Transportation Problems

- Transportation is considered as a “special case” of LP

- Reasons?
  - it can be formulated using LP technique so is its solution

(to p2)

- Here, we attempt to firstly define what are them and then studying their solution methods:

(to p3)

Transportation Problem

- We have seen a sample of transportation problem on slide 29 in lecture 2 (to p4)

- Here, we study its alternative solution method

- Consider the following transportation tableau (to p6)

Review of Transportation Problem

Warehouse supply of televisions sets:

1- Cincinnati 300

2- Atlanta 200

3- Pittsburgh 200

total 700

Retail store demand for television sets:

A - New York 150

B - Dallas 250

C - Detroit 200

total 600

From Warehouse	To Store		
	A	B	C
1	$16	$18	$11
2	14	12	13
3	13	15	17

(to p5)

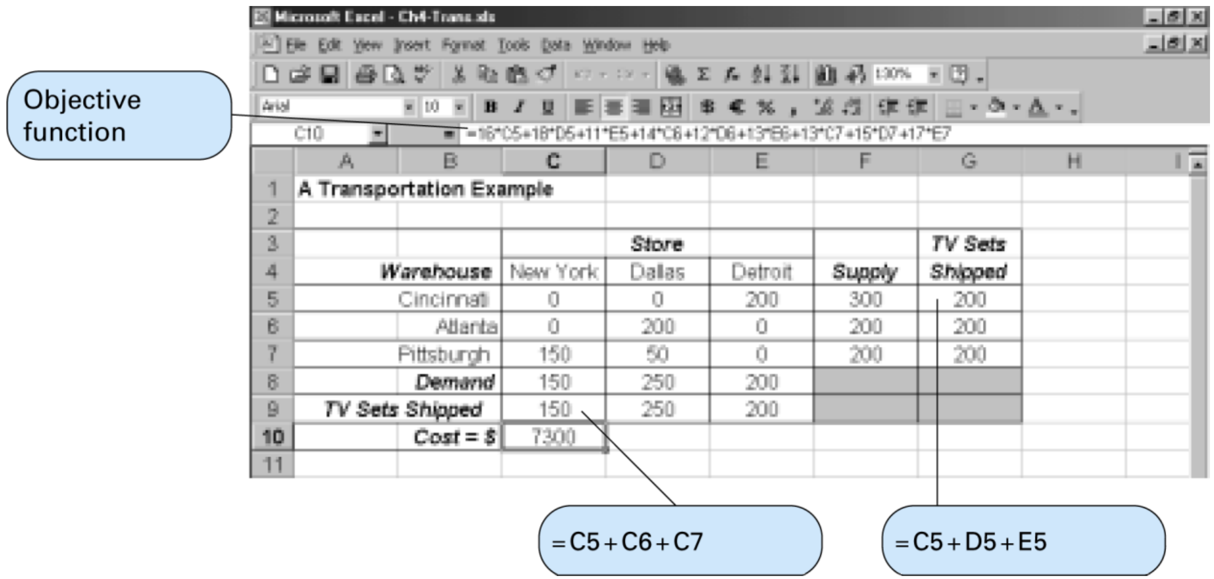
LP formulation ▶

A Transportation Example (2 of 3)
Model Summary and Computer Solution with Excel

Minimize $Z = \$16x_{1A} + 18x_{1B} + 11x_{1C} + 14x_{2A} + 12x_{2B} + 13x_{2C} + 13x_{3A} + 15x_{3B} + 17x_{3C}$

subject to

$$x_{1A} + x_{1B} + x_{1} \leq 300$$
$$x_{2A} + x_{2B} + x_{2C} \leq 200$$
$$x_{3A} + x_{3B} + x_{3C} \leq 200$$
$$x_{1A} + x_{2A} + x_{3A} = 150$$
$$x_{1B} + x_{2B} + x_{3B} = 250$$
$$x_{1C} + x_{2C} + x_{3C} = 200$$
$$x_{ij} \geq 0$$



A Transportation Example						
		Store				TV Sets
	Warehouse	New York	Dallas	Detroit	Supply	Shipped
	Cincinnati	0	0	200	300	200
	Atlanta	0	200	0	200	200
	Pittsburgh	150	50	0	200	200
	Demand	150	250	200		
	TV Sets Shipped	150	250	200		
	Cost = $	7300				

Exhibit 4.15

(to p3)

Transportation Tableau

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

- We know how to formulate it using LP technique
  - Refer to lecture 2 note
- Here, we study its solution by firstly attempting to determine its initial tableau (to p7)
  - Just like the first simplex tableau!

Solution to a transportation problem

- Initial tableau (to p8)
- Optimal solution (to p23)
- Important Notes (to p43)
- Tutorials (to p53)

initial tableau

- Three different ways:
  - Northwest corner method (to p9)
  - The Minimum cell cost method (to p12)
  - Vogel's approximation method (VAM) (to p16)

- Now, are these initial tableaus given us an Optimal solution? (to p7)

Northwest corner method

Steps:

1. assign largest possible allocation to the cell in the upper left-hand corner of the tableau
2. Repeat step 1 until all allocations have been assigned
3. Stop. Initial tableau is obtained

Example (to p10)

Northeast corner

Step 1

Step 2

Max (150,200)

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply	
1	[6] 150	[8] --	[10] --	~~150~~	0
2	[7]	[11]	[11]	175	
3	[4]	[5]	[12]	275	
Demand	~~200~~	100	300	600	
	150				

→

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply	
1	[6] 150	[8] --	[10] --	~~150~~	0
2	[7] 50	[11]	[11]	~~175~~	125
3	[4] --	[5]	[12]	275	
Demand	~~200~~	100	300	600	
	0				

(to p11)

Initial tableau of NW corner method

- Repeat the above steps, we have the following tableau.
- Stop. Since all allocated have been assigned

Table B-2
The Initial NW Corner Solution

From \ To	A	B	C	Supply
1	[6] 150	[8]	[10]	150
2	[7] 50	[11] 100	[11] 25	175
3	[4]	[5]	[12] 275	275
Demand	200	100	300	600

Ensure that all columns and rows added up to its respective totals.

(to p8)

The Minimum cell cost method

Here, we use the following steps:

Steps:

- Step 1 Find the cell that has the least cost
- Step 2: Assign as much as allocation to this cell
- Step 3: Block those cells that cannot be allocated
- Step 4: Repeat above steps until all allocation have been assigned.

Example: (to p13)

Step 1 Find the cell that has the least cost
Step 2: Assign as much as allocation to this cell

Step 1:

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

Step 2:

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5	12	275
Demand	200	100	300	600

The min cost, so allocate as much resource as possible here

Step 3 (to p14)

Step 3: Block those cells that cannot be allocated
Step 4: Repeat above steps until all allocation have been assigned.

Step 3:

Table B-1
The Transportation Tableau

From \ To	A	B	C	Supply
1	[6] --	[8]	[10]	150
2	[7] --	[11]	[11]	175
3	[4] 200	[5]	[12]	~~275~~ 75
Demand	~~200~~ 0	100	300	600

Second iteration, step 4

Table B-4
The Second Minimum Cell Cost Allocation

From \ To	A	B	C	Supply
1	[6]	[8]	[10]	150
2	[7]	[11]	[11]	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

(to p15)

The initial solution

Table B-5
The Initial Solution

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

- Stop. The above tableau is an initial tableau because all allocations have been assigned

(to p8)

Vogel's approximation method

Operational steps:

Step 1: for each column and row, determine its penalty cost by subtracting their two of their least cost

Step 2: select row/column that has the highest penalty cost in step 1

Step 3: assign as much as allocation to the selected row/column that has the least cost

Step 4: Block those cells that cannot be further allocated

Step 5: Repeat above steps until all allocations have been assigned

Example (to p17)

subtracting their two of their least cost

Step 1

Table B-6
The VAM Penalty Costs

From \ To	A	B	C	Supply	
1	6	8	10	150	2 (8-6)
2	7	11	11	175	4 (11-7)
3	4	5	12	275	1 (5-4)
Demand	200	100	300	600	
	2 (6-4)	3 (8-5)	1 (11-10)		

(to p18)

Steps 2 & 3

Step 2:

Highest penalty cost

Table B-6
The VAM Penalty Costs

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7	11	11	175	4
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

(to p19)

Step 3: this has the least cost

Step 4

Table B-7
The Initial VAM Allocation

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7 175	11 ---	11 ---	~~175~~	
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

(to p20)

Step 5
Second Iteration

Table B-8
The Second VAM Allocation

From \ To	A	B	C	Supply	
1	6	8 ---	10	150	4
2	7 175	11 ---	11 ---	175	
3	4	5 100	12	275	8
Demand	200	100	300	600	
	2		2		

(to p21)

3rd Iteration of VAM

Table B-9
The Third VAM Allocation

From \ To	A	B	C	Supply
1	6 ---	8 ---	10	150
2	7 175	11 ---	11 ---	175
3	4 25	5 100	12	275
Demand	200	100	300	600
			2	

(to p22)

Initial tableau for VAM

Table B-10

The Initial VAM Solution

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

(to p8)

Optimal solution?

Initial solution from:

Northeast cost, total cost =$5,925

The min cost, total cost =$4,550

VAM, total cost = $5,125

(note: here, we are not saying the second one always better!)

It shows that the second one has the min cost, but is it the optimal solution? (to p24)

Solution methods

- We need a method, like the simplex method, to check and obtain the optimal solution
- Two methods:

1. Stepping-stone method (to p25)
2. Modified distributed method (MODI) (to p30)

(to p7)

Stepping-stone method

Let consider the following initial tableau from the Min Cost algorithm

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

There are
Non-basic variables

These are basic variables

(to p26)

Question: How can we introduce
a non-basic variable into basic variable?

Introduce
a non-basic variable into basic variables

- Here, we can select any non-basic variable as an entry and then using the “+ and –” steps to form a closed loop as follows:

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	(6)	(8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	(5) 75	(12)	275
Demand	200	100	300	600

let consider this non basic variable (→ cell 1A)

Then we have

(to p27)

Stepping stone

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	+ 6	− 8 25	10 125	150
2	7	11	11 175	175
3	− 4 200	+ 5 75	12	275
Demand	200	100	300	600

(to p28)

The above saying that, we add min value of all –ve cells into cell that has “+” sign, and subtracts the same value to the “-ve” cells
Thus, max –ve is min (200,25) = 25, and we add 25 to cell A1 and A3, and subtract it from B1 and A3

Stepping stone

Table B-19

The Second Iteration of the Stepping-Stone Method

From \ To	A	B	C	Supply
1	6 25	8	10 125	150
2	7	11	11 175	175
3	4 175	5 100	12	275
Demand	200	100	300	600

The above tableaus give min cost = 25*6 + 120*10 + 175*11
175*4 + 100* 5 = $4525

(to p29)

We can repeat this process to all possible non-basic cells in that above tableau until one has the min cost! ➡ NOT a Good solution method

Getting optimal solution

- In such, we introducing the next algorithm called Modified Distribution (MODI)

(to p24)

Modified distributed method (MODI)

- It is a modified version of stepping stone method
- MODI has two important elements:
  1. It determines if a tableau is the optimal one
  2. It tells you which non-basic variable should be firstly considered as an entry variable
  3. It makes use of stepping-stone to get its answer of next iteration

  – How it works? ▶ (to p31)

Procedure (MODI)

Step 0: let $u_{i,}$ v , c_{ij} variables represent rows, columns, and cost in the transportation tableau, respectively

Step 1: (a) form a set of equations that uses to represent all basic variables

$$u_i + vj = c_{ij}$$

(b) solve variables by assign one variable = 0

Step2: (a) form a set of equations use to represent non-basic variable (or empty cell) as such

$$c_{ij} – u_i – vj = k_{ij}$$

(b) solve variables by using step 1b information

Step 3: Select the cell that has the most –ve value in 2b

Step 4: Use stepping-stone method to allocate resource to cell in step 3

Step 5: Repeat the above steps until all cells in 2a has no negative value

Example (to p32)

(to p24)

MODI

Consider to this initial tableau:

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

Step 0: let ui, v , cij variables represent rows, columns, and cost in the transportation tableau, respectively

(to p33)

Step 0

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	(6)	(8) 25	(10) 125	150
$u_2 =$	2	(7)	(11)	(11) 175	175
$u_3 =$	3	(4) 200	(5) 75	(12)	275
	Demand	200	100	300	600

C_{3A}

Step 1: (a) form a set of equations that uses to represent all basic variables

$u_i + v_j = c_{ij}$

(to p34)

ui + vj = cij

$$u_i + v_j = c_{ij}$$

The value c_{ij} is the unit transportation cost for cell *ij*. For example, the formula for cell 1B is

$$u_1 + v_B = c_{1B}$$

and, since $c_{1B} = 8$,

$$u_1 + v_B = 8$$

The formulas for the remaining cells that presently contain allocations are

x_{1C}: $u_1 + v_C = 10$

x_{2C}: $u_2 + v_C = 11$

x_{3A}: $u_3 + v_A = 4$

x_{3B}: $u_3 + v_B = 5$

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	6	8 25	10 125	150
$u_2 =$	2	7	11	11 175	175
$u_3 =$	3	4 200	5 75	12	275
	Demand	200	100	300	600

(to p35)

(b) solve variables by assign one variable = 0

Set one variable = 0

Now there are five equations with six unknowns. To solve these equations, it is necessary to assign only one of the unknowns a value of zero. Thus, if we let $u_1 = 0$, we can solve for all remaining u_i and v_j values.

$$
\begin{aligned}
x_{1B}:\quad u_1 + v_B &= 8 \\
0 + v_B &= 8 \\
v_B &= 8 \\
x_{1C}:\quad u_1 + v_C &= 10 \\
0 + v_C &= 10 \\
v_C &= 10 \\
x_{2C}:\quad u_2 + v_C &= 11 \\
u_2 + 10 &= 11 \\
u_2 &= 1 \\
x_{3B}:\quad u_3 + v_B &= 5 \\
u_3 + 8 &= 5 \\
u_3 &= -3 \\
x_{3A}:\quad u_3 + v_A &= 4 \\
-3 + v_A &= 4 \\
v_A &= 7
\end{aligned}
$$

Because we added an non-basic variable

(to p36)

Step2: (a & b)

Step2: (a & b)

	v_j	$v_A = 7$	$v_B = 8$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	[6]	[8] 25	[10] 125	150
$u_2 = 1$	2	[7]	[11]	[11] 175	175
$u_3 = -3$	3	[4] 200	[5] 75	[12]	275
	Demand	200	100	300	600

Next, we use the following formula to evaluate all *empty cells:*

$$c_{ij} - u_i - v_j = k_{ij}$$

where k_{ij} equals the cost increase or decrease that would occur by allocating to a cell.

For the *empty cells* in Table B-26, the formula yields the following values:

$$x_{1A}: \quad k_{1A} = c_{1A} - u_1 - v_A = 6 - 0 - 7 = -1$$
$$x_{2A}: \quad k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 7 = -1$$
$$x_{2B}: \quad k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 - 8 = +2$$
$$x_{3C}: \quad k_{3C} = c_{3C} - u_3 - v_C = 12 - (-3) - 10 = +5$$

Note this may look difficult and complicated, however, we can add these V=values into the above tableau as well (to p37)

Step2: (a & b), alternative

u_i	v_j / From \ To	$v_A = 7$ A	$v_B = 8$ B	$v_C = 10$ C	Supply
$u_1 = 0$	1	[6] -1	[8] 25	[10] 125	150
$u_2 = 1$	2	[7] -1	[11] +2	[11] 175	175
$u_3 = -3$	3	[4] 200	[5] 75	[12] +5	275
	Demand	200	100	300	600

Next, we use the following formula to evaluate all *empty cells:*

$$c_{ij} - u_i - v_j = k_{ij}$$

6-0-7

(to p38)

Step 3: Select the cell that has the most –ve value in 2b

Step3

	v_j	$v_A = 7$	$v_B = 8$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	[6] -1	[8] 25	[10] 125	150
$u_2 = 1$	2	[7] -1	[11] +2	[11] 175	175
$u_3 = -3$	3	[4] 200	[5] 75	[12] +5	275
	Demand	200	100	300	600

Next, we use the following formula to evaluate all *empty cells:*

$$c_{ij} - u_i - v_j = k_{ij}$$

(to p39)

Select either one, (Why?)
These cells mean, introduce it will reduce the min z to -1 cost unit

Step 4: Use stepping-stone method

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	+ [6]	− [8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	− [4] 200	+ [5] 75	[12]	275
Demand	200	100	300	600

From here we have ….

(to p40)

Step 4: Use stepping-stone method

Table B-19
The Second Iteration of the Stepping-Stone Method

From \ To	A	B	C	Supply
1	[6] 25	[8]	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 175	[5] 100	[12]	275
Demand	200	100	300	600

(to p41)

Step 5: we repeat steps 1-4 again for the above tableau, we have

Step 5

Table B-27
The Second Iteration of the MODI Solution Method

u_i	v_j	$v_A =$	$v_B =$	$v_C =$	
	From \ To	A	B	C	Supply
$u_1 =$	1	[6] 25	[8]	[10] 125	150
$u_2 =$	2	[7]	[11]	[11] 175	175
$u_3 =$	3	[4] 175	[5] 100	[12]	275
	Demand	200	100	300	600

$$x_{1A}: \quad u_1 + v_A = 6$$
$$0 + v_A = 6$$
$$v_A = 6$$
$$x_{1C}: \quad u_1 + v_C = 10$$
$$0 + v_C = 10$$
$$v_C = 10$$
$$x_{2C}: \quad u_2 + v_C = 11$$
$$u_2 + 10 = 11$$
$$u_2 = 1$$

$$x_{3A}: \quad u_3 + v_A = 4$$
$$u_3 + 6 = 4$$
$$u_3 = -2$$
$$x_{3B}: \quad u_3 + v_B = 5$$
$$-2 + v_B = 5$$
$$v_B = 7$$

(to p42)

Step 5 cont

Table B-28
The New u_i and v_j Values for the Second Iteration

	v_j	$v_A = 6$	$v_B = 7$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	(6) 25	(8)	(10) 125	150
$u_2 = 1$	2	(7)	(11)	(11) 175	175
$u_3 = -2$	3	(4) 175	(5) 100	(12)	275
	Demand	200	100	300	600

The cost changes for the empty cells are now computed using the formula $c_{ij} - u_i - v_j = k_{ij}$.

$$x_{1A}:\quad k_{1B} = c_{1B} - u_1 - v_B = 8 - 0 - 7 = +1$$
$$x_{2A}:\quad k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 6 = 0$$
$$x_{2B}:\quad k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 - 7 = +3$$
$$x_{3C}:\quad k_{3C} = c_{3C} - u_3 - v_C = 12 - (-2) - 10 = +4$$

All positives
STOP

Because none of these values is negative, the solution shown in Table B-28 is optimal. However, as in the stepping-stone method, cell 2A with a zero cost change indicates a multiple optimal solution.

(to p31)

Important Notes

- When start solving a transportation problem using algorithm, we need to ensure the following:

1. Alternative solution (to p44)
2. Total demand ≠ total supply (to p45)
3. Degeneracy (to p48)
4. others (to p52)

(to p7)

Alternative solution

- When on the following k = 0

$$c_{ij} – u_i – vj = k_{ij}$$

Why?

(to p43)

Total demand ≠ total supply

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	350	

Note that, total demand=650, and total supply = 600

How to solve it?

(to p46)

We need to add a dummy row and assign o cost to each cell as such ..

Dd≠ss

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Dummy	0	0	0	50
Demand	200	100	350	650

Extra row, since Demand > supply

Other example ……

(to p47)

Dd≠ss

Table B-30
An Unbalanced Model
(Supply > Demand)

From \ To	A	B	C	Dummy	Supply
1	6	8	10	0	150
2	7	11	11	0	175
3	4	5	12	0	375
Demand	200	100	300	100	700

Extra column is added

(to p43)

Degeneracy

In all the tableaus showing a solution to the wheat transportation problem, the following condition was met.

$$m \text{ rows} + n \text{ columns} - 1 = \text{the number of cells with allocations}$$

ie, basic variables in the tableau

Example …..

(to p49)

Degeneracy

Table B-11
The Minimum Cell Cost Solution

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

m rows + n column – 1 = the number of cells with allocations
3 + 3 - 1 = 5 (five basic variables, and above has 5 as well!)

It satisfied.

If failed? …… considering ….. (to p50)

Degeneracy

Table B-31
he Minimum Cell Cost Initial Solution

From \ To	A	B	C	Supply
1	[6]	[8] 100	[10] 50	150
2	[7]	[11]	[11] 250	250
3	[4] 200	[5]	[12]	200
Demand	200	100	300	600

The tableau shown in Table B-31 does not meet the condition

$$m + n - 1 = \text{the number of cells with allocations}$$
$$3 + 3 - 1 = 5 \text{ cells}$$

(note above has only 4 basic variable only!)

If not matched, then we select an non-basic variable with least cheapest cost and considered it as a new basic variable with assigned 0 allocation to it

(to p51)

Degeneracy

Table B-32
The Initial Solution

From \ To	A	B	C	Supply
1	6 / 0	8 / 100	10 / 50	150
2	7	11	11 / 250	250
3	4 / 200	5	12	200
Demand	200	100	300	600

Added this
Note: we pick this over others because it has the least cost for the Min Z problem!

(to p43)

others

1. When one route cannot be used
   - Assign a big M cost to its cell

If 10 changed to
Cannot delivered
Then we assigned
M value here

Table B-32
The Initial Solution

From \ To	A	B	C	Supply
1	[6] 0	[8] 100	[10] 50	150
2	[7]	[11]	[11] 250	250
3	[4] 200	[5]	[12]	200
Demand	200	100	300	600

(to p43)

Tutorials

- Module B
  - 1, 5, 8, 13, 21, 34
  - these questions are attached in the following slides

1

1. Green Valley Mills produces carpet at plants in St. Louis and Richmond. The carpet is then shi
to two outlets located in Chicago and Atlanta. The cost per ton of shipping carpet from each o
two plants to the two warehouses is as follows.

	To	
From	*Chicago*	*Atlanta*
St. Louis	$40	$65
Richmond	70	30

The plant at St. Louis can supply 250 tons of carpet per week; the plant at Richmond can supply 400 tons per week. The Chicago outlet has a demand of 300 tons per week, and the outlet at Atlanta demands 350 tons per week. The company wants to know the number of tons of carpet to ship from each plant to each outlet in order to minimize the total shipping cost. Solve this transportation problem.

5. Given a transportation problem with the following costs, supply, and demand, find the initial solution using the minimum cell cost method and Vogel's approximation model. Is the VAM solution optimal?

	To			
From	*1*	*2*	*3*	Supply
A	$ 6	$ 7	$ 4	100
B	5	3	6	180
C	8	5	7	200
Demand	135	175	170	

8. Consider the following transportation problem.

	To			
From	*1*	*2*	*3*	**Supply**
A	$ 6	$ 9	$ 7	130
B	12	3	5	70
C	4	11	11	100
Demand	80	110	60	

a. Find the initial solution using the minimum cell cost method.
b. Solve using the stepping-stone method.

13. A manufacturing firm produces diesel engines in four cities—Phoenix, Seattle, St. Louis, and Detroit. The company is able to produce the following numbers of engines per month.

Plant	*Production*
1. Phoenix	5
2. Seattle	25
3. St. Louis	20
4. Detroit	25

Three trucking firms purchase the following numbers of engines for their plants in three cities.

Firm	*Demand*
A. Greensboro	10
B. Charlotte	20
C. Louisville	15

The transportation costs per engine ($100s) from sources to destinations are shown in the following table. However, the Charlotte firm will not accept engines made in Seattle, and the Louisville firm will not accept engines from Detroit; therefore, these routes are prohibited.

	To		
From	*A*	*B*	*C*
1	$ 7	$ 8	$ 5
2	6	10	6
3	10	4	5
4	3	9	11

a. Set up the transportation tableau for this problem. Find the initial solution using VAM.

b. Solve for the optimal solution using the stepping-stone method. Compute the total minimum cost.

c. Formulate this problem as a linear programming model.

21. A large manufacturing company is closing three of its existing plants and intends to transfer some of its more skilled employees to three plants that will remain open. The number of employees available for transfer from each closing plant is as follows.

Closing Plant	Transferable Employees
1	60
2	105
3	70
Total	235

The following number of employees can be accommodated at the three plants remaining open.

Open Plants	Employees Demanded
A	45
B	90
C	35
Total	170

Each transferred employee will increase product output per day at each plant as shown in the following table. The company wants to transfer employees so as to ensure the maximum increase in product output.

From	To *A*	*B*	*C*
1	5	8	6
2	10	9	12
3	7	6	8

a. Find the initial solution using VAM.
b. Solve using MODI.

34. Orient Express is a global distribution company that transports its clients' products to customers in Hong Kong, Singapore, and Taipei. All of the products Orient Express ships are stored at three distribution centers, one in Los Angeles, one in Savannah, and one in Galveston. For the coming month the company has 450 containers of computer components available at the Los Angeles center, 600 containers available at Savannah, and 350 containers available in Galveston. The company has orders for 600 containers from Hong Kong, 500 containers from Singapore, and 500 containers from Taipei. The shipping costs per container from each U.S. port to each of the overseas ports are shown in the following table.

U.S. Center Distribution	Overseas Port		
	Hong Kong	*Singapore*	*Taipei*
Los Angeles	$300	$210	$340
Savannah	490	520	610
Galveston	360	320	500

The Orient Express as the overseas broker for its U.S. customers is responsible for unfulfilled orders, and it incurs stiff penalty costs from overseas customers if it does not meet an order. The Hong Kong customers charge a penalty cost of $800 per container for unfulfilled demand, Singapore customers charge a penalty cost of $920 per container, and Taipei customers charge $1,100 per container. Formulate and solve a transportation model to determine the shipments from each U.S. distribution center to each overseas port that will minimize shipping costs. Indicate what portion of the total cost is a result of penalties.