

THERMODYNAMICS RELATIONS

At temperatures in excess of critical temperature of a fluid, and low pressure, the vapour of fluid tends to obey the equation.

$$\frac{Pv}{T} = \text{constant} = R \text{ or } \frac{P}{\rho} = RT$$

An imaginary ideal gas which obeys this law is called perfect gas and equation $\frac{Pv}{T} = R$, is called the characteristic equation of a perfect gas.

R is the gas constant; each perfect gas has its own gas constant. The unit of R is Nm/Kg^k
or Nm/Kg k.

$$Pv = RT \text{ for } mKg, \text{ and } Vm^3$$

$$Pv = mRT$$

$$P = \frac{m}{v}RT = \rho RT.$$

Taking log. on both sides, we have

$$\log_{\rho} P = \log_{\rho} \rho + \log_{\rho} R + \log_{\rho} T$$

By differentiation, we have

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dR}{R} + \frac{dT}{T}$$

Since R is constant for a particular gas, its derivation is zero.

$$\therefore \frac{dP}{P} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0 - \text{Differential equation of a perfect gas.}$$

3.1.1 SPECIFIC HEATS

The specific heat of a solid or liquid is usually defined as the heat required to raise temperature of a unit mass through one degree temperature rise.

Since gas can add heat between any two temperatures through an infinite number of ways, hence gas could have an infinite number of specific heats.

Let us consider: (i) Specific heat at constant volume C_v and (ii) Specific heat at constant pressure, C_p real gases, C_p and C_v vary with temperature, but a suitable average value may be used for most practical purposes.

$$C_p = C_v + R \rightarrow \frac{C_p}{C_v} = \gamma (\text{gamma}).$$

3.1.2 INTERNAL ENERGY

This is the heat energy stored in a gas. When heat is supplied to a gas the result is that the temperature or volume of gas may increase.

Thereby doing some work, also it could be that both temperature and volume may increase.

During addition of heat if the temperature increases, its internal energy will also increase.

'Joule's law' of internal energy states that the internal energy of a perfect gas is a function of temperature only. That is internal energy of gas is dependent only on temperature change and not affected by the change in pressure and volume. In engineering it is the change of internal energy that is required.

Enthalpy

In thermodynamics, enthalpy is the sum of internal energy (u) and pressure volume product (Pv).

$$h = u + Pv.$$

The total enthalpy of mass, m, of a fluid is given by $H = U + Pv$ where $H = mh$

Basic Thermodynamic Process:

1. **Isochoric Process:** This is a change in the state of a (gas (working fluid)) system at constant volume is called isochoric process. This occurs when gas system is heated or cooled in an enclosed space (e.g. a rigid vessel)

$$Q = C_v (T_2 - T_1)$$

$$\text{Work done, } W = 0$$

$$\text{P, V, T relations } \frac{T_2}{T_1} = \frac{P_2}{P_1}$$

2. **Constant Pressure (isobaric) Process:** In this process the change in the state of a gas system taken place at constant pressure.

For a constant pressure process, the boundary must move against an external resistance as heat is supplied:

$$\text{Heat added, } Q = C_p (T_2 - T_1)$$

$$\text{Work done, } W = P (V_2 - V_1)$$

$$\text{P, V, T relations } \frac{T_2}{T_1} = \frac{V_2}{V_1}$$

3. **Isothermal Process, Pv or $\frac{P}{\rho} = \text{constant}$, $T = \text{constant}$:** A process at a constant temperature is called an isothermal process. In an isothermal expansion heat must

be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.

$$\text{Heat added, } Q = P_1 V_1 \log_{\rho} \frac{V_2}{V_1} = RT_1 \log_{\rho} \frac{\rho_1}{\rho_2}$$

$$\text{Work done, } W = P_1 V_1 \log_{\rho} \frac{V_2}{V_1} = RT_1 \log_{\rho} \frac{\rho_1}{\rho_2}$$

$$\text{Remember Work} = \int p dv$$

$$= p \int dv \text{ but } Pv = c \text{ for isothermal process}$$

$$\text{Work} = \frac{c}{v} \int dv \quad \therefore P = \frac{c}{v}$$

$$= C \int \frac{1}{v} dv$$

$$= C \log_{\rho} \frac{V_2}{V_1}$$

$$= P_1 V_1 \log_{\rho} \frac{V_2}{V_1}$$

$$P, V, T \text{ relation: } = P_1 V_1 = P_2 V_2 \left(\text{or } \frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \right)$$

4. Adiabatic Process ($Pv^\gamma = \text{constant}$)

An adiabatic process is one in which no heat is transferred to or from the gas during the process.

For an adiabatic process to take place, perfect thermal insulation for the system must be available.

$$\text{Heat added, } Q = 0$$

$$\text{Work done, } W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$P, V, T \text{ relations: } = P_1V_1 = P_2V_2$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

If the adiabatic process is reversible (or frictionless) it is known as isentropic process.

3.1.3 Basic Equations of Compressible Fluid Flow

(i) Continuity equation

In cases of non-dimensional flow, mass per sec. $= \rho AV$.

(where, ρ = mass density, A = cross-sectional area, V = Velocity)

Since mass or mass per second is constant according to law of conservation of mass, therefore,

$$\rho AV = \text{constant}$$

By differentiation, we have

$$d(\rho AV) = 0$$

$$\rightarrow \rho d(AV) + AV d\rho = 0$$

$$\rightarrow \rho AdV + \rho V dA + AV d\rho = 0$$

Dividing both sides by ρAV

$$\rightarrow \frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$$

Rearranging, we have,

$$\rightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad (\text{Equation of Continuity in differential form.})$$

3.1.4 Momentum Equation

The momentum equation for compressible fluid is similar to the one for incompressible fluids.

This is because in momentum flux is equated to force required to cause this change.

$$\begin{aligned} \text{Momentum flux} &= \text{mass flux} \times \text{velocity} \\ &= \rho AV \times V \end{aligned}$$

But, the mass flux, ρAV is constant.....by continuity equation.

The momentum equation is independent of the compressibility effects and hence for compressible fluids too, the momentum equation, say in X – direction, may be expressed as:

$$\begin{aligned} \sum F_x &= (\rho AVV_x)_2 - (\rho AVV_x)_1 \\ &= \rho AV_2 - \rho AV_1 \end{aligned}$$

3.1.5 Bernoulli's or Energy Equation

Euler's equation is obtained as:

$$\frac{dp}{\rho} + Vdv + gdz = 0$$

Integrating both sides, we have

$$\int \frac{dp}{\rho} + \int Vdv + \int gdz = \text{constant.}$$

$$\rightarrow \int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant.}$$

In compressible flow ρ is not constant, therefore it cannot be taken outside the integration sign.

Bernoulli's equation for isothermal process.

For isothermal process,

$$Pv = \text{constant or } \frac{P}{\rho} = \text{constant}$$

Where special volume, $V = 1/\rho$

$$\rho = \frac{P}{c}$$

$$\begin{aligned} \text{Hence } \int \frac{dp}{\rho} &= \int \frac{dp}{P/c_1} \\ &= \frac{c_1 dp}{P} = C_1 \int \frac{dp}{P} = C_1 \log_{\rho} P = \frac{P}{\rho} \log_{\rho} P \end{aligned}$$

$$\left\{ \dots C_1 = \frac{P}{\rho} \right\}$$

Substituting the value of $\int \frac{dp}{P}$ into equation

$$\int \frac{dp}{P} + \frac{V^2}{2} + gz = \text{constant, we have}$$

$$\frac{P}{\rho} \log_{\rho} P + \frac{V^2}{2} + gz = \text{constant.}$$

Dividing both sides by g, we have

$$\frac{P}{\rho g} \log_{\rho} P + \frac{V^2}{2} + gz = \text{constant}$$

Bernoulli's equation for compressible flow that is undergoing isothermal process.

Bernoulli's equation for adiabatic process:

For adiabatic process, $Pv^2 = \text{constant}$

$$\text{Or } \frac{P}{\rho^\gamma} = \text{constant} = C_2 \text{ (say).}$$

$$\rightarrow \frac{P}{C_2} = \rho^\gamma \text{ or } \left(\frac{P}{C_2}\right)^{1/\gamma} = \rho$$

$$\begin{aligned} \text{Hence, } \int \frac{dp}{\rho} &= \int \frac{dp}{\left(\frac{P}{C_2}\right)^{1/\gamma}} \\ &= (C_2)^{1/\gamma} \int \frac{1}{P^{1/\gamma}} dp = (C_2)^{1/\gamma} \int \frac{1}{P^{1/\gamma}} dp. \\ &= (C_2)^{1/\gamma} \left[\frac{P^{-1/\gamma}}{(-1/\gamma + 1)} \right] = \frac{(C_2)^{1/\gamma} \cdot P^{\left(\frac{\gamma-1}{\gamma}\right)}}{\left[\frac{\gamma-1}{\gamma}\right]} \\ &= \frac{\gamma}{\gamma-1} \cdot C_2^{1/\gamma} \cdot P \left[\frac{\gamma-1}{\gamma} \right] \end{aligned}$$

$$\text{But } C_2 = \frac{P}{\rho^\gamma}$$

$$\begin{aligned} \text{Therefore, } \int \frac{dp}{\rho} &= \frac{\gamma}{\gamma-1} * \left[\frac{P}{\rho^\gamma}\right]^{1/\gamma} \cdot P \left[\frac{\gamma-1}{\gamma} \right] \\ &= \left[\frac{\gamma}{\gamma-1}\right] \frac{P^{1/\gamma}}{\rho} \cdot P \left[\frac{\gamma-1}{\gamma} \right] \\ &= \left[\frac{\gamma}{\gamma-1}\right] \frac{P \left[\frac{1}{\gamma} + \frac{\gamma-1}{\gamma} \right]}{\rho} \\ &= \left[\frac{\gamma}{\gamma-1}\right] \cdot \frac{P}{\rho} \end{aligned}$$

Substituting the value of $\int \frac{dp}{\rho}$ in

$$\int \frac{dp}{\rho} + \frac{v^2}{2} + gz = \text{constant, we have}$$

$$\left[\frac{\gamma}{\gamma-1} \right] \frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\left[\frac{\gamma}{\gamma-1} \right] \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant} - \text{Bernoulli's equation for compressible flow undergoing adiabatic process.}$$