The Assignment Model

The situation can be illustrated by the assignment of workers to jobs, where any worker may undertake any job, albeit with varying degrees of skill

A job that happens to match a worker's skill costs less than that in which the operator is not as skillful

The objective of the model is to determine the optimum (least-cost) assignment of workers to jobs

The Assignment Model

The general assignment model with n workers and n jobs

The element c_{ij} represents the cost of assigning worker i to job j

If #workers != #jobs, add fictitious workers or fictitious jobs to make them equal

$$
\begin{matrix}
c_{11}, & c_{12}, & c_{13}, & c_{14}, & \cdots & c_{1n} \\
c_{21}, & c_{22}, & c_{23}, & c_{24}, & \cdots & c_{2n} \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
c_{n1}, & c_{n2}, & c_{n3}, & c_{n4}, & \cdots & c_{nn}
\end{matrix}
$$

The Assignment Model

The assignment model is a special case of the transportation model

- Workers represent sources
- Jobs represent destinations
- Supply at each source = 1
- Demand at each destination = 1

The cost of transporting each worker is c_{ij}

As all supplies and demands are 1, can be solved in a simpler way

The Assignment Model

Three children, John, Karen, and Tim, want to earn some pocket money

Their father has chosen three chores:

- mowing the lawn
- painting the garage
- washing the family cars

To avoid arguments, he asks them to submit (secret) bids for what they feel was a fair pay for each of the three chores.

The Assignment Model

The bids he receives are

	Mow	Paint	Wash
John	$15	$10	$9
Karen	$9	$15	$10
Tim	$10	$12	$8

How do we assign the chores ?

Solve this special instance of the transportation model using the Hungarian method

The Hungarian Method

Step 1. For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row

Step 2. For the matrix resulting from step 1, identify each column's minimum, and sub-tract it from all the entries of the column

Step 3. Identify the optimal assignment as the one associated with the zero elements of the matrix obtained in step 2

The Hungarian Method

Compute the row minimum for each row

	Mow	Paint	Wash	min
John	$15	$10	$9	$9
Karen	$9	$15	$10	$9
Tim	$10	$12	$8	$8

Subtract the minimum from each respective row

The Hungarian Method

Compute the row minimum for each row

	Mow	Paint	Wash	min
John	$6	$1	$0	$9
Karen	$0	$6	$1	$9
Tim	$2	$4	$0	$8

Subtract the minimum from each respective row

The Hungarian Method

Compute the column minimum for each column

	Mow	Paint	Wash
John	\$6	\$1	\$0
Karen	\$0	\$6	\$1
Tim	\$2	\$4	\$0
min	\$0	\$1	\$0

Subtract the minimum from each respective column

The Hungarian Method

Compute the column minimum for each column

	Mow	Paint	Wash
John	$6	$0	$0
Karen	$0	$5	$1
Tim	$2	$3	$0
min	$0	$1	$0

Underlined zeroes give optimal assignment

Jon – paint, Karen – mow, Tim – wash

Cost is $9 + $10 + $8 = $27

The Hungarian Method

Suppose we had 4 children and 4 chores:

	1	2	3	4
1	$1	$4	$6	$3
2	$9	$7	$10	$9
3	$4	$5	$11	$7
4	$8	$7	$8	$5

Row minima are $1, $7, $4, $5 - subtract

The Hungarian Method

Suppose we had 4 children and 4 chores:

	1	2	3	4
1	\$0	\$3	\$5	\$2
2	\$2	\$0	\$3	\$2
3	\$0	\$1	\$7	\$3
4	\$3	\$2	\$3	\$0

Column minima are \$0, \$0, \$3, \$0 - subtract

The Hungarian Method

Suppose we had 4 children and 4 chores:

	1	2	3	4
1	$0	$3	$2	$2
2	$2	$0	$0	$2
3	$0	$1	$4	$3
4	$3	$2	$0	$0

Is there an optimal assignment ?

The Hungarian Method

Add the following step to the procedure:

Step 2a.

If no feasible assignment can be secured from steps 1 and 2,

(i) Draw the *minimum* number of straight lines in the last reduced matrix that will cover *all* the zero entries

(ii) Select the *smallest* uncovered element;
subtract it from every uncovered element;
add it to every element at the intersection of two lines

(iii) If no feasible assignment can be found, repeat step 2a. Otherwise, go to step 3

The Hungarian Method

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0

There are six zeroes – no more than two in any one row or column – we need at least three lines

Subtract 1 from each uncovered element

The Hungarian Method

	1	2	3	4
1	0	2	1	1
2	2	0	0	2
3	0	0	3	2
4	3	2	0	0

There are six zeroes – no more than two in any one row or column – we need at least three lines

Subtract 1 from each uncovered element

The Hungarian Method

	1	2	3	4
1	$\underline{0}$	2	1	1
2	2	0	$\underline{0}$	2
3	0	$\underline{0}$	3	2
4	3	2	0	$\underline{0}$

Underscores give optimal assignment

Subtract 1 from each uncovered element