VARIOUS METHOD OF FM DEMODULATION

Demodulation: It is the process of deriving the original modulation signal from a modulated carrier wave. It is a detection technique of a received modulated signal. It is exactly opposite to that of frequency modulation. The FM demodulation (detector or discriminator) operates on a different principle compared with the AM detector. The AM detector is basically a rectifier. But FM detector is basically a frequency to amplitude converter. It is expected to convert the frequency variations in FM wave at its input into amplitude variations at its output to recover the original modulating signal. Let’s briefly see the modulated wave we are to demodulated or detect.

Fig. 4.17 AM, FM, and PM waveforms.
Fig. 1 — A 10-MHz carrier modulated by a 1-kHz sine wave.

Fig. 3 — Amplitude variation of the carrier and sideband pairs with modulation index. This is a graphical representation of mathematical functions developed by Fred Bessel. Note that the carrier completely disappears at modulation indexes of 2.405, 5.52 and 8.664.

Fig. 4 — The top photo is an oscilloscope pattern of an unmodulated RP carrier. The vertical (Y) axis shows amplitude, and the horizontal (X) axis time. The bottom photo is a spectrum analyzer display, where amplitude is shown on the Y axis and frequency on the X axis. Notice that there is only one line that represents the amplitude at the carrier frequency.
**Requirement of FM Demodulator (Detector)**

The FM demodulator must satisfy the following requirements:

(i) It must convert frequency variations into amplitude variations.
(ii) This conversion must be linear and efficient.
(iii) The demodulator circuit should be insensitive to amplitude changes. It should respond only to the frequency changes.
(iv) It should not be too critical in its adjustment and operation.

**Classification of FM Demodulators**

<table>
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<tr>
<th>FM Demodulator</th>
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<tr>
<td>Direct type</td>
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<tr>
<td>Indirect Type</td>
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</table>

1. Direct type

   - Frequency discrimination

2. Indirect Type

   - Phase locked loop
   - Zero crossing detector

**Fig 2: Classification of FM Demodulators**

**VARIOUS METHODS OF FM DEMODULATION**

**Fig 3 Tuned cct**

Frequency discrimination operates on the principle of the slope detection as shown in Fig 3 above where a frequency modulated signal is applied to the tuned circuit. We have

- Fc – The center frequency
- Δf – Frequency deviation
The resonant frequency of the tuned circuit is adjusted as \( (f_c + \Delta f) \).

Note: The amplitude of the O/P Voltage of the tank cct depends on the frequency deviation of the input FM signal.

**SIMPLE SLOPE DETECTOR (FREQ DISCRIMINATOR)**

![Fig 4 Simple slope Detector](image)

*Fig 4 Simple slope Detector*

![Fig 5 Characteristics of a slope Detector](image)

*Fig 5 Characteristics of a slope Detector*

From fig 4 above, the output voltage of the tank circuit is then applied to a simple diode detector with an RC load with proper time constant. This detector is identical to AM diode detector.

The above slope Detector has the following **Draw backs**:

(i) It is inefficient

(ii) It is linear only over a limited freq range.

(iii) It is difficult to adjust because the primary and secondary windings of transformer must be tuned to slightly different frequencies.
Advantage of slope Director
It is a simple circuit. We can correct the draw backs of the slope detector by building a balanced slope Detector as shown in the circuit diagram of Figure 6.

BALANCED SLOPE DETECTOR
The balanced frequency detector circuit diagram is shown below in Figure 6.

Fig6 Balanced slope Detector

It consists of two detector circuits. The input transformer has a center tapped secondary winding. Hence the input voltages to the two slope detectors are 180° out of phase. It consists of 3 tuned circuits: the primary with frequency f_c, the upper tuned circuit of the secondary (f_c + Δf). It is tuned above f_c by a resonant frequency (f_c + Δf). The lower tuned circuit is (f_c - Δf). R_1C_1 and R_2C_2 are the filters used to bypass the RF ripple. V_{o1} & V_{o2} are output voltages. Total output voltage is given by

\[ V_0 = V_{o1} + V_{o2} \]

PRINCIPLE OF OPERATION
(i) \( f_{in} = f_c \)
The induced voltages at T_1 & T_2 is exactly equal
Thus the voltages at the inputs of D_1 & D_2 are the same: - The output voltages V_{o1} & V_{o2} are identical but opposite in polarization. Hence the net output voltage V_0 = 0
(ii) \( f_c < f_{in} < (f_c + Δf) \)
The induced voltage in the T_1 is higher than that induced in T_2. The input voltage to D_1 is higher than D_2. Hence, the +ve output V_{o1} of D_1 is higher than –ve output V_{o2} of D_2. Thus, the output voltage V_0 is +ve.
As the frequency increases towards (f_c +Δf), the +ve output voltage increases as shown in fig7 below:
Characteristics of the balanced slope detector

Due to the typical shape as shown in figure 7 above, it is called the S- shape characteristics.

**Advantages**

(i) The circuit is more efficient than simple slope detector.

(ii) It has better linearity than the simple slope detector.

**Draw backs**

(i) Even through linearity is okay, it is not good enough.

(ii) The cct is difficult to tune since the three tuned at different frequencies i.e fc, (fc +Δf), and (fc - Δf).

(iii) Amplitude limiting is not provided.
ZERO CROSSING DETECTOR

The zero crossing detector operates on the principle that the instantaneous frequency of FM wave is approximately given as a linear function of the message signal.

Where Δt is the time difference between adjacent zero cross over points of the FM as shown above. Consider the time duration T, the time T is chosen such that it satisfies the following two conditions:

(i) T should be small compared to (I/W) wheel, W is the band width of the message signal.

(ii) T should be large as compared to (1/ f_{c}), where fc is carrier frequency of the FM wave.

Let the number of zero crossings during interval T be denoted by n_{0}. Hence, Δt i.e. the time between the adjacent zero crossing points is given by

Δt = T/ n_{0} \hspace{1cm} (ii)

Therefore, the instantaneous frequency is given by:

f_{1} = 1/ (2Δt) = n_{0} / 2T

By definition of the instantaneous frequency, we know that there is a linear relation between f_{1} and message signal x(t). This can be achieved by using a zero crossing detector below:

Fig.9 Block Diagram of zero crossing detector
PHASE DISCRIMINATOR (FOSTER SEELEY DISCRIMINATOR)

Fig10 phase Discriminator

If we compare this phase discriminator with the balanced slope detector circuit, the load arrangement is the same in both circuits, but the method of applying the input voltages (which is proportional to the frequency deviation) to the diodes, is entirely different. Foster Seeley’s discriminator is derived from the balance modulator. The primary and the secondary windings are both tuned to the same center frequency $f_c$ of the incoming signal. These simplify the tuning process and yield better linearity than the balanced slope detector.

**Operations**

Even though the primary and secondary tuned circuits are tuned to the same center frequency, the voltages applied to the two diodes $D_1$ $\alpha$ $D_2$ are not constant. They may vary depending on the frequency of the input signal. This is due to the change in phase shift between the primary and secondary windings depending on the input frequency.

The result as follow:

(i) At $f_{in}=f_c$, the individual output voltage of the two diodes will be equal and opposite. The output voltage is zero as

$$V_o = V_o_1 - V_o_2$$

(ii) For $f_{in} > f_c$, the phase shift between the primary and secondary winding is such that the output of $D_1$ is higher than $D_2$, hence the output voltage will be $+$ve

(iii) For $f_{in} < f_c$, the phase shift between the primary and secondary winding is such that the o/p of $D_2$ is higher than that of $D_1$, making the output voltage $-$ve

Because the output is dependent on the primary-secondary phase relationship, this circuit is called “phase Discriminator”
Phase Diagram:- the diagram below shows the phasor diagram at different i/p frequencies.

\[ \frac{1}{2} V_{ab} \]
equal voltage
are induced in the
two halves of
secondary \(- \frac{1}{2} V_{ab} \)

Fig.11 (a) Relationship between primary & secondary.

Fig 11 (b) secondary equivalent CCt

\[ \frac{1}{2} V_{ab} (90^\circ, 0) V_{ao} \]
\[ V_{1} \]
\[ V_{bo} \]

Fig 11(c) phazor diagram for \( f_{in}=f_{c} \)

Fig 12 below show frequency response of phase discriminator

\[ \text{o/p voltage} \]

useful range

\[ \text{i/p freq, } f \]

9
Advantages of phase discriminator

(i) it is more easy to align (tune) than the balanced slope detector as there are only two tuned circuits and both are to be tuned at the same frequency $f_c$.
(ii) linearity is better

Draw backs
It does not provide amplitude limiting

RATIO DETECTOR

Fig 13 ratio detector cct

If you compare the above circuit with Foster Seedy discriminator, the two circuits are identical except for the following changes:

(i) the direction of divide $D_2$ is reversed
(ii) A large value capacitor $C_5$ has been included in the cct.
(iii) The o/p is taken somewhere else.
(iv) It has large capacitor $C_3$ & $C_4$ for amplitude limiting

Advantages of ratio detector
(i) Easy to align
(ii) Very good linearity, due to linear phase relationship between primary and secondary transformer.
(iii) Amplitude limiting is provided.
**Performance comparison of Fm Demodulators**

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<th>Phase discriminator</th>
<th>Ratio detector</th>
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<td>Not provided inherently</td>
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<td>Fm radio, satellite station receiver</td>
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</tr>
</tbody>
</table>

**PRE-EMPHASIS**-
This is a technique employed to limit noise interference pre-modulation. We are to achieve noise immunity at higher modulating frequencies. We can boost higher frequency modulating signal by using pre-emphasis circuit as shown in Fig 14 below. The modulating Af signal is passed through a high pass RC filter, before applying it to the Fm modulator.
As fm increase, reaction of C decrease and modulating voltage applied to Fm modulator goes on increasing. Modulating Af signal

**DE- EMPHASIS** – Reverting the process of emphasis by bring down the artificially boosted high frequency signals to their original amplitude using the de-emphasis cct as shown in fig 16 below

![De-emphasis cct & its xtics](image)

The 75 µ sec de-emphasis cct is standard 75 µ sec de-emphasis corresponds to a freq response curve at 3db

\[ f = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 75 \times 10^{-6}} = 2,122 \text{ HZ} \]

**PHASE LOCK LOOP** it is all about cct design technique to achieve system stability

A phase-locked loop (PLL) is primarily used in tracking the phase and frequency of the carrier component of an incoming fm signal. PLL is useful for demodulating fm signal in presence of large noise and low signal power it is basically a negative feedback system.
Useful Applicable Areas

(i) Synchronous demodulation of AM-SC
(ii) Demodulates FM signal in the presence of large noise and low signal power

Note. The ability of a PLL to provide frequency selectivity and filtering gives it a signal to noise S/N ratio superior to that of any other type of FM detector.

Example 1. Determine the permissible range in maximum modulation index for

(i) Commercial FM which has 30Hz to 15Hz mod frequencies
(ii) Narrow band FM system which allows maximum deviation of 10 KHz and 100 Hz to 3 KHz modulating frequencies.

Solution. We know that maximum deviation in commercial FM is given as

\( \Delta f = 75 \text{KHz} \)

Modulating index in FM is

\( M_f = \frac{\Delta f}{F_m} \)

Modulating index for commercial FM at \( F_m = 30 \text{Hz} \)

\[ M_f = \frac{\Delta f}{F_m} = \frac{75 \times 10^3}{30} = 2500 \]

Modulating index for commercial FM at \( F_m = 15 \text{kHz} \)

\[ M_f = \frac{\Delta f}{F_m} = \frac{75 \times 10^3}{15 \times 10^3} = 5 \]

Hence modulating index for commercial FM varies between 2500 and 5

(ii) For a given narrow band FM system, the maximum frequency deviation is given as \( \Delta f = 10 \text{ KHz} \).

Hence modulating index for a given NBFM system varies between

\[ M_f = \frac{\Delta f}{F_m} = \frac{10 \times 10^3}{100} = 100 \]
\[ M_f = \frac{\Delta f}{F_m} = \frac{10 \times 10^3}{3 \times 10^3} = 3.33 \text{ Ans.} \]

Example 2 A 100 MHz carrier wave has a peak voltage of 5 volt. The carrier is frequency modulated (FM) by a sinusoidal modulating signal or wave form of
frequency 2KHz such that the frequency deviation $\Delta f$ is 75 KHz. The modulated wave for passes through zero and is increasing at $+ = 0$

Determine the expression for the modulated carrier wave form.

**Solution.** Because the frequency modulated carrier wave form passes through zero and is increasing at $t = 0$ therefore the FM signal must be sine wave signal. Thus:

$$S(t) = A \sin \left[ 2\pi f_c (t) + mf \sin (2\pi f_m(t)) \right] \quad \ldots \quad (1)$$

Where $Mf = \text{modulating index of } F_m \text{ wave} = \frac{\Delta f}{F_m} \quad \ldots \quad (2)$

Where the following parameters given

$F_c = \text{carrier wave frequency} = 100 \times 10^6 = 10^8 \text{ Hz}$

$\Delta f = \text{frequency deviation} = 75 \text{ KHz} = 75 \times 10^3 \text{ Hz}$

$F_m = \text{modulating frequency} = 2 \text{ KHz} = 2 \times 10^3 \text{ Hz}$

$A = \text{peak voltage of carrier wave} = 5 \text{ volt}$.$$

\Rightarrow mf = \frac{\Delta f}{F_m} = \frac{75 \times 10^3}{2 \times 10^3} = 37.5$

Substituting all the above values in eqn 1

$$S(t) = 5 \sin \left[ 2\pi \times 10^8 t + 37.5 \sin (2\pi \times 2 \times 10^3 t) \right]$$

Or $$S(t) = 5 \sin \left[ 2\pi \times 10^8 t + 37.5 \sin (4\pi \times 10^3 t) \right] \quad \text{Ans.}$$

**Noise**

In electrical terms, noise may be defined as an unwanted for energy which tend to interfere with proper reception and reproduction of transmitted signal. Noise always limits performance of a communication system. Noise is any random interference to a weak signal.

Classification of noise we have two broad groups

(i) External noise
(ii) Internal noise

(i) **EXTERNAL NOISE** – The sources of are external to the communication system but can be controlled. It can further be classified as:-

(a) Atmospheric Noise
(b) Extra terrestrial Noise
(c) Industrial Noise

(a) Atmospheric Noise which is static in nature, is produced by lightning discharges in thunderstorm and other natural electrical disturbance which occur in the atmosphere. The noise energy spread along the entire range of frequency spectrum. Due to this reason, at any receiving point, the receiving antenna picks up not only the required signal but also the static from the thunder storms.

From observation, atmospheric noise varies inversely with the frequency. This implies that large atmospheric noise is produced in low and medium frequency bands where as small noise is produced in the VHF and UHF bands.
Thus Atmospheric noise becomes less severe at frequencies above about 30 MHz.

(b) EXTRA TERRESTIAL NOISE OR SPACE NOISE CONTAINS

(i) solar noise
(ii) cosmic noise

(i) Solar noise is the electrical noise emanating from the sun. The sun is a very massive body and at extremely high temperature, it radiates energy in the form of noise over a very wide frequency spectrum, including that of radio communication. Sun has 11 years cyclic, the electrical disturbance are caused.

(ii) Cosmic noise:- Distant stars just as sun cause noise. The distance stars have high temperature and therefore radiates noise in the same manner as the sun. the noise receive from distant stars is also refer to as Thermal noise and is evenly distributed across the sky. The noise from distant stars also received from the centre of our galaxy and other galaxies.

The space noise is well pronounced at 1.43 GH and in the frequency range of 20 to 12 MHz the space noise becomes strongest noise next to industrial noise.

(iii) Industrial noise or otherwise called man-made noise which is produced from sources such as, Automobile, air craft ignition, electrical motor, switch gears, and leakage from high voltage transmission lines, fluorescent lights, and other heavy electrical equipments. It affects frequency between 1 MHz to 600 MHz.

INTERNAL NOISE

Internal noise in general within the communication systems or receiver. The internal noise may be treated quantitatively and can be reduced or minimized by proper system design. Since internal noise is randomly distributed over the entire frequency spectrum, the noise present in a given band width B is the same at any frequency in the frequency spectrum. Thos implies that random noise power is proportional to the bandwidth over which it is measured.

The internal noise may be classified as:

(i) Short noise
(ii) Thermal noise

(i) SHORT NOISE

Short noise arises in active devices due to random behavior of charge carriers. In electron tubes, short noise is generated due to the random emission of electron from cathode, whereas in semi-conductor devices short noise is generated due to the random diffusion of minority carriers or simply random generation and recombination of electron – hole pairs. In fact, the current electron devices (i.e. tubes or solid state) flows in the form of discrete pulses.

Hence, although the current appears to be continuous, it is still a discrete phenomenon. Fig 20 below shows the nature of current variation with time.
Fig 20 current variation with time.
The current fluctuate about a mean value $I_0$. The current $i_n(t)$ wiggles around the mean value, but it assumed that the current is a constant equal to $I_0$.
Therefore, the total current $i(t)$ can be expressed as

$$i(t) = I_0 + i_n(t) \quad \text{eqn 1}$$

$i_n(t)$ is always random and is in deterministic function but can be specified by its power density spectrum.

Power density spectrum of the statically independent non-interacting random noise current $i_n(t)$ is expressed as

$$S_i(\omega) = qI_0 \quad \text{eqn 2}$$

Where $q$ is the electronic charge and $I_0$ is the mean value of the current in amperes. Please note that the power density spectrum ($S_i(\omega)$) in eqn 2 is frequency independent.

The frequency independent is only up to frequency range determined by the transit time ($\tau$) of an electron to reach to from the anode to the cathode.
The transit time ($\tau$) of an electron in a diode depends upon anode voltage $V$, expressed as

$$\tau = 3.36 \times \frac{d}{\sqrt{V}} \mu \text{sec} \ldots \text{eqn 3}$$

d is the spacing between anode & cathode.

**THERMAL NOISE**

The thermal noise or white noise is the random noise which is generated in a resistor or the resistive component, of complex impedance due to rapid and random motion of the molecule atoms and electronic. According to the kinetic theory of thermodynamic the temperature of a particle demotes its internal kinetic energy. This means that the temperature of a body expresses the rms value of the velocity of motion of the partition in body. This implies that at zero velocity, the kinetic energy of the particle is absolute zero; therefore the noise power produced in a resistor is proportional to its absolute temperature. Also the noise power ($P_n$) is proportional to the band with over which the noise is measured.

$$P_n \propto T \cdot B \text{----------------------Equal}$$

Or

$$P_n = K \cdot T \cdot B \text{----------------------Equal}$$

Where $K = \text{Botzmann's constant}$

$$= 1.38 \times 10^{-23} \text{ Joule / deg . K}$$

$T = \text{absolute temperature}$

$B = \text{Band width in Hz.}$

**VOLTAGE & CURRENT MODEL OF A NOISY RESISTOR**

Fig 22

According to maximum power transfer Theorem for maximum transfer of power from noise voltage Source $V_n$ to load resistor $R_L$, we must have

$$R_L = R$$

Then the maximum noise power so transferred is given as

$$P_n = \frac{V^2}{R_L} \text{-------------------equation 4}$$

But $R_L = R$
\[ P_n = \frac{V^2}{R} \quad \text{equation 5} \]

Applying Voltage divide method in Fig 22, we get
\[ V = \frac{V_n}{2} \quad \text{equation 6} \]

So that
\[ P_n = \frac{V^2}{R} = \frac{(V_n/2)^2}{R} \]

Or
\[ P_n = \frac{V_n^2}{R} \quad \text{equation 7} \]

Or
\[ V_n^2 = 4RP_n \quad \text{equation 8} \]

But we know that \( P_n = k \cdot T \cdot B \)
Putting eqn 2 in eqn 8, we have
\[ V_n^2 = 4R(KTB) \]
Or
\[ V_n^2 = 4RKTB \]
Or
\[ V_n = \sqrt{4RKTB} \quad \text{equation 9} \]

Let’s consider the current model of a resistor

\[ V_n^2 = 4RKTB \quad \Rightarrow I_n^2 = 4KTB \]

\[ G = \frac{1}{R} \quad \text{Fig 23} \quad \text{Fig 24} \]

Using conductance \( G = \frac{1}{R} \)

The rms noise current
\[ I_n^2 = 4GKT \quad \text{equation 10} \]

**Example 1** An amplifier operation over the frequency range from 18 to 20 MHZ has 10k\( \Omega \) input resistor. Calculate the rms Voltage to this amplifier if the ambient temperature is 27°C

**Solution**
\[ V_n = \sqrt{4RKTB} \]

Given the following Parameters
\[ R = 10 \, \text{k}\Omega \]
\[ T = 27 + 273 = 300 \, ^\circ \text{k} \]
Bandwidth = 20 – 18 = 2MHZ
\[ K = 1.38 \times 10^{-23} \text{ Joule / deg. K} \]
\[ V_n = \sqrt{4 \times 10 \times 10^3 \times 1.38 \times 300 \times 2 \times 10^6} \]
\[ \sqrt{4 \times 1.38 \times 3 \times 2 \times 10^{11}} \]
\[ 1.82 \times 10^5 \text{ Volts} \]
18.2 \mu \text{V. Ans.} 

**NOISE IN REACTIVE CCTS**

\[ \text{Fig 2} \]

The mean Square Value of noise Voltage from fig 25 above is given to

\[ \overline{V_{n_i}^2} = 4KRT (\Delta f) \] eqn 11

The mean Square value of output noise voltage Value is given as

\[ V_{n_o}^2 = 4KTR_p (\Delta f) \]

Where \( R_p \) in the equivalent parallel resistance.

**Example2:** - A parallel tuned cct is made to resonate at a frequency of 100 MHZ. The Parallel tuned circuit uses a coil h having quality factor Q of and capacitance of 10pf. The temperature of the circuit is maintained at 17^\circ C. Determine the output voltage across the circuit measured by a wide band voltmeter.

**Solution:** Quality factor of the coil is given by

\[ Q = \frac{f_r}{\Delta f} \]

Where \( f_r = \) resonant frequency
\[ \Delta f = \text{bandwidth of the tuned cct.} \]
\[ \Delta f = 100 \frac{10}{10} \]
\[ \Delta f = 10 \text{MHZ} \]

The quality factor is given by

\[ Q = \frac{1}{\omega c R} \]

Or \[ R = \frac{1}{Q c \omega} = \frac{1}{10 \times 10 \times 10^{-12}} \]

= 1
\[
10 \times 2\pi \times 100 \times 10^6 \times 10 \times 10^{-12}
\]

\( R = 16 \text{ Ohms} \)

The output voltage (rms) is given as

\[
V_{no} = (\overline{V_{no}^2})^{1/2}
\]

But mean Square value \( \overline{V_{no}^2} \) is given by

\[
\overline{V_{no}^2} = 4 \frac{KT}{Q^2} R
\]

There

\[
4 \quad V_{no} = \sqrt{4KT(\Delta f)Q^2R}
\]

\[
= \sqrt{4 \times 1.38 \times 10^{-23} \times (273+17) \times 10 \times 10^5 \times 10^2 \times 16}
\]

\( V_{no} = 160 \times 10^{-7} = 16 \mu \text{V} \quad \text{Ans.} \)

**SIGNAL TO NOISE RATIO (SNR)**

S/N Ratio Is Expressed As Signal Power To The Associated Noise Power At The Same Point In A System.

Thus, it a signal voltage \( V_s(t) \) is associated with a noise voltage source \( V_n(t) \)

Then the ratio of signal power to the noise power will be

\[
S = \frac{V_s^2}{\overline{V_n^2}}
\]

Because the power spectrum density is power per unit bandwidth, the above expression can be given as

\[
S = S_s(\omega)
\]

\[
N \quad S_n(\omega) = \frac{\text{power spectrum density of signal voltage}}{\text{Power spectrum density of noise voltage}}
\]

**NOISE FIGURE**

Noise figure is a figure of merit used to indicate how much the signal –to- noise ratio deteriorates as a signal passes through a circuit or series of circuits.

Mathematically

Noise figure

\[
F = \frac{\text{input SNR}}{\text{output SNR}} = \frac{(\text{SNR})_i}{(\text{SNR})_o}
\]

**NOISE TEMPERATURE**

According to thermodynamics of any system, the available thermal noise power is expressed as

\[
P_n = KT \cdot B \text{ watts}
\]

And

\[T_n = P_n\]
Where $T =$ absolute temperature

$B =$ band width in HZ

$K =$ Botzman's constant

$= 1.38 \times 10^{-23}$ Joule/deg.K

**EFFECT OF NOISE ON AM AND FM SYSTEM**

Noise as any random interference signal in radio transmission and receiver causes distortion, there is always the need to introduce filter before demodulator in order to eliminate or reduce it to tolerable level.

Fig 26 A Filter is connected before a demodulator to reduce the power input.

The noise signal is an additive to receive signal of $s(t)$ and $\omega(t)$ as shown in figure below.

Fig 27 Noisy Receiver model

At the output of the band pass filter, the signal present is $x(t)$ which is given by:

$X(t) = S(t) + n(t)$ ...............eqn1

Therefore $n(t) = x(t) - S(t)$ ...............eqn2

From fig27, $f_c >> BT$

$n(t) = n_I(t) \cos (2\pi f_c t) - n_Q(t) \sin (2\pi f_c t)$ ...............eqn3

Where $n_I(t)$ is the in phase no component and $N_Q(t)$ is the quadrature noise Component.
Fig 28 ideal xtic of band pass filtered noise
Average noise power = $2 \times \frac{N_o B_T}{2} = N_o B_T$ .............. eqn 4

Where $B_T$ = Transmission band width

Signal to noise ratio at the demodulator input (signal sensitivity)
A measure of a receiver noise performance is its signal to noise (S/N) ratio

$(SNR)_t = \frac{\text{Average power of } S(t)}{\text{Average power of filtered noise } n(t)}$

= Average signal power at receiver input
Average noise power at receiver input .................. eqn5

**Factor Affecting SNR**

(i) Type of modulation used at the transmitter
(ii) Type of demodulation used at the receiver.

Figure merit = $SNR_O = SNR_O$ .............. eqn 6

$SNR_c = \frac{SNR_I}{SNR_O}$

Figure of merit can be less than or equal to 1 depending on the type of modulation.

**NOISE IN AM RECEIVER**

The transmitted AM wave is expressed mathematically as.
$S(t) = v_c (1 + m(t)) \cos (2\pi f_c t)$ .............. eqn 7

Hence, $V_c \cos (2\pi f_c t)$ is the carrier wave
$X(t) =$ message signal
M = modulation index

The average power of the carrier component is given by $P_c = \frac{V_c^2}{2}$ ........ eqn 8

The information bearing component in equation 8 is given by $mV_c x(t) \cos (2\pi f_c t)$

the total average power associated with it is:
$\frac{V_c^2 M^2}{2} \frac{P}{V_m^2}$ ........ eqn 9

Where $P$ = average power of the message signal $x(t)$ the average power of the full

AM signal $x(t)$ is therefore equal to $\frac{V_c^2}{2} (\frac{m^2 P}{V_m^2})$ ........ eqn 10
The DSB –SC system has the message band width of fm No. The channel signal to noise ratio of AM is therefore given by

\[ \text{SNR} = \frac{\text{SNR}_c = \frac{V_c^2 (1+m^2P)}{2f_m N_o}}{V_m^2} \quad \text{Eqn 11} \]

AM signal \( s(t) \) → Band-pass Filter (BPF) → f(t) → Envelope Detector → o/p \( y(t) \)

Thus, we have

\[ f(t) = S(t) + n(t) \]

Where \( n(t) \) = filtered noise, \( n_L(t)n_Q(t) = \) quadrature component

\[ f(t) = S(t) + n(t) \quad \text{eqn 12} \]

\[ = [V_c + V_c m x(t) + n_1(t) \cos(2\pi f_c(t) - n_Q(t) \sin2\pi f_c(t)) \quad \text{eqn 1}] \]

Resultant

\[ y(t) = V_c + V_c m x(t) + n_1(t) \quad \text{eqn 15} \]

Envelop Detector

From phasor diagram, the receiver o/p \( y(t) \)

\[ y(t) = \text{envelope of } f(t) \]

\[ = \sqrt{V_c^2 \left[1 + m x(t) + n_1(t) \right]^2 + n_Q^2(t)} \left\{ \frac{1}{2} \right\} \quad \text{eqn 14} \]

This is the o/p an ideal detector, it is completely insensitive to phase

\[ y(t) = V_c + V_c m x(t) + n_1(t) \quad \text{eqn 15} \]

DC terms msg signal Noise

Note Vc can be ignored because no coherent relationship with message signal. Hence, the output signal noise ration is given as:

\[ \text{SNR}_o = \frac{V_c^2 m^2 P}{2f_m N_o} \quad \text{eqn 16} \]
**NOISE IN FM RECEIVER**

The noise $\omega(t)$ in Fig 31 is a white Gaussian noise with Zero mean value. Its power spectra density is $N_0/2$.

$S(t)$ represent received FM signal having carrier frequency $f_c$ and transmission bandwidth $B_{T2}$.

**Note**

We assume that almost all the transmitted power lies inside the frequency band $f_c \pm (B_{T2}/2)$

In an FM system, the base band signal varies only the frequency of the carrier. Hence, any amplitude variation of the carrier must be due to noise alone.

The limiter is used to suppress such amplitude variation noise.

**Noise**

$n(t)$ represent the filtered version of the received signal noise $\omega(t)$. The $n(t)$ in phase quadrature components are $n_L(t)$ and $n_Q(t)$ respectively.

**Analysis of Noise performance of FM System**

The filtered noise $n(t)$ can be expressed in terms of the phase and quadrature component as:

$$n(t) = n_1(t) \cos \left(2\pi f_c(t) - n_Q(t) \sin(2\pi f_c(t))\right) \ldots \ldots 17$$

When we expressed $n(t)$ in terms of its envelope and phase

$$n(t) = r(t) \cos \left[2\pi f_c(t) + \psi(t)\right] \ldots \ldots 18$$

Where the envelope is given by

$$r(t) = \left[n_1^2(t) + n_Q^2(t)\right]^{1/2} \ldots \ldots 19$$

And the phase is given by
\[
\Psi(t) = \tan^{-1} \left[ \frac{n_0(t)}{n_1(t)} \right] \quad \ldots \text{eqn 20}
\]

It may be noted that the envelope \( r(t) \) has a Raleigh distribution and phase \( \psi(t) \) is distributed uniformly over \( 2\pi \) radius. The FM wave at the input is given by

\[
S(t) = V_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) \, dt \right] \quad \ldots \text{eqn 21}
\]

Where \( V_c = \) carrier amplitude,
\( f_c = \) carrier frequency
\( K_f = \) frequency sensitivity
\( m(t) = \) msg signal.

Let \( 2\pi k_f \int_0^t m(t) \, dt = \Phi(t) \), hence the expression for FM wave is given by

\[
S(t) = V_c \cos \left[ 2\pi f_c t + \Phi(t) \right] \quad \ldots \text{eqn 22}
\]

The noisy signal at Bp filter o/p is given by

\[
X(t) = S(t) + n(t) = V_c \cos \left[ 2\pi f_c t + \Phi(t) \right] + r(t) \cos \left[ 2\pi f_c t + \psi(t) \right] \quad \ldots \text{eqn 23}
\]

Fig 33 phasor diagram of FM wave plus narrow band noise signal

The resultant phase \( x(t) \) has a phase \( \theta(t) \)

Hence, \( \theta(t) - \Phi(t) = \tan^{-1} \left\{ \frac{r(t) \sin[\psi(t) - \Phi(t)]}{V_c + r(t) \cos[\psi(t) - \Phi(t)]} \right\} \quad \ldots \text{eqn 24}

Therefore, we have

\[
\theta(t) - \Phi(t) = \tan^{-1} \left\{ \frac{r(t) \sin[\psi(t) - \Phi(t)]}{V_c + r(t) \cos[\psi(t) - \Phi(t)]} \right\}
\]

Note that envelope variation of \( x(t) \) is removed by the limiter. The target focus is on filtered noise \( n(t) \).