . \sigma \qquad 2.14$$

$$\sigma = -pI + \tau \qquad 2.15$$

Substituting eqn(2.15) in eqn (2.14) yield

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \nabla . \tau \qquad 2.16$$

Where p = pressure, I = unit tensor and τ = viscous force.

For a Newtonian fluid, the viscous stress relation is given as

$$\tau = -\mu \left[\nabla V + (\nabla V)^T \right] + \left(\frac{2}{3}\mu - k \right)(\nabla V)I \qquad 2.17$$

Where $\nabla V = \frac{\partial \upsilon_i}{\partial x_j}$, the subscript T indicates the transpose matrix, i.e $(\nabla V)^T = \frac{\partial \upsilon_j}{\partial x_i}$

μ and k are the coefficient of shear viscosity and bulk viscosity respectively.

For constant ρ and μ, eqn (2.16) yield

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \mu \nabla^2 V \qquad 2.18$$

Eqn (2.18) is known as ***Navier–Stokes Equations***

2.3.3 Energy equation

From eqn (2.10), $\dot{Q} = -\iint q.ndA = -\iiint \nabla .qd\upsilon$ 2.19

$$\dot{W} = -\iiint \rho g.Vd\upsilon - \iint V.(\sigma n)dA \qquad 2.20$$

Substituting eqns (2.19) and (2.20) in (2.10) and neglecting the potential energy contribution yield

$$\rho \frac{D}{Dt}\left(u + \frac{1}{2}V^2 \right) = -\nabla .q + \rho g.V + \nabla .(V\sigma) \qquad 2.21$$

Substituting eqn (2.15) in (2.21) yield

$$\rho \frac{De}{Dt} = -\nabla.q + V.\rho g - \nabla.(pV) + \nabla.[V.\tau] \qquad 2.22$$

In summary, for incompressible flow, equations can be written as

Continuity equation: $\nabla.V = 0$ 2.23

Momentum equation: $\rho \frac{DV}{Dt} = \rho g - \nabla p + \mu \nabla^2 V$ 2.24

Energy equation: $\rho c_v \frac{DT}{Dt} = k\nabla^2 T + \Phi$ 2.25

Where k = thermal conductivity, Φ = dissipation function

$$V = ui + \upsilon j + wk \qquad 2.26$$

$\frac{D}{Dt}$ = substantial derivative operator = $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \upsilon \frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ 2.27

2.4 Boundary conditions

The applications of boundary conditions at the boundary of a fluid in contact with another medium depends on the nature of this other medium — solid, liquid, or gas.

For solid surface, ***V*** and ***T*** are continuous. In the case viscous flow, the ‘non-slip’ condition is applied i.e the tangential velocity of the fluid in contact with the solid boundary is equal to that of the boundary (zero). In the case of inviscide flow, the ‘non slip’ condition cannot be applied, and only the normal component of the velocity is continuous.

However, if the wall is permeable, the tangential velocity is continuous and the normal velocity is arbitrary; the temperature boundary condition for this case depends on the nature of the injection or suction at the wall.

3.0 Angular velocity, Vorticity and Irrotational flow

The local velocity field of fluid particle or element consists of translation, rotation with angular velocity and velocity rate of deformation.

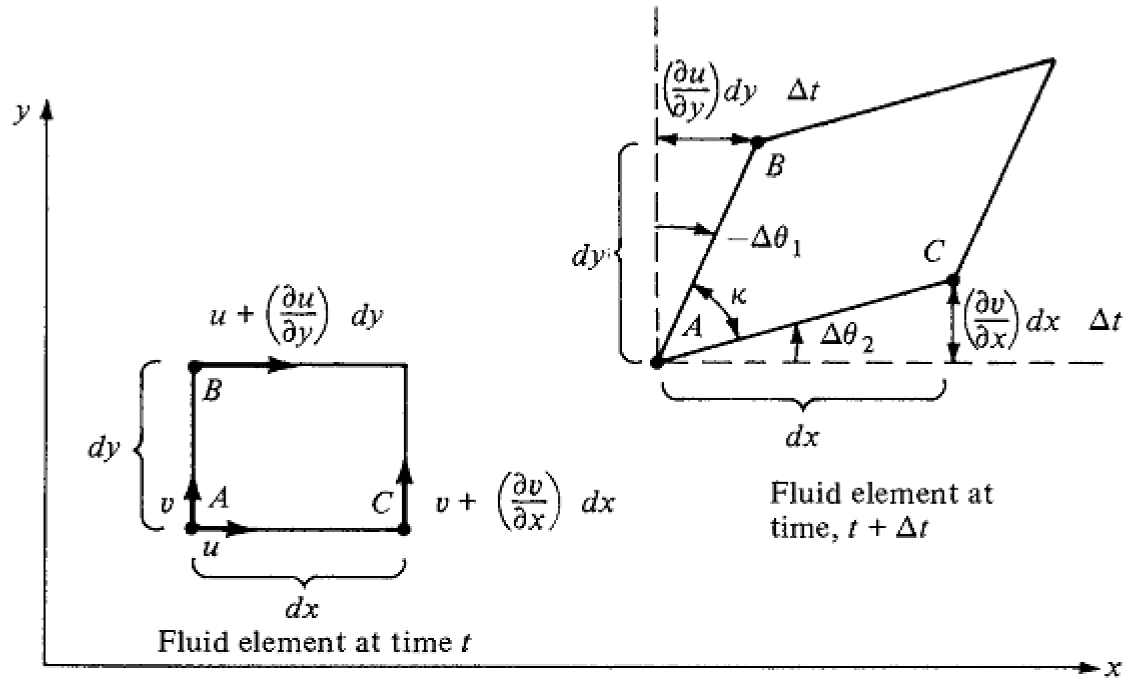


Fig 3.1: Rotation and distortion of a fluid element

Consider fluid particle moving in two-dimensional *xy* plane as shown in fig 3.1. At time *t* the shape of this fluid element is rectangular, as shown at the left of Fig. 3.1. As the fluid element moves upward and to the right; its position and shape at time $t + \Delta t$ are shown at the right in Fig. 3.1. During the time increment Δt, the sides *AB* and *AC* rotated through the angular displacements $-\Delta\theta_1$ and $\Delta\theta_2$, respectively. (Note that by convention, counterclockwise rotation is positive while clockwise rotation negative).

For the y direction:

Distance move by A at time Δt = $\upsilon \nabla t$ 3.1

Distance move by C at time Δt = $\left(\upsilon + \frac{\partial \upsilon}{\partial x} dx\right)\Delta t$ 3.2

Displacement of C relative to A = $\left(\upsilon + \frac{\partial \upsilon}{\partial x}dx\right)\Delta t$ - $\upsilon \Delta t$

$$= \left(\frac{\partial \upsilon}{\partial x}dx\right)\Delta t \qquad 3.3$$

From the geometry in fig.3.1, $\tan\theta_2 = \frac{[(\partial \upsilon / \partial x)dx]\Delta t}{dx} = \frac{\partial \upsilon}{\partial x}\Delta t$ 3.4

Since $\Delta\theta_2$ is small, $\tan\Delta\theta_2 \approx \theta_2$, $\therefore \Delta\theta_2 = \frac{\partial \upsilon}{\partial x}\Delta t$ 3.5

For the x direction:

Distance move by A at time Δt = $u\nabla t$ 3.6

Distance move by B at time Δt = $\left(u + \frac{\partial u}{\partial y}dy\right)\Delta t$ 3.7

Displacement of B relative to A = $\left(u + \frac{\partial u}{\partial y}dy\right)\Delta t$ - $u\Delta t$

$$= \left(\frac{\partial u}{\partial y}dy\right)\Delta t \qquad 3.8$$

From the geometry in fig.3.1, $\tan(-\theta_1) = \frac{[(\partial u / \partial y)dy]\Delta t}{dy} = \frac{\partial u}{\partial x}\Delta t$ 3.9

Since $-\Delta\theta_1$ is small, $\tan(-\Delta\theta_1) \approx \theta_1$, $\therefore \Delta\theta_1 = -\frac{\partial u}{\partial y}\Delta t$ 3.10

The angular rotation of the fluid immediately adjacent to point A is given by

$$\frac{1}{2}(\Delta\theta_2 - \Delta\theta_1) = \frac{1}{2}\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta t \qquad 3.11$$

The rate of rotation of the fluid about the z-axis is defined by the angular velocity as

$$\omega_z = \frac{\frac{1}{2}\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta t}{\Delta t} = \frac{1}{2}\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad 3.12$$

The resulting angular velocity in three-dimensional space is represented as

$$\omega = \omega_x i + \omega_y j + \omega_z k \qquad 3.13$$

$$\omega = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial \upsilon}{\partial z}\right)i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)j + \left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)k\right] \qquad 3.14$$

$$2\omega = \left(\frac{\partial w}{\partial y} - \frac{\partial \upsilon}{\partial z}\right)i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)j + \left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)k \qquad 3.15$$

eqn (3.15) is called ***vorticity*** and is denoted as

$$\xi = 2\omega = \nabla \times V \qquad 3.16$$

Vorticity is defined as twice the angular velocity.

The preceding eqn (3.16) leads to two important definitions:

1. If $\nabla \times V \neq 0$ at every point in a flow, the flow is called ***rotational***. This implies that the fluid elements have a finite angular velocity.

2. If $\nabla \times V = 0$ at every point in a flow, the flow is called ***irrotational***. This implies that the fluid elements have no angular velocity; rather, their motion through space is a pure translation.

3.1 Circulation

Mathematically, circulation is express, $\Gamma = -\oint V.dS$ i.e a line integral of flow velocity integrated about the closed curve drawn in the flow.

It is used in aerodynamics specifically in the analysis of low-speed airfoils and wings.

In vortex flow, the vortex strength Γ is the circulation taken about any closed curve that encloses the central point.

3.2 Inviscid Flow models

Inviscid flow is one in which the transport phenomena of viscosity, thermal conduction and mass diffusion are negligible. This approximation applied to flows at high Reynolds number that contain only small regions of negligible separated flow. Inviscid model adequately predicts the pressure distribution and lift on the body and give a valid representation of the streamlines and flow field away from the body. However, because friction (shear stress) is a major source of aerodynamic drag, inviscid theories by themselves cannot adequately predict total drag.

The equations describing inviscid flows can be obtained by neglecting the viscous terms of the **Navier-Stokes equations**.

$$\rho \frac{DV}{Dt} = \rho g - \nabla p \quad 3.17$$

Equation (3.17) is called **Euler equation.** It consists of hyperbolic system of partial differential equations. Due to the absence of viscous term in the equation, the resulting solutions are discontinuous across the solid surfaces or walls in the flow. Thus, such solution must be interpreted within the context of generalized or weak solutions.

3.3 Bernoulli equation

The Bernoulli equation can derive by integrating the Euler's equation as follows:

x-direction $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\rho \partial x} + Bx \quad 3.18a$$

y-direction $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\rho \partial y} + By \quad 3.18b$$

z-direction $$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\rho \partial x} + Bx \quad 3.18c$$

For potential flow; $$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0, \because \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \quad 3.19$$

Similarly, $\frac{\partial u}{\partial z}=\frac{\partial w}{\partial x}$ and $\frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}$ 3.20

Substituting eqns (3.19) and (3.20) in (3.18) yield

x-direction $\frac{\partial^2 \phi}{\partial x \partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}+w\frac{\partial w}{\partial x}=-\frac{\partial p}{\rho \partial x}+Bx$ 3.21a

y-direction $\frac{\partial^2 v}{\partial y \partial t}+u\frac{\partial u}{\partial y}+v\frac{\partial v}{\partial y}+w\frac{\partial w}{\partial y}=-\frac{\partial p}{\rho \partial y}+By$ 3.21b

z-direction $\frac{\partial^2 w}{\partial z \partial t}+u\frac{\partial u}{\partial z}+v\frac{\partial v}{\partial z}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\rho \partial x}+Bx$ 3.21c

integrating eqns (3.21a), (3.21b) and (3.21c) with respect to x, y and z respectively

x-direction $\frac{\partial \phi}{\partial t}+\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}+\frac{p}{\rho}+\Omega= f_1(y,z,t)$ 3.22a

x-direction $\frac{\partial \phi}{\partial t}+\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}+\frac{p}{\rho}+\Omega= f_2(x,z,t)$ 3.22a

x-direction $\frac{\partial \phi}{\partial t}+\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}+\frac{p}{\rho}+\Omega= f_3(x,y,t)$ 3.22c

since the LHS of eqns (3.22) are same, therefore $f_1(y,z,t) = f_2(x,z,t) = f_3(x,y,t) = f(t)$

let $|V|=\left(u^2+v^2+w^2\right)^{1/2}$ magnitude of velocity vector.

$$\frac{\partial \phi}{\partial t}=\frac{|V|^2}{2}+\Omega+\frac{p}{\rho}+f(t) \quad 3.23$$

Where Ω = gz = body force

Eqn (3.23) is known as ***Bernoulli equation*** for unsteady incompressible flow.

For steady flow eqn (3.23) can be written as

$$\frac{|V|^2}{2g}+z+\frac{p}{\rho g} = \text{constant} \quad 3.24$$

4.0 Potential flow

This is used to describe frictionless irrotational flow as shown in fig 4.1.

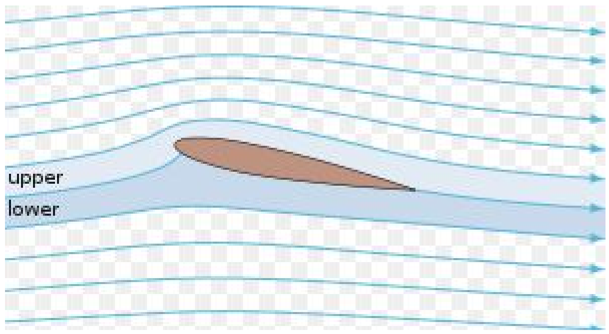


Fig 4.1: Potential flow streamlines over airfoil

4.1 Velocity potential

In fluid dynamics, potential flow describes the ***velocity field*** as the gradient of a scalar function: ***the velocity potential***. Potential flow is characterized by an irrotational velocity field as shown in fig 4.1, which is a valid approximation for several applications. The irrotationality of a potential flow is due to the curl of the gradient of a scalar always being equal to zero. This implies that that the individual particles of fluid moving along a streamline are in translational motion only.

From vector calculus, the curl of a gradient is equal to zero

Potential flow, $V = \nabla\phi$ 4.1

Irrotational flow, $\nabla \times V = 0$ 4.2

Substituting eqn (4.1) in (4.2) yield

$\nabla \times \nabla\phi = 0$ 4.3

Eqn (4.3) shows that the curl of a gradient is equal to zero

Where $\phi = \phi(x, y, z)$ is the velocity potential function, V is the velocity vector field, '$\times$' curl, ∇ is 'del' or gradient.

Eqn (4.1) can be express in component form with respect to Cartesian coordinates as

$$u = \frac{\partial \phi}{\partial x},\ \upsilon = \frac{\partial \phi}{\partial y} \text{ and } w = \frac{\partial \phi}{\partial z} \qquad 4.4$$

4.2 Laplace equation

For incompressible flow, the continuity equation in Cartesian coordinates can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad 4.5$$

Substituting eqn (4) in (5) yield

$$\frac{\partial^2 \phi}{\partial x} + \frac{\partial^2 \phi}{\partial y} + \frac{\partial^2 \phi}{\partial 2} = 0 \qquad 4.6$$

Or $\nabla^2 \phi = 0$

Eqn (4.6) is know as **Laplace equation**

4.3 Stream function

Besides potential function, stream function ψ is sometime use for obtaining solutions of inviscid flow. Stream function is constant along a given streamline but change between two streamlines. The change in stream function $\Delta\psi$ is equal to the mass flow between two streamlines.

Stream function is defined as

$$u = \frac{\partial \psi}{\partial y} \text{ and } \upsilon = -\frac{\partial \psi}{\partial x} \qquad 4.7$$

Substituting the expressions in continuity equation for compressible flow will give

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad 4.8$$

4.4 Potential flow equation in coordinates systems

4.4.1. Cartesian coordinates

Velocity components:

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \upsilon = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad 4.9$$

Corresponding Laplace equations:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad 4.10$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \qquad 4.11$$

4.4.2. Polar coordinates

Velocity components:

$$V_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r} \qquad 4.12$$

Corresponding Laplace equations:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \qquad 4.13$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0 \qquad 4.14$$

5.0 Elementary flows

1. Uniform flow: It is a potential flow in which the straight streamlines are oriented in a single direction i.e x-direction as shown in fig 5.1.

$$\phi = V_\infty x = V_\infty r \cos\theta \,; \qquad \psi = V_\infty y = V_\infty r \sin\theta \qquad 5.1$$

Where r and θ are polar coordinates

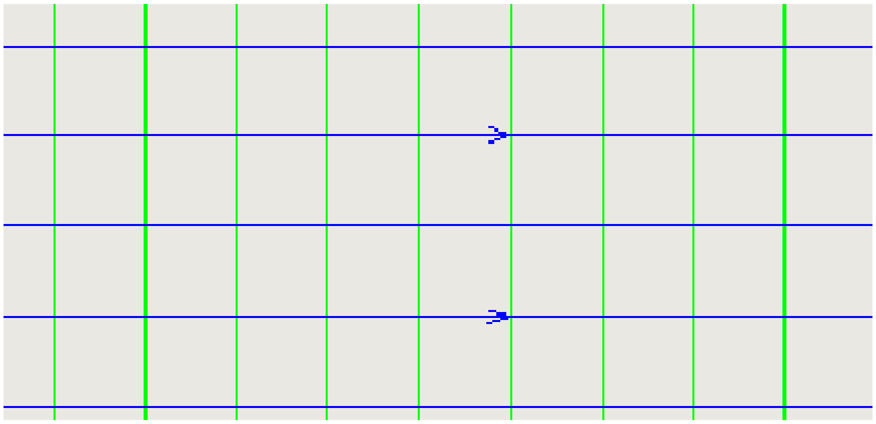
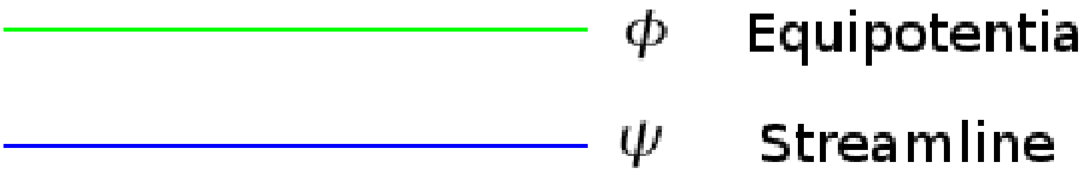


Fig.5.1: Equipotential lines and streamlines for uniform flow



2. Source and sink flow (2D): A source consists of streamlines emanating from a central point as shown in fig 5.2. The velocity along the streamline varies inversely with distance from the origin. This is completely radial flow with no component velocity in the tangential direction, i.e. $v_\theta = 0$.

$$\phi = \frac{\Lambda}{2\pi}\ln r \, ; \qquad \psi = \frac{\Lambda}{2\pi}\theta \qquad 5.2$$

Where Λ is the source strength defined as the rate of volume flow from the source. A negative value of Λ depict a sink.

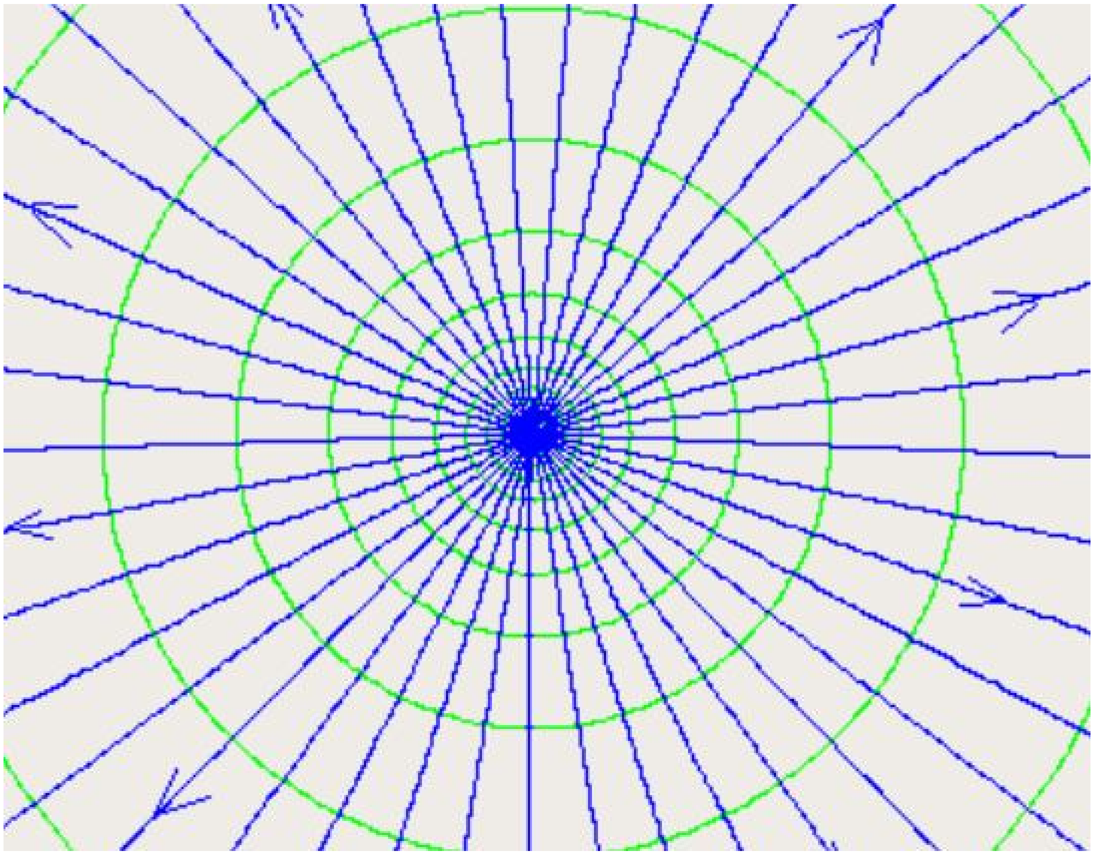


Fig 5.2: Equipotential lines and streamlines for source flow

3. Source and sink flow (3D): It is a flow with straight streamlines originating in three dimensions from the central point. Here the velocity varies inversely as the square of the distance from the origin, and

$$\phi = -\frac{\lambda}{4\pi r} \qquad 5.3$$

Where λ is the source strength, and it is defined as the rate of volume flow from the origin. For a sink, λ is negative.

4. Doublet flow (2D): A doublet is formed by the superposition of source and sink of equal but opposite strength and the distance l between the two approaches zero at the same time that the product $k = \Lambda l$ remain constant as shown in fig 5.3. In polar coordinates,

$$\phi = K\frac{\cos\theta}{r}; \qquad \psi = -K\frac{\sin\theta}{r} \qquad 5.4$$

Where $K = \frac{k}{2\pi}$

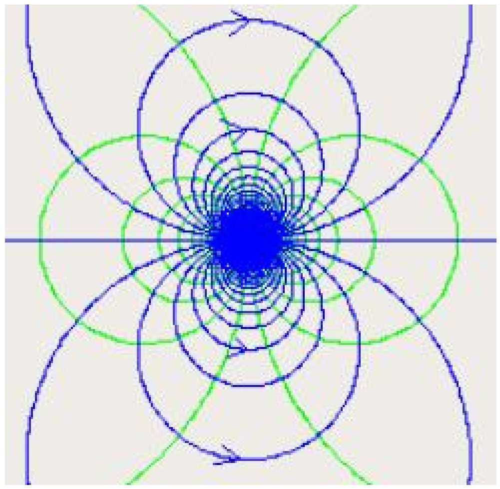


Fig 5.3: Equipotential lines and streamlines for 2D doublet

5. Doublet flow (3D): It formed by the superimposition of a three dimensional source and sink of equal and opposite strength, and the distance l between the two approaches zero at the same time that the product $\mu = \lambda l$ remain constant. In spherical coordinates,

$$\phi = -\left(\frac{\mu}{4\pi}\right)\frac{\cos\theta}{r^2} \qquad 5.5$$

6. Vortex flow (in 2D): This is concern with flows that go in circumferential direction as shown in fig 5.4. The radial velocity is equal to zero. In polar coordinates with an origin at the central point,

$$\phi = \frac{\Gamma}{2\pi}\theta; \qquad \psi = -\frac{\Gamma}{2\pi}\ln r \qquad 5.6$$

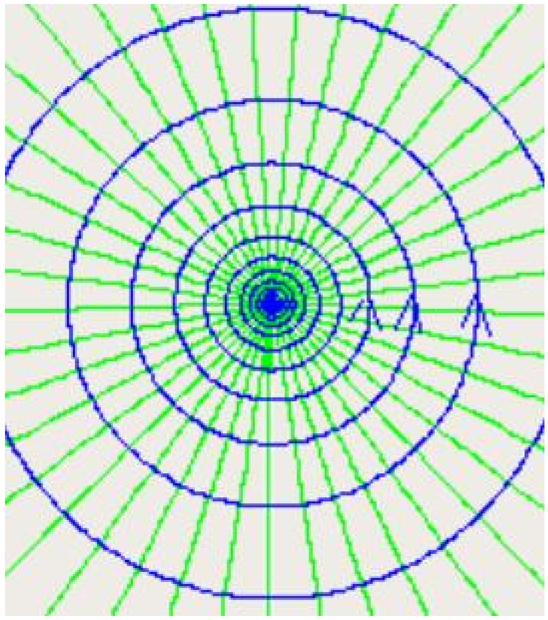


Fig 5.4: Equipotential lines and streamlines for 2D vortex

5.1 Applications of elementary flows

The six elementary flows describe above are not practical flow fields. However, they can be superimposed to synthesize practical flows in two and three dimensions, such as flow over cylinders, spheres, airfoils, wings, and whole airplanes.

1. ***Flow over a circular cylinder without circulation***

This flow is synthesized by the superposition of a uniform flow with a doublet; yielding

$$\phi = V_\infty r\cos\theta + K\frac{\cos\theta}{r}; \qquad \psi = V_\infty r\sin\theta - K\frac{\sin\theta}{r} \tag{5.7}$$

The velocity components are given by

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(V_\infty - \frac{K}{r^2}\right)\cos\theta \tag{5.8}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = \left(V_\infty + \frac{K}{r^2}\right)\sin\theta \tag{5.9}$$

The radial velocity is zero when

$$\frac{K}{r^2} = V_\infty \qquad 5.10$$

Or when $\theta = \frac{\pi}{2},\ 3\frac{\pi}{2}$ 5. 5.11

Let R = radius of circular cylinder

If $r = R$

$$K = V_\infty R^2 \qquad 5.12$$

The potential and stream functions for flow over a circular cylinder can be re-written in terms of cylinder radius as

$$\phi = V_\infty\left(r + \frac{R^2}{r}\right)\cos\theta\,; \qquad \psi = V_\infty\left(r - \frac{R^2}{r}\right)\sin\theta \qquad 5.13$$

$$v_r = V_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta \qquad 5.14$$

$$v_\theta = -V_\infty\left(1 + \frac{R^2}{r}\right)\sin\theta \qquad 5.15$$

The velocity components on the surface of the cylinder are obtained by setting r = R. On the surface of the cylinder, the velocity is necessarily tangential and is expressed as

$$v_\theta\big|_{r=R} = -V_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta\Bigg|_{r=R} = -2V_\infty\sin\theta \qquad 5.16$$

The velocity is zero at $\theta = 0$, π and has maximum values of $2V_\infty$ at $\theta = \frac{\pi}{2},\ 3\frac{\pi}{2}$

The pressure distribution can be obtained by substituting the tangential velocity in Bernoulli equation, yielding

$$p = \frac{1}{2}\rho V_\infty^2\left(1 - 4\sin^2\theta\right) \qquad 5.17$$

The pressure coefficient is

$$C_p = 1 - 4\sin^2\theta \qquad 5.18$$

The maximum pressure occurs at stagnation point where $\theta = 0$, π and the minimum pressure occurs at points where $\theta = \frac{\pi}{2}, 3\frac{\pi}{2}$.

Because the pressure variation is symmetrical, the lift and drag theoretically predicted for the cylinder is zero.

i.e

The drag on the cylinder is

$$D = \int_0^{2\pi} pR\cos\theta d\theta = \int_0^{2\pi} \frac{\rho R V_\infty^2}{2}\left(1 - 4\sin^2\theta\right)\cos\theta d\theta = 0 \qquad 5.19$$

Similarly, the lift on the cylinder is zero.

2. Lifting flow over a circular cylinder.

Lifting flow over a circular cylinder is formed by the superposition of a vortex to the doublet and the uniform flow. The stream function and the velocity potential now become,

$$\phi = V_\infty\left(r + \frac{R^2}{r}\right)\cos\theta - \frac{\Gamma}{2\pi}\theta; \qquad \psi = V_\infty\left(r - \frac{R^2}{r}\right)\sin\theta + \frac{\Gamma}{2\pi}\ln r \qquad 5.20$$

The velocity components are given by

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta \qquad 5.21$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta - \frac{\Gamma}{2\pi r} \qquad 5.22$$

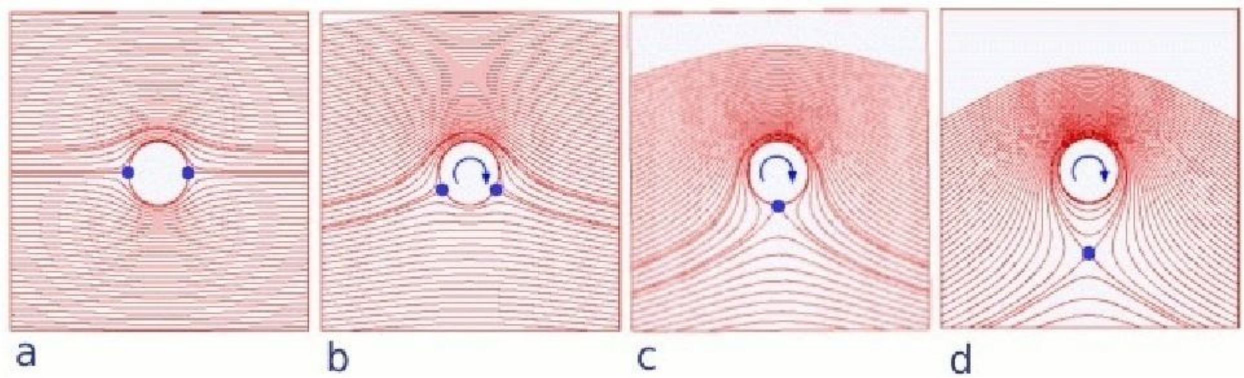
On the surface of the cylinder, the velocity is necessarily tangential and is expressed as

$$v_\theta\big|_{r=R} = -V_\infty\left(1+\frac{R^2}{r^2}\right)\sin\theta\Big|_{r=R} - \frac{\Gamma}{2\pi r}\Big|_{r=R} = -2V_\infty\sin\theta - \frac{\Gamma}{2\pi R} \quad 5.23$$

The stagnation points occur when $v_\theta = 0$; so that

$$\sin\theta = \frac{\Gamma}{4\pi V_\infty R} \quad 5.24$$

When the circulation is $4\pi V_\infty R$, the two stagnation points coincide at r = R, i.e at $\theta = -\frac{\pi}{2}$ as shown in fig 5.5c. For larger circulation, the stagnation point moves away from the cylinder as shown in fig 5.5d.



$\Gamma = 0$ $\qquad$ $\Gamma < 4\pi V_\infty R$ $\qquad$ $\Gamma = 4\pi V_\infty R$ $\qquad$ $\Gamma > 4\pi V_\infty R$

Fig 5.5: Lift generation

The pressure at the surface of the cylinder is

$$p = \frac{1}{2}\rho V_\infty^2\left[1-\left(4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right)\right] \quad 5.25$$

The resulting pressure coefficient distribution is not symmetrical and is given by

$$C_p = 1-\left[4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right] \quad 5.26$$

The drag is also zero. The lift however becomes

$$L = -\int_0^{2\pi} pR\sin\theta d\theta \; = \; \frac{1}{2}\rho V_\infty^2 R\int_0^{2\pi}\left[1-\left(4\sin^2\theta+\frac{2\Gamma\sin\theta}{\pi R V_\infty}+\left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right)\right]\sin\theta d\theta \qquad 5.27$$

$$\therefore\ L = \rho V_\infty \Gamma \qquad 5.28$$

This shows that the lift is directly proportional to the fluid density, the freestream velocity and circulation.

5.2 Magnus Effect/ Kutta-Joukowsky Theorem

We have shown that a force is produced when circulation is imposed upon a cylinder placed in uniform flow. This force is the lift. This effect is called **Magnus Effect**.

This result is a general result for inviscide incompressible flow over cylindrical body of any arbitrary shape and is called the **Kutta – Joukowski theorem**. It states that the lift per unit span along the body is directly proportional to the circulation about the body.

6.0 VISCOUS FLOW

The inviscid, incompressible flow considered in the preceding section assumed the fluid is frictionless or disregarding the viscosity. In such situations, losses were assumed without probing into the underlying causes. In reality inviscid flows are theoretical. Any real flow in nature is viscous.

Viscosity is the fluid property that causes shear stresses in moving fluid; it is also one means by which irreversibility or losses developed. It gives rise to many of the interesting physical features of a flow such as boundary layer development, laminar, transition and turbulent flow, pressure gradient and flow separation, etc.

5.1 Practical Effects of Viscous Flow

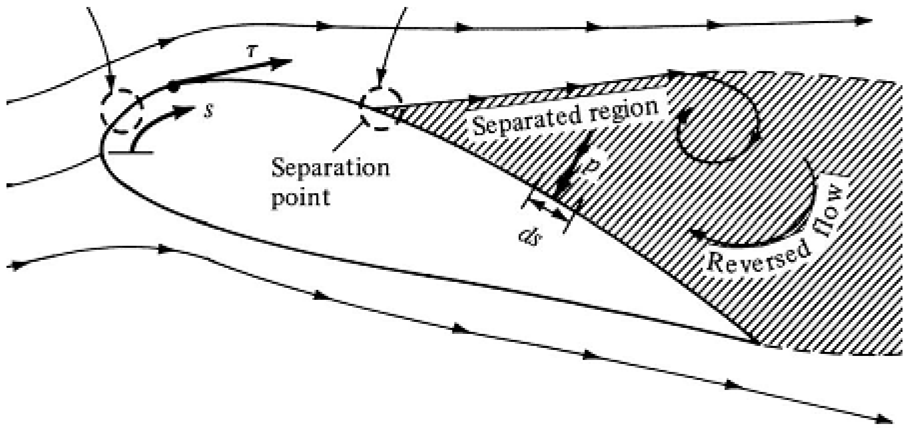


Fig 6.1: Effect of viscosity body

1. *Skin friction:* The action of viscosity produces shear stress at the solid surface, which in turn create drag called skin friction drag as shown in fig 6.1.

2. *Flow separation:* Shear stress acting on the surface tends to slow the flow velocity near the surface. If the flow is experiencing an adverse pressure gradient (a region where the pressure increases in the flow direction), then the low-energy fluid elements near the surface cannot negotiate the adverse pressure gradient, result to flow separation from the surface as shown in fig.6.1.

Flow separation alters the pressure distribution over the surface in such a fashion to increase the drag; this is called **pressure drag due to flow separation or form drag**. In addition, if the body is producing lift, then the flow separation can greatly reduce the lift. This is the mechanism that limits the lift coefficient on airfoil, wing, or lifting body to some maximum value.

For flow over an aerofoil, lift coefficient increase with angle of attack until a maximum value is achieved. As the angle of attack is further increased, massive flow separation occurs, which cause the lift to rapidly decrease. Under this condition, the airfoil is said to be **stalled**.

3. *Reverse flow:* This occurs at the wake region with attendant effect on poor mixing.

4. ***Aerodynamic heating:*** This occur when the kinetic energy of the fluid elements near the surface is converted to thermal energy in the flow near the surface due to reduce effect of friction, resulting to increase in temperature of the flow. For high-speed flow at supersonic and hypersonic speeds, this dissipative phenomenon can create very high temperature near the surface. Through the mechanism of thermal conduction on the surface, large aerodynamic heating rates can result.

Note: The magnitude of the skin friction and aerodynamic heating and the extent of flow separation are greatly influenced by the nature of the viscous flow; i.e whether the flow is laminar or turbulent.

6.2 Boundary layer

When a viscous fluid flows along a stationary impermeable wall or rigid surface immersed in fluid, the velocity at any point on the wall or surface is zero. Since the effect of viscosity is to resist fluid motion, the velocity close to the solid surface continuously decreases towards downstream. But away from the wall or surface the speed is equal to the freestream value of U_∞. Consequently a velocity gradient is set up in the fluid in a direction normal to flow. Thus a thin layer called ***Boundary Layer*** is established adjacent to the wall. As this layer moves along the body, the continuous action of shear stress tends to slow down additional fluid particles, causing the thickness of the boundary layer to increase with distance from the upstream point as shown in fig 6.2. Within such layers the fluid velocity changes rapidly from zero to its main-stream value, and this may imply a steep gradient of shearing stress.

Boundary layer thickness is defined as the height from the solid surface where we first encounter 99% of free stream speed (i.e $\frac{u}{u_\infty} = 0.99$).

Boundary layer has a pronounced effect upon any body such as aeroplane, ship, airfoil, wing and pipe immersed or moving in a fluid. Such effect is manifested as drag or friction.

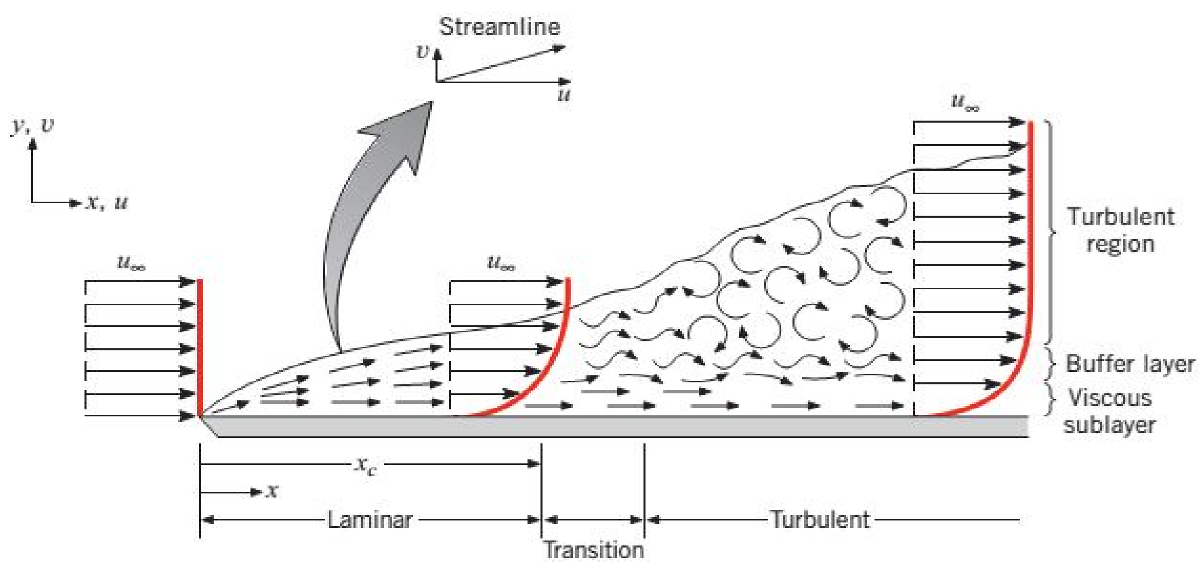


Fig 6.2: Velocity boundary layer development of flow over immersed body.

6.2.1 Boundary Layer Equations

Viscous flow problems are analyzed by solving the Navier-Stokes equations. For a Newtonian fluid with constant density and viscosity, the boundary layer equations in Cartesian Coordinates are:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \rho \nabla . V = 0 \qquad 6.1$$

Momentum equation:

$$\rho \frac{DV}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \nabla^2 V$$

$$\rho \frac{DV}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \nabla^2 V \qquad 6.2$$

$$\rho \frac{DV}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \nabla^2 V$$

6.3 Flow regimes

6.3.1 Laminar flow: It is a flow pattern in which the streamlines are smooth and regular and a fluid element moves smoothly along a streamline. The boundary layer development is shown in fig.6.2. In laminar boundary layer exchange of mass, momentum and energy take place only between adjacent layers on a microscopic scale. Consequently molecular viscosity is able to predict the shear stress associated. Laminar flow occur only when the Reynolds numbers are low i.e

flow over flat plate, Re < 5 x 10^{4}

flow in smooth pipe, Re < 2000

The solutions to boundary layer problem development are very complex and require advanced mathematical treatment. However, some assumptions can be made to obtain approximate solutions whose results agree closely with exact approach from differential equation.

Solution of laminar flow over flat plate

Assumed the flow distribution satisfies the following boundary conditions:

u = 0, y = 0 and u = U, y = δ 6.3

the velocity distribution is given as

$$\frac{u}{U} = F\left(\frac{y}{\delta}\right) = F(\eta) \qquad 6.4$$

Where $\eta = \dfrac{y}{\delta}$, δ = boundary layer thickness

Prandtl shows that, $\dfrac{u}{U} = F = \dfrac{3}{2}\eta - \dfrac{\eta^3}{2}$ 6.5

$0 \le y \le \delta$ and $F = 1 \quad \delta \le y$

The shear stress is given as

$$\tau_0 = \rho U^2 \frac{\partial \delta}{\partial x} \int_0^1 \left(1 - \frac{u}{U}\right) \frac{u}{U} d\eta \qquad 6.6$$

Substituting eqn (6.5) in (6.6), we obtained

$$\tau_0 = \rho U^2 \frac{\partial \delta}{\partial x} \int_0^1 \left(1 - \frac{3}{2}\eta + \frac{\eta^3}{2}\right)\left(\frac{3}{2}\eta - \frac{\eta^3}{2}\right) d\eta = 0.139 \rho U^2 \frac{\partial \delta}{\partial x} \qquad 6.7$$

At the wall

$$\tau_0 = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu \frac{U}{\delta} \frac{\partial F}{\partial \eta}\bigg|_{\eta=0} = \mu \frac{U}{\delta} \frac{\partial}{\partial \eta}\left(\frac{3}{2}\eta - \frac{\eta^3}{2}\right)\bigg|_{\eta=0} = \frac{3}{2}\mu \frac{U}{\delta} \qquad 6.8$$

Equating equations (6.8) and (6.7)

$\dfrac{3}{2}\mu \dfrac{U}{\delta} = 0.139 \rho U^2 \dfrac{\partial \delta}{\partial x}$ and rearranging gives

$$\delta d\delta = 10.78 \mu \frac{dx}{\rho U} \qquad 6.9$$

Since δ is a function of x only in equation (6.9), integrating gives

$$\frac{\delta^2}{2} = 10.78 \frac{\nu}{U} x + \text{const.} \qquad 6.10$$

If $\delta = 0$ and $x = 0$, the integration constant is zero. Therefore equation (6.10) becomes

$$\frac{\delta}{x} = 4.65 \sqrt{\frac{\nu}{Ux}} = \frac{4.65}{\sqrt{\text{Re}_x}} \qquad 6.11$$

or

boundary layer thickness

$$\delta = \frac{4.65x}{\sqrt{\mathrm{Re}_x}} \approx \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$$

Friction coefficient, $C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$ 6.12

Shear stress $\tau = \frac{1}{2}\rho U^2 C_f$ 6.13

Drag per unit width is given as:

$$D = \frac{1}{2} x\rho U^2 C_f$$ 6.14

Reynolds number, $\mathrm{Re}_x = \frac{\rho U x}{\mu} = \frac{Ux}{\nu}$ 6.15

6.3.2 Turbulent flow: It is a flow pattern in which the streamlines break up and the fluid element moves in a random, irregular, and tortuous manner. The resulting boundary layer development is shown in fig.6.2. In turbulent boundary layer, mass, momentum and energy are exchange on a much larger scale compared to laminar boundary layer. A turbulent boundary layer occurs at high Reynolds numbers i.e

flow over a flat plate, Re > 5 x 10^4

flow in pipe, Re > 2300

Solution of turbulent flow over flat plate

The Prandtl's one-seventh power law for flow through smooth pipes can be used to determine the turbulent boundary layer growth.

This can be express numerically as $\frac{u_\infty}{u} = \left(\frac{y}{r_0}\right)^{1/7} = \eta^{1/7}$. 6.16

Applying it to flat plate, we have

$$F = \frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7} \qquad 6.17$$

Substituting eqn (6.17) in (6.6), the shear stress is

$$\tau_0 = \rho U^2 \frac{\partial \delta}{\partial x} \int_0^1 \left(1-\eta^{1/7}\right)\eta^{1/7} d\eta = \frac{7}{72} \rho U^2 \frac{\partial \delta}{\partial x} \qquad 6.18$$

At the wall, the shear stress is given as

$$\tau_0 = 0.022 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4} \qquad 6.19$$

Equating eqns (1.68) and (6.19), and integrating the resulting boundary layer over the whole length of the plate with initial conditions $x = 0$, and $\delta = 0$, we obtain

$$\delta^{5/4} = 0.292 \left(\frac{\nu}{U}\right)^{1/4} x \qquad 6.20$$

$$\delta = 0.37 \left(\frac{\nu}{U}\right)^{1/5} x^{4/5} = \frac{0.37x}{(Ux/\nu)^{1/5}} = \frac{0.37x}{\mathrm{Re}_x^{0.2}} \qquad 6.21$$

Friction coefficient, $C_f = \dfrac{0.058}{\mathrm{Re}_x^{0.2}}$

Drag per unit width is given as:

$$D = \frac{1}{2} x \rho U^2 C_f \qquad 6.22$$

6.4 Flow Separation mechanism

Flow separation occurs in flow over a body such as airfoils, wing of aircraft, sphere etc due to adverse pressure gradient. The effect of adverse pressure gradient is felt more strongly in the regions close to the wall where the momentum is lower than in the regions near the free stream. As indicated in the fig 6.3, the velocity near the wall reduces and the boundary layer thickens. A continuous retardation of flow brings the wall shear stress at the point S on the wall to zero. From this point onwards the shear stress becomes negative and the flow reverses and a region of recirculating flow known as **wake** is develops. For this reason, the flow can no longer follow the contour of the body. This implies that the flow has separated. The point S where the shear stress is zero is called the **point of Separation.**

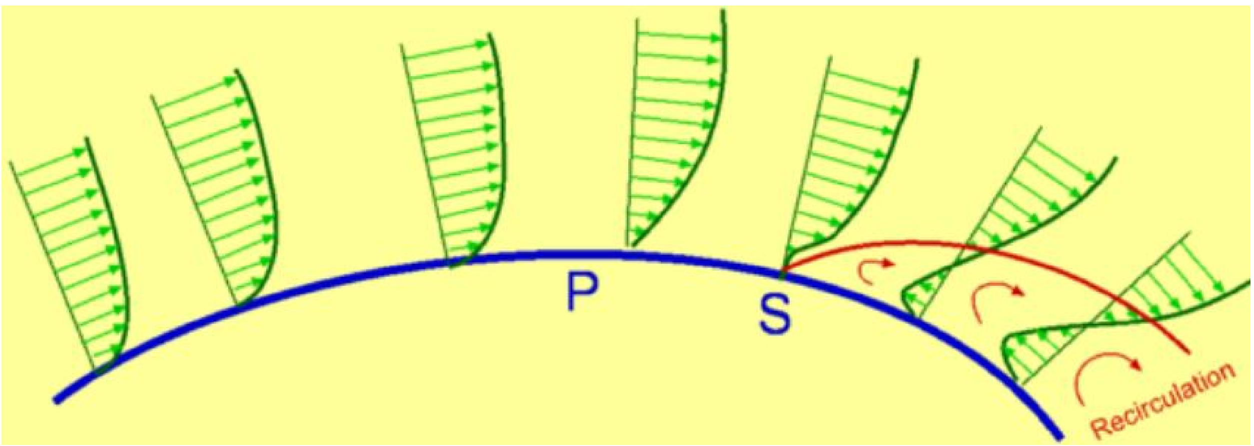


Fig 6.3: Separation of flow over a curved surface.

Depending on the flow conditions the recirculating flow terminates and the flow may become reattached to the body. There are a variety of factors that could influence this reattachment; 1) the pressure gradient may become favourable due to body geometry and other reasons, 2) the flow initially laminar may undergo transition within the wake and may become turbulent. A turbulent flow has more energy and momentum than a laminar flow. This can eliminate separation and the flow may reattach.

During flow over aerofoil separation could occurs near the leading edge and gives rise to a short vortex. But in a situation when flow separation occurs towards the trailing edge without reattaching could be very dangerous. In this situation the separated region merges with the wake and may result to stall of the aerofoil (loss of lift).

7.0 Aerodynamics

It is a branch of fluid dynamics the deals with the study of air and its interactions with solid surface in the flow. These surfaces may be aerodynamic bodies like airplanes and missiles or the inside walls of ducts such as inside rocket nozzles and wind tunnels.

7.1 Applications of aerodynamics

1. Predict force and moment on bodies moving through fluid usually air.

2. Predict heat transfer from body or to body in air.

3. Determine flows phenomena in ducts such as flow through wind tunnels and jet engines.

7.2 Classification of aerodynamics

1. Incompressible versus compressible flow.

2. Inviscid versus viscous flow.

3. Steady versus unsteady flow.

4. Natural versus forced flow.

5. Laminar versus turbulent flow.

6. one-, two- and three-dimensional flows.

A flow is said to be one-, two- or three- dimensional if the velocity varies in one-, two- or three-dimensional space.

7.3 Aerodynamic forces: Lift and Drag

Drag is a force that opposes motion. It is defined as the force component parallel to the relative approach velocity, exerted on a body by fluid in motion. An aircraft flying has to overcome the drag force upon it, a ball in flight, a sailing ship and an automobile at high speed are some of the examples drag applications. Drag can be express as

$$D = C_D A \frac{\rho U^2}{2} \qquad 7.1$$

Lift is defined as the force component perpendicular or normal to the relative approach velocity exerted on a body in moving fluid. It is express as

$$L = C_L A \frac{\rho U^2}{2} \qquad 7.2$$

Where C_D is the drag coefficient, C_L is the lift coefficient and A is the projected area of the body.

7.4 Aerodynamic Requirements for Aircraft Design and Performance Analysis

The most significant aerodynamic forces acting on aircraft moving through the atmosphere are drag and lift. For a well designed aircraft, the wing is the major source of lift and drag. Therefore, it is of paramount importance to maximize the lift and minimize the drag.

Drag is a function of the drag coefficient $\mathbf{C_D}$ which is, in turn, a function of a parasite drag and an induced drag. ***Induced drag*** occurs due to an alteration of pressure distribution over the wing by strong vortices trailing downstream from the wing tips. The wing tip vortices induce a general downward component velocity over the wing, which in turn changes the pressure distribution in such a manner as to increase the drag. Induced drag is directly proportional to the square of the lift coefficient: therefore, induced drag rapidly increases as lift increases.

$$C_D = C_{D0} + C_{Di} \qquad 7.3$$

$$C_{Di} = KC_L^2 \qquad 7.4$$

$$C_D = C_{D0} + KC_L^2 \qquad 7.5$$

$$K = \frac{1}{\pi AR\varepsilon} \qquad 7.6$$

Eqn (7.3) in called the polar drag.

Where C_{D0} is the parasite drag coefficient at zero lift; parasite drag is produced by the net effect of skin friction over the body surface plus the extral pressure drag produced by regions of flow separation over the surface (sometimes called form drag) and ε is the Oswald span efficiency. An ideal wing with infinite span has a value of unity for ε. Practical value of ε ranges from 0.6 – 0.9

The drag of the aircraft is found from the drag coefficient, the dynamic pressure and the wing platform area:

$$D = C_D \rho \frac{V^2}{2} S \qquad 7.7$$

Therefore, $D = \left(C_{D0} + KC_L^2\right)\rho \frac{V^2}{2} S$ 7.8

The lift is given as

$$L = C_L \rho \frac{V^2}{2} S \qquad 7.9$$

$$S = bc \qquad 7.10$$

$$AR = \frac{b}{c} = \frac{b^2}{S} \qquad 7.11$$

Thickness ratio $= \frac{t}{c}$ 7.12

In these equations, ρ is the atmospheric density, V is the airspeed, S is the wing area i.e the area of one side of the wing to include area occupied by the fuselage, C_L and C_D are the dimensionless lift and drag coefficients respectively, b is the wing span i.e distance from the wing tip to wing tip, c is chord i.e distance between from the leading edge of wing to the trailing edge, and t is wing thickness. The lift and drag coefficients are functions of the angle of attack, Mach number, Reynolds number, and the wing shape. C_L increases with increasing angle of attack until maximum value occurs at the stall and then decreases, usually sharply as shown in fig 7.1. However, conventional aircraft do not fly beyond stall point.

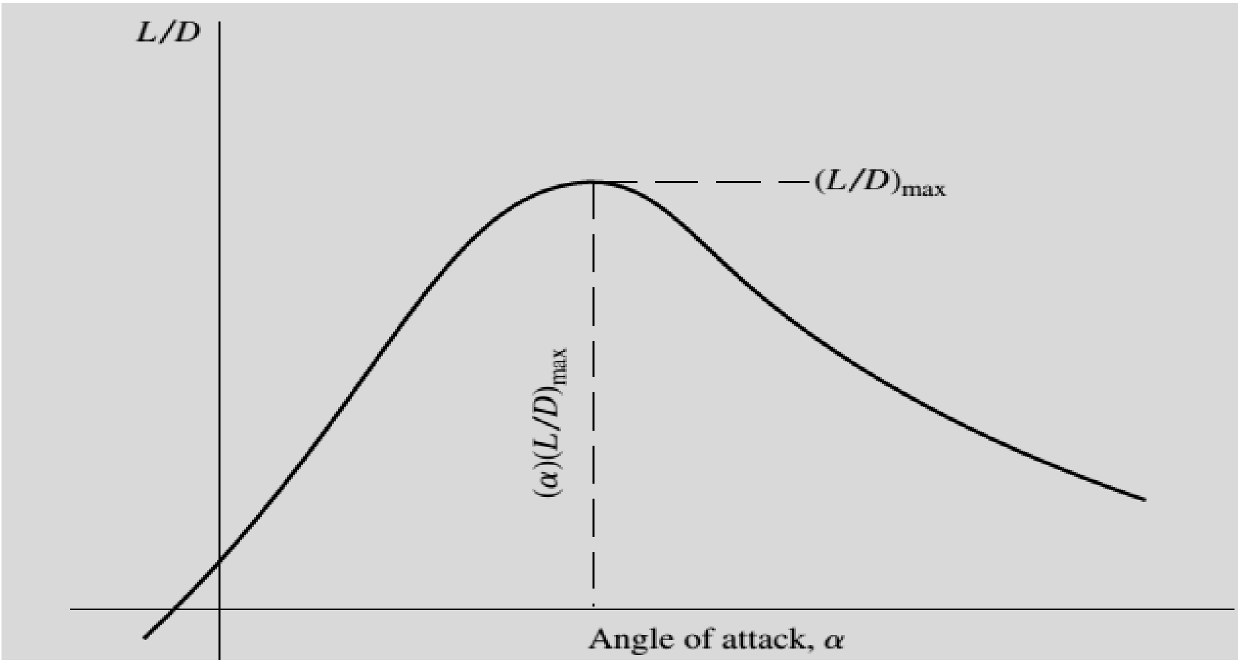


Fig 7.1: Effect of angle of attack on lift and drag.

7.5 MAXIMUM LIFT-DRAG RATIO (E_{max})

Lift to drag ratio (L/D) or (C_L/C_D) also called aerodynamic efficiency is a very important performance parameter of aircraft. For a parabolic polar drag, the maximum value of lift-drag ratio occurs when the aircraft is flying so that the zero to lift drag is equal to drag due to lift coefficient. At maximum lift-drag ratio, the minimum drag occurs.

Recall the parabolic polar drag

$$C_D = C_{D0} + KC_L^2 \qquad 7.13$$

Divide both side of equation (7.13) by C_L

$$\frac{C_D}{C_L} = \frac{C_{D0} + KC_L^2}{C_L} \qquad 7.14$$

Differentiate eqn (7.14) with respect to C_L and equate to zero to determine the conditions for minimum ratio of drag coefficient to lift coefficient, which is a condition for minimum drag.

$$\frac{d\left(\frac{C_D}{C_L}\right)}{dC_L} = \frac{C_L(2KC_L) - C_{D0} - KC_L^2}{C_L^2} = 0 \qquad 7.15$$

$$K = \frac{C_{D0}}{C_L^2} \qquad 7.16$$

$$C_{D0} = KC_{LMD}^2 \qquad 7.17$$

Eqn (7.13) can be rewritten in terms of minimum drag as

$$C_{DMD} = C_{D0} + KC_{LMD}^2 \qquad 7.18$$

Substituting eqn (7.17) in (7.18) yield

$$C_{DMD} = 2C_{D0} = 2KC_{LMD}^2 \qquad 7.19$$

$$C_{LMD} = \sqrt{\frac{C_{D0}}{K}} \qquad 7.20$$

From this we can find the value of the maximum lift-to-drag ratio in terms of basic drag parameters as

$$E_{\max} = \left(\frac{L}{D}\right)_{\max} = \frac{C_{LMD}}{C_{DMD}} = \frac{\sqrt{C_{D0}/K}}{2C_{DO}} \qquad 7.21$$

$$E_{\max} = \frac{1}{2\sqrt{C_{D0}K}} \qquad 7.22$$

Example

1. An aircraft has a wing span of 58 m, an average chord of 7.24 m, a C_{D0} = 0.016, and an $\varepsilon = 0.85$. a) Write the expression for the parabolic drag polar and b) find $E_{\max}$ and the corresponding C_L.

Solution

a) $AR = \frac{b}{c}$ = 8.01

$K = 1/(\pi \times 8 \times 0.85) = 0.0468$

$C_D = 0.016 + 0.0468 C_L^2$

b) $E_{\max} = \frac{1}{2\sqrt{C_{D0}K}} = \frac{1}{2(0.0468 \times 0.016)^{1/2}}$ = 18.27

corresponding $C_L = \left(\frac{0.016}{0.0468}\right)^{1/2}$ = 0.585

2. Find C_D and $E_{\max}$ for a C_L a of 0.3

$C_D = 0.016 + 0.0468(0.3)^2 = 0.020$

$E_{\max} = \left(\frac{C_L}{C_D}\right) = \frac{0.3}{0.02} = 15$

8.0 COMPRESSIBLE FLOW

Compressible flow differs from incompressible flow in the following respects:

i. variation in flow density is significant.

ii. flow speeds are high enough that the kinetic energy becomes important and therefore energy changes in the flow must be considered.

iii. shock waves occur, which completely dominate the flow.

8.1 Governing equations of inviscid compressible Flow

The governing equations for inviscid, compressible flow are

Continuity $\frac{\partial \rho}{\partial t} = \nabla.(\rho V) = 0$ 8.1

Momentum (x component) $\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x$ 8.2a

Momentum (y component) $\frac{D\upsilon}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y$ 8.2b

Momentum (z component) $\frac{D\omega}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z$ 8.2c

Energy $\rho \frac{D\left(e + V^2/2\right)}{Dt} = p\dot{q} - \nabla.(pV)$ 8.3

Equation of state $p = \rho RT$ 8.4

8.2 Speed of sound wave and Mach number

Speed of sound is the rate of propagation of a pressure pulse or wave of infinitesimal strength through a still fluid. It is given

$$a = \sqrt{kRT} \qquad 8.5$$

Where a = speed of sound, ρ = density of gas, p = pressure, T = temperature, and k = specific heat ratio.

Eqn (14) shows that the speed of sound is a function of absolute temperature only.

8.3 Mach number, M, is a measure of the importance of compressibility. It is defined as the ratio of velocity of fluid to the local velocity of sound in the medium.

$$M = \frac{V}{a} \qquad 8.6$$

Example: 1.What is the speed of sound in air at sea level when t = 20^{o}C and in stratosphere when t = -55^{o}C?

2. An airplane flying at a velocity of 250 m/s. Calculate its Mach number if it is flying at a standard altitude of (*a*) sea level, (*b*) 5 km, (*c*) 10 km.

8.4 ISENTROPIC LOW

Isentropic flow is a reversible adiabatic flow which occur when the $dq_H = 0$ and $ds = 0$.

Recall from thermodynamic isentropic relation;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = \left(\frac{\rho_2}{\rho_1}\right)^{(k-1)} \qquad 8.7$$

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} \qquad 8.8$$

$$\Delta h = c_p T_1 \left(\frac{T_2}{T_1} - 1 \right) = c_p T_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] \qquad 8.9$$

For one-dimensional steady, reversible, adiabatic flow of perfect gas, the relation between the stagnation (total) and static properties is a function of k and M only, as

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2 \qquad 8.10$$

$$\frac{p_o}{p} = \left(1 + \frac{k-1}{2} M^2 \right)^{k/(k-1)} \qquad 8.11$$

$$\frac{\rho_o}{\rho} = \left(1 + \frac{k-1}{2} M^2 \right)^{1/(k-1)} \qquad 8.12$$

Where subscript o denotes the isentropic stagnation condition reached by the stream when stopped isentropically

8.5 Compressible flow in ducts

Nozzle and diffuser are ducts of varying cross-section for producing supersonic flow as shown in fig 8.1. Compressible flows through these devices are of profound importance in the design of high-speed wind tunnels, jet engines and rocket engines, etc.

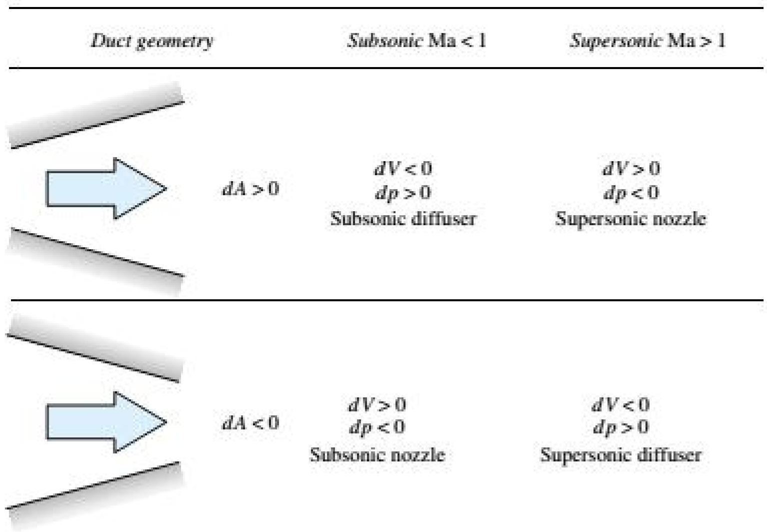


Fig 8.1 Nozzle and diffuser cross section

For steady one-dimensional flow, the variation of area with Mach number is obtained by use of continuity, momentum and energy equations for compressible flow. Consider a duct with cross sectional area, A, changing along the length of the duct, the expression is given as

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{dV}{V} \qquad 8.13$$

Where A is the steam-tube cross section normal to the velocity, dA is the local change in area, dV is the corresponding change in velocity and M is local Mach number.

The following can be inferred from eqn (1)

i. For subsonic flow $\left(0 \le M < 1\right)$, V increases as A decreases, and V decreases as A increases. Therefore, to increase the velocity, a nozzle (convergent duct) must be used, whereas to decrease the velocity, a diffuser (divergent duct) must be employed.

ii. For supersonic flow $\left(M > 1\right)$, V increases as A increases, and V decreases as A decreases. Thus the velocity at the minimum area of a duct with supersonic compressible flow is a minimum. This is the principle underlying the operation of diffusers on jet engines for supersonic aircraft. The purpose of the diffuser is to decelerate the flow so that there is sufficient time for combustion in the chamber. Then the diverging nozzle accelerates the flow again to achieve a larger kinetic energy of the exhaust gases and an increased engine thrust. Hence, an increase in velocity is realized by using nozzle (divergent duct) whereas a decrease in velocity can be achieved through the application of diffuser (convergent duct).

iii. Sonic flow $(M = 1)$, when M = 1, $dA = 0$. Sonic flow occurs in that location inside a variable-area duct where the area variation is a minimum. Such location is called sonic throat. Thus, for given stagnation conditions, the maximum possible mass flow passes through a duct when its throat is at the critical or sonic condition. The duct is then said to be *choked* and can carry no additional mass flow unless the throat is widened. If the throat is constricted further, the mass flow through the duct must decrease. Under sonic condition the following relations exist:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M^2\right)\right]^{(k+1)/(k-1)} \qquad 8.14$$

The maximum mass flow rate at the throat is

$$\dot{m}_{\max} = \rho^* V^* A^* = \frac{A^* p_o}{\sqrt{T_o}}\sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \qquad 8.15$$

For $k = 1.4$

$\dot{m}_{\max} = 0.686\dfrac{A^* p_o}{\sqrt{RT_o}}$ which implies that the mass flow rate varies linearly as A^* and p_o, and inversely as square root of the absolute temperature.

For subsonic flow through a converging-diverging duct, the velocity at the throat must be less than sonic velocity. The mass flow rate is given as

$$\dot{m} = \rho VA = A\sqrt{2 p_o \rho_o \frac{k}{k-1}\left(\frac{p}{p_o}\right)^{2/k}\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \qquad 8.16$$

Examples:

1. A preliminary design of wind tunnel to produce Mach number 3.0 at exit is desired. The mass flow rate is 1 kg/s for p_o = 90 kPa abs, t_o = 25^oC. Determine (a) the throat area, (b) the outlet area, (c) the velocity, pressure, temperature, and density at outlet.

2. A converging-diverging duct in an air line downstream from a reservoir has 50 mm diameter throat. Determine the mass rate of flow when p_o = 0.8 MPa abs, t_o = 33^oC, and p = 0.5 MPa abs at the throat.

8.6 Shock wave

Shock waves are very thin regions in a supersonic flow through which the flow physical properties change. They can be normal or oblique to the flow. The formation of shock waves is strongly dependent on the Mach number.

As the speed of a body increased from low subsonic value to transonic value, shock waves appear and are attached to the sides of the body. At still higher transonic speeds, the shock wave detached and appears as bow shock wave ahead of the body, and the earlier side shocks either disappear or move to the rear. For a sharp-nosed body, the head wave move back and becomes attached. At this point the flow is generally supersonic everywhere and the transonic regime is replace by supersonic regime. As flow transverse the shock wave, it experiences a sudden increase in pressure, density, temperature, and entropy, and a decrease in Mach number, flow velocity, and stagnation pressure.

Due to the formation of shock wave, there occur a significant alteration in pressure, and the center of pressure of airfoil section is displaced from one-fourth chord point back toward the one-half chord point. There is an associate increase of drag, and often flow separation at the base of the shock.

Examples:

1. Using the velocity distribution $\frac{u}{U} = \sin\left(\frac{y\pi}{2\delta}\right)$, determine the equation for growth of the laminar boundary layer and shear stress along a smooth flat plate in two-dimension.

2. Derive the equations for turbulent boundary layer growth over a smooth flat plate based on exponential law $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/9}$, and $f = \frac{0.185}{\text{Re}^{0.2}}$ and $\tau_0 = f\frac{\rho V^2}{8}$.

3. Air at 20^{o}C and 100 kPa abs flows along a smooth plate with velocity 150 km/h. What plate length is required to obtain a boundary layer thickness of 8mm?

4. Estimate the skin friction drag on an airship 100 m long, average diameter 20 m, with velocity of 130 km/h travelling through air at 90 kPa abs and 25^{o}C.

5. The wing on a Boeing 77 aircraft is rectangular, with a span of 9.75 m and a chord of 1.6 m. The aircraft is flying at cruising speed (141 mi/h) at sea level. Assume that the skin friction drag on the wing can be approximated by the drag on a flat plate of the same dimensions. Calculate the skin friction drag:

a. If the flow were completely laminar (which is not the case in real life)

b. If the flows were completely turbulent (which is more realistic) Compare the two results.

6. For the case in Problem 5, calculate the boundary-layer thickness at the trailing edge for

a. Completely laminar flow

b. Completely turbulent flow

7. For the case in Problem 5, calculate the skin friction drag accounting for transition. Assume the transition Reynolds number = 5×10^5.

Take the standard sea level value of viscosity coefficient for air as $\mu = 1.7894\times10{-5}$ kg/(m · s).