1.4 Characterization of an Isotropic Material that is Linearly Elastic

The responses in the linearly elastic region of a material are characterized with four numerical properties which are enough to establish constitutive equations. The four properties are: the Shear Modulus (*G*), elastic modulus (*E*), Poisson's ratio (*v*) and the coefficient of thermal expansion (α). These constitutive equations are associated to Hooke's law. They are experimentally determined and they are used to construct the thermo-elastic constitutive equations that are associated with stresses and strains.

1.4.1 Shear Modulus Determination in One-Dimensional Stress State

The determination of the shear modulus *G* experimentally, employs a torsion test specimen. **Shear modulus** is also known as **modulus of rigidity**. The shear modulus G connects shear strain (γ) to the corresponding shear stress (τ) as shown in equation (1.4):

$$\textbf{Shear Modulus } G = \frac{Shear\_stress(\tau)}{Shear\_strain(\gamma)} \qquad \textbf{(1.4)}$$

From the definition, if $\gamma = \gamma_{xz}$, a corresponding $\tau = \tau_{xz}$ is used for the determination of shear modulus *G*. The shear modulus is basically obtained from a torsion test.

1.4.2 Elastic Modulus and Poisson's Ratio in Stress State

1D normal stress to 1D extensional strain are associated through two constitutive equations which relates to the one-dimensional Hooke's law: the modulus of elasticity ***E***, also known as Young's modulus and Poisson's ratio ***v***.

The modulus of elasticity (***E***) links the axial stress (σ) and axial strain (ε):

$$\text{Modulus of Elasticity } (\boldsymbol{E}) = \frac{Axial\_Stress(\sigma)}{Axial\_Strain(\varepsilon)} \qquad \textbf{(1.5)}$$

Poisson's ratio (***v***) is defined as ratio of lateral strain to axial strain:

$$= \left| \frac{Lateral\_Strain(\varepsilon_L)}{Axial\_Strain(\varepsilon)} \right| = -\frac{Lateral\_Strain(\varepsilon_L)}{Axial\_Strain(\varepsilon)} \qquad \textbf{(1.6)}$$

Note: The − sign is introduced for convenience so that v comes out positive. For structural materials v lies in the range $0.0 \leq v < 0.5$. For most metals, $v \approx 0.25$–0.35. For concrete and ceramics, $v \approx 0.10$. For cork, $v \approx 0$. For rubber, $v \approx 0.5$ to 3 places. A material for which $v = 0.5$ is called *incompressible*.

It turns out that the 3 material properties E, ν and G for an elastic isotropic material are not independent, but are connected by the relation shown in equation (**1.7(a-c)**), which means that if two of them are known by measurement, the third one can be obtained from the relations.

$$G = \frac{E}{2(1+\nu)}, \quad \textbf{(1.7a)}$$

$$E = 2(1+\nu)G, \quad \textbf{(1.7b)}$$

$$\nu = \frac{E}{2G} - 1 \quad \textbf{(1.7c)}$$

1.4.3 Thermal Strains in a One-Dimensional Stress State

The supply of thermal energy to a body leads to increase in the body's temperature which increases the average kinetic energy of the body and this generates thermal strain in the body. The measure of thermal strain undergone by a body, which takes place as a result of a change in temperature, is expressed as:

$$\text{Thermal strain } \varepsilon_T = \alpha \Delta T \quad \textbf{(1.8)}$$

α is the coefficient of thermal dilatation, measured in $1/^{\circ}F$ or $1/^{\circ}C$.

Note: $\alpha > 0$ and very small: $\alpha << 1$, of the order 10^{-6} for most structural materials.

For an isotropic material that is linearly elastic, the total strain the material undergoes is expressed as:

$$\varepsilon = \varepsilon_{Axial} + \varepsilon_T = \frac{\sigma}{E} + \alpha \Delta T \quad \textbf{(1.9)}$$

Example1. The bar AB shown in Figure 1.7 is precluded from extending axially. It has elastic modulus E and coefficient of dilatation $\alpha > 0$. The stress σ is zero when the bar is at the reference temperature $Tref$. Find the axial stress σ developed if the temperature changes to $T = Tref + T$.

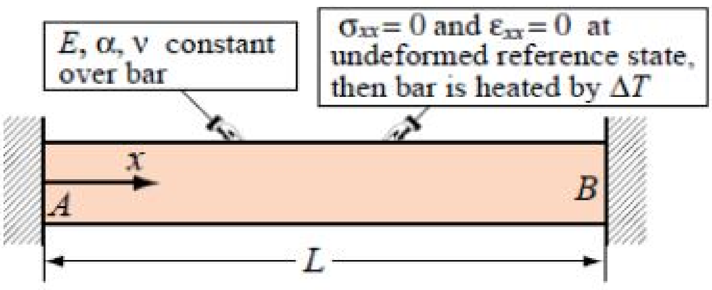

Figure 1.7: A constrained bar at both edges

Solution

Since the bar length cannot change because it is constrained at both edges, the combined axial strain must be zero:

$$\varepsilon_{xx} = \varepsilon = \frac{\sigma}{E} + \alpha\Delta T = 0 \qquad \textbf{(1.10)}$$

Solving for stress σ

Since the bar is constrained at both edges, as the temperature rises, that is $\Delta T > 0$, compression will arise which prevents a change in length and this produces a negative axial stress. The stress induced in the bar is referred to a *thermally induced stress* or *thermal stress.*

$$\sigma = -E\alpha\Delta T \qquad \textbf{(1.11)}$$

1.4.4 Application of the knowledge of thermal stress in Design

The effect of thermal stress on systems like space orbiting vehicles is considered in its design. This is because of their exposure to extreme temperature changes as a result of being fully exposed to the sun and also being shaded by the earth at night. This knowledge is also applied in the creation of expansion joints during the construction of rails, bridges, pavements, etc.

1.5 Hook's Law in Three-Dimension (3D)

1.5.1 Relationship between Strain-To- Stress

Considering the three-dimensional (3D) strain-stress relation of a body subjected to external force, with the assumption that the material is elastic and isotropic. Assuming that the body is a cube of material subjected to tension stresses as shown in Figures 1.8 (b-d) shown. The tension tests are conducted in the x, y and z axes as in b, c and d respectively.

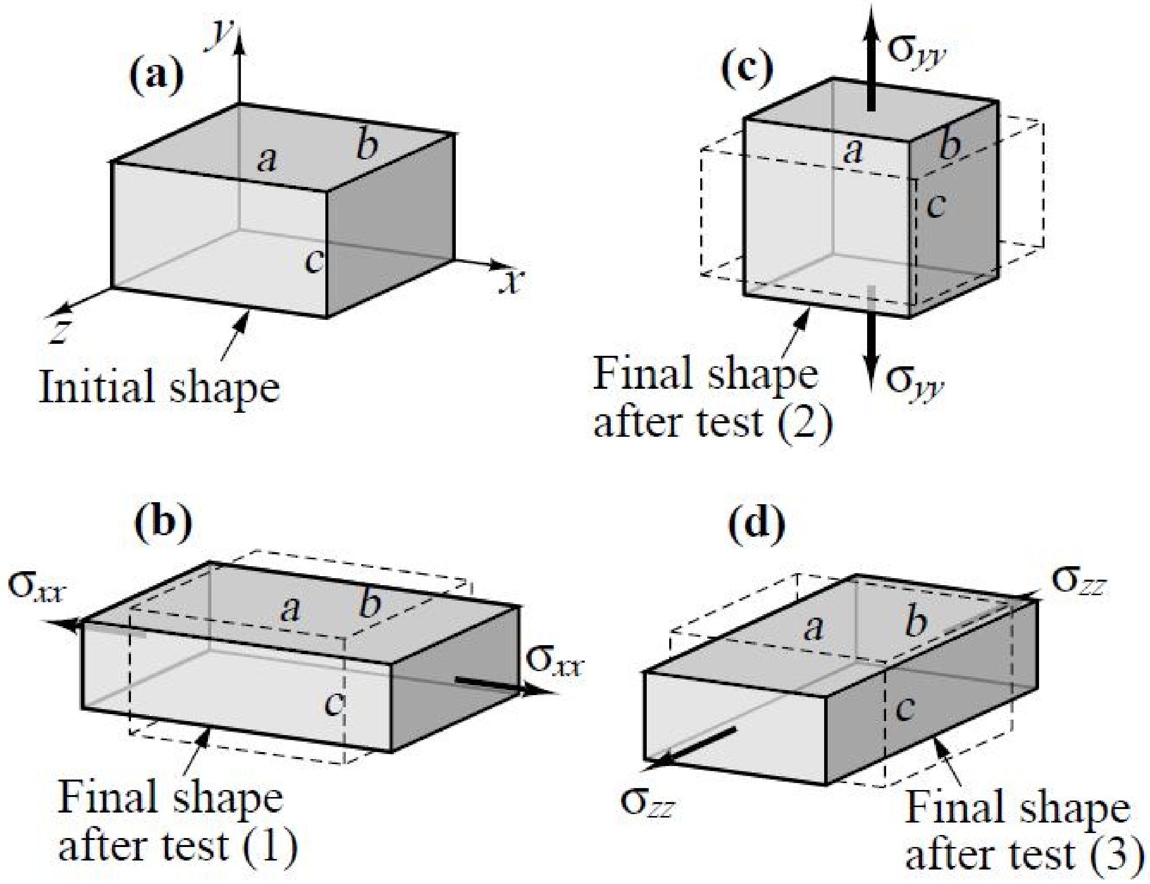

Figure 1.8: A cube subjected to tension on the x, y and z axes

For Figure 1.8 (b), the strain-stress relation, for the tension test is given as:

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}, \varepsilon_{yy} = -\frac{\nu\sigma_{xx}}{E}, \varepsilon_{zz} = -\frac{\nu\sigma_{xx}}{E} \qquad \textbf{(1.12a)}$$

For Figure 1.8 (c), the strain-stress relation, for the tension test is given as:

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E}, \varepsilon_{xx} = -\frac{\nu\sigma_{yy}}{E}, \varepsilon_{zz} = -\frac{\nu\sigma_{yy}}{E} \qquad \textbf{(1.12b)}$$

For Figure 1.8 (c), the strain-stress relation, for the tension test is given as:

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E}, \varepsilon_{xx} = -\frac{\nu\sigma_{zz}}{E}, \varepsilon_{yy} = -\frac{\nu\sigma_{zz}}{E}$$

(1.13c)

The cube is subjected to a combined normal stress σ_{xx}, σ_{yy} and σ_{zz}. The material is assumed to be linearly elastic, therefore, the combined strain can be superimposed and this gives the following expressions:

$$\varepsilon_{xx} = \varepsilon_{xx}^{(b)} + \varepsilon_{xx}^{(c)} + \varepsilon_{xx}^{(d)} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} - \frac{\nu\sigma_{zz}}{E} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}\right) \quad \textbf{(1.13a)}$$

$$\varepsilon_{yy} = \varepsilon_{yy}^{(b)} + \varepsilon_{yy}^{(c)} + \varepsilon_{yy}^{(d)} = \frac{-\nu\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \frac{\nu\sigma_{zz}}{E} = \frac{1}{E}\left(-\nu\sigma_{xx} + \sigma_{yy} - \nu\sigma_{zz}\right) \quad \textbf{(1.13b)}$$

$$\varepsilon_{zz} = \varepsilon_{zz}^{(b)} + \varepsilon_{zz}^{(c)} + \varepsilon_{zz}^{(d)} = \frac{\sigma_{xx}}{E} - \frac{\nu\sigma_{yy}}{E} - \frac{\nu\sigma_{zz}}{E} = \frac{1}{E}\left(-\nu\sigma_{xx} - \nu\sigma_{yy} + \sigma_{zz}\right) \quad \textbf{(1.13c)}$$

Recall that the shear modulus connects the shear strains and stresses and it is expressed as:-

$$\gamma_{xy} = \gamma_{yx} = \frac{\tau_{xy}}{G} = \frac{\tau_{yx}}{G}, \gamma_{yz} = \gamma_{zy} = \frac{\tau_{yz}}{G} = \frac{\tau_{zy}}{G}, \gamma_{xy} = \gamma_{zx} = \frac{\tau_{zx}}{G} = \frac{\tau_{xz}}{G} \quad \textbf{(1.14)}$$

When the equations (1.13) and (1.14) are merged or added, the summation gives the matrix in equation (1.15):-

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} \quad \textbf{(1.15)}$$

1.6.2 Stress –To-Strain Relations

The stress to strain relation can be easily gotten by inverting the matrix in equation (1.15) of strain-to-stress relation and this gives:-

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} \hat{E}(1-\nu) & \hat{E}\nu & \hat{E}\nu & 0 & 0 & 0 \\ \hat{E}\nu & \hat{E}(1-\nu) & \hat{E}\nu & 0 & 0 & 0 \\ \hat{E}\nu & \hat{E}\nu & \hat{E}(1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \quad \textbf{(1.16)}$$

$\hat{E}$ denotes an effective elastic modulus modified by Poisson's ratio:

$$\hat{E} = \frac{E}{(1-2\nu)(1+\nu)} \quad \textbf{(1.17)}$$

and for the six relations. The expressions can be written as follows:

$$\sigma_{xx} = \frac{E}{(1-2\nu)(1+\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy} + \nu\varepsilon_{zz}\right] \quad \textbf{(1.18a)}$$

$$\sigma_{yy} = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\varepsilon_{xx} + (1-\nu)\varepsilon_{yy} + \nu\varepsilon_{zz}\right] \quad \textbf{(1.18b)}$$

$$\sigma_{zz} = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\varepsilon_{xx} + \nu\varepsilon_{yy} + (1-\nu)\varepsilon_{zz}\right] \quad \textbf{(1.18c)}$$

$$\tau_{xy} = G\gamma_{xy}, \quad \textbf{(1.18d)}$$

$$\tau_{yz} = G\gamma_{yz}, \quad \textbf{(1.18e)}$$

$$\tau_{zx} = G\gamma_{zx} \quad \textbf{(1.18f)}$$

Average or mean stress is defined as

$$\alpha_{me} = \frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right). \quad \textbf{(1.19)}$$

The negative value of σ_{me} is the pressure.

Pressure $P = -\sigma_{me}$ **(1.20)**

The summation $\varepsilon_{xx}, \varepsilon_{yy}$ and ε_{zz} gives the **volumetric strain** ε_{vol} or **dilatation**.

$$\varepsilon_{vol} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \quad \textbf{(1.21)}$$

Condensation is defined as $= -\varepsilon_{vol}$. **(1.22)**

It should be noted that the pressure and volumetric strains do not change when the axes [x, y, z] are rotated. They are invariants. An important relation between pressure and volumetric strain can be obtained by adding the three stress relations σ_{xx}, σ_{yy}, and σ_{zz} which upon simplification and accounting for the mean stress and pressure $P = -\sigma_{me}$, relates pressure and volume strain as:-

$$\Pr essure(P) = -\frac{E}{3(1-2v)}\varepsilon_v = -K\varepsilon_v \qquad \textbf{(1.23)}$$

Where,

the coefficient ***K*** is called the bulk modulus.

If Poisson's ratio approaches 0.5, which occurs in near incompressible materials, $K \to \infty$.

1.7 Thermal Effects in a Body in Three-Dimension (3D)

A change in temperature (ΔT) from the base or reference temperature to a current temperature, adds $\alpha\Delta T$ to the normal strain, as expressed below:-

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - v\sigma_{yy} - v\sigma_{zz}\right) + \alpha\Delta T, \qquad \textbf{(1.24a)}$$

$$\varepsilon_{yy} = \frac{1}{E}\left(-v\sigma_{xx} + v\sigma_{yy} - v\sigma_{zz}\right) + \alpha\Delta T, \qquad \textbf{(1.24b)}$$

$$\varepsilon_{zz} = \frac{1}{E}\left(-v\sigma_{xx} - v\sigma_{yy} + \sigma_{zz}\right) + \alpha\Delta T, \qquad \textbf{(1.24c)}$$

No change in the shear strain-stress relation is needed because if the material is linearly elastic and isotropic, a temperature change only produces normal strains. The stress-to-strain matrix relation expands to

$$
\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} + \alpha \Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(1.25)}
$$

Inverting this relation provides the stress-strain relations that account for a temperature change in which $\hat{E}$ is defined in:

$$
\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} \hat{E}(1-\nu) & \hat{E}\nu & \hat{E}\nu & 0 & 0 & 0 \\ \hat{E}\nu & \hat{E}(1-\nu) & \hat{E}\nu & 0 & 0 & 0 \\ \hat{E}\nu & \hat{E}\nu & \hat{E}(1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} - \frac{E\alpha\Delta T}{(1-2\nu)} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(1.26)}
$$

Note that if all mechanical normal strains ε_{xx}, ε_{yy} and ε_{zz} vanish, but $\Delta T \neq 0$, the normal stresses given above are nonzero. Those are called *initial thermal stresses*, and are important in engineering systems exposed to large temperature variations, such as rails, turbine engines, satellites or re-entry vehicles.

1.8 Generalized Hook's Law in Two-Dimension (2D)

Two specializations of the foregoing 3D equations to two dimensions are of interest in the applications: *plane strain* and *plane stress*. Plane stress is more important in Aerospace structures, which tend to be thin.

1.8.1 Plane Strain

In this case all stress components with a *z* component are assumed to vanish. For a linearly elastic isotropic material, the strain and stress matrices take on the form

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \textbf{(1.27)}$$

Note that the ε_{zz} strain, often called the *transverse strain* or *thickness strain* in applications, in general will be nonzero because of Poisson's ratio effect. The strain-stress equations are easily obtained by going to (1.13) and (1.14) and setting

$$\sigma_{zz} = \tau_{yz} = \tau_{zx} = 0. \qquad \textbf{(1.28)}$$

This gives:

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy}\right), \qquad \textbf{(1.29a)}$$

$$\varepsilon_{yy} = \frac{1}{E}\left(-\nu\sigma_{xx} + \sigma_{yy}\right), \qquad \textbf{(1.29b)}$$

$$\varepsilon_{zz} = -\frac{\nu}{E}\left(\sigma_{xx} + \sigma_{yy}\right), \qquad \textbf{(1.29c)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \gamma_{yz} = \gamma_{zx} = 0 \qquad \textbf{(1.29d)}$$

The matrix formed by the above expression, while omitting the zero components is:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{v}{E} & 0 \\ -\frac{v}{E} & \frac{1}{E} & 0 \\ -\frac{v}{E} & -\frac{v}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \quad \textbf{(1.30)}$$

Inverting the matrix composed by the first, second and fourth rows of the above relation gives the stress-strain equations

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \tilde{E} & \tilde{E}v & 0 \\ \tilde{E}v & \tilde{E} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \quad \textbf{(1.31)}$$

Where $\tilde{E} = \dfrac{E}{(1-v^2)}$ **(1.32)** and normal stresses and shear stress are expressed as:-

$$\sigma_{xx} = \frac{E}{(1-v^2)}\left(\varepsilon_{xx} + v\varepsilon_{yy}\right), \quad \textbf{(1.33a)}$$

$$\sigma_{yy} = \frac{E}{(1-v^2)}\left(\varepsilon_{yy} + v\varepsilon_{xx}\right), \quad \textbf{(1.33b)}$$

$$\tau_{xy} = G\gamma_{xy} \quad \textbf{(1.33c)}$$

If there are changes in temperature ΔT, the equations acquire extra terms as shown below:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{v}{E} & 0 \\ -\frac{v}{E} & \frac{1}{E} & 0 \\ -\frac{v}{E} & -\frac{v}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} + \alpha\Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{1.34}$$

and

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \tilde{E} & \tilde{E}v & 0 \\ \tilde{E}v & \tilde{E} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} - \frac{E\alpha\Delta T}{(1-v)} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{1.35}$$

1.8.2 Plane Stress

In this situation, it is assumed that all the strain components with a z component are ignored. For an isotopic material which is linearly elastic the expressions below show the forms the strain and stress matrices take:

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.36a}$$

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \tag{1.36b}$$

It should be noted that the transverse stress, represented by σ_{zz}, will remain in the matrix. The strain-to-stress relations can easily be obtained by setting

$$\varepsilon_{zz} = \gamma_{yz} = \gamma_{zx} = 0. \quad \textbf{(1.37)}$$

This results to:

$$\sigma_{xx} = \frac{E}{(1-2\nu)(1+\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy}\right] \quad \textbf{(1.38a)}$$

$$\sigma_{yy} = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\varepsilon_{xx} + (1-\nu)\varepsilon_{yy}\right] \quad \textbf{(1.38b)}$$

$$\sigma_{zz} = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\varepsilon_{xx} + \nu\varepsilon_{yy}\right] \quad \textbf{(1.38c)}$$

$$\tau_{xy} = G\gamma_{xy}, \tau_{yz} = 0, \tau_{zx} = 0 \quad \textbf{(1.38d)}$$

When the zero components are removed, the expression in matrix form becomes:-

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \hat{E}(1-\nu) & \hat{E}\nu & 0 \\ \hat{E}\nu & \hat{E}(1-\nu) & 0 \\ \hat{E}\nu & \hat{E}\nu & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \quad \textbf{(1.39)}$$

Inverting the expression gives the stress-to-strain relation.

Note: the temperature change effect can be accounted for by adding its term.