MCE415
Heat and Mass Transfer

Lecture 03: 18/09/2017

Dr. Ayokunle O. Balogun
balogun.ayokunle@lmu.edu.ng

Class: Monday (12 – 2 pm)
Venue: B13

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Forced Convection

- Flow across cylinders and spheres
- Thermal Entrance Region
- Laminar flow in Pipes
- Turbulent Flow in Pipes

Recommended textbook

- Fundamentals of Thermal-Fluid Sciences by Cengel Y.A., Turner R.H., & Cimbala J.M. 3rd edition

Conceptual Understanding

Which is more likely to have a higher heat transfer coefficient?

Flow Across Cylinders and Spheres

- The flow across cylinders and spheres are very complex due to ***flow separation*** and can not be handled analytically.

- The complexity in the local Nusselt number for a flow across a cylinder is demonstrated in the graph below

- Among several others, the average Nu for a flow across a cylinder has proposed by Churchill and Bernstein is given below

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- For flow over sphere we have

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

- For $3.5 \leq \mathrm{Re} \leq 80{,}000$ and $0.7 \leq \mathrm{Pr} \leq 380$

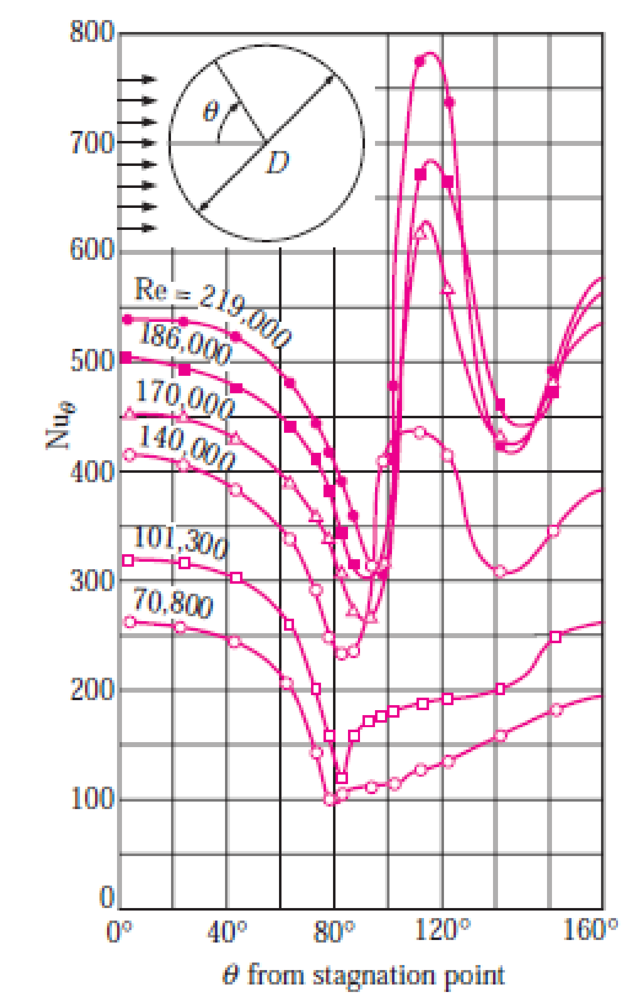

Fig 1: Variation of the local heat transfer coefficient along the circumference of a cylinder in a cross-flow of air

Flow Across Cylinders and Spheres

- The average Nu for a cylinder can be expressed compactly as

$$\mathrm{Nu}_{cyl} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

- Where n = $^1/_3$ and the experimentally determined constants C and *m* are given in the table below. The characteristics length *D* for Reynolds and Nusselt numbers are also given.

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross-flow

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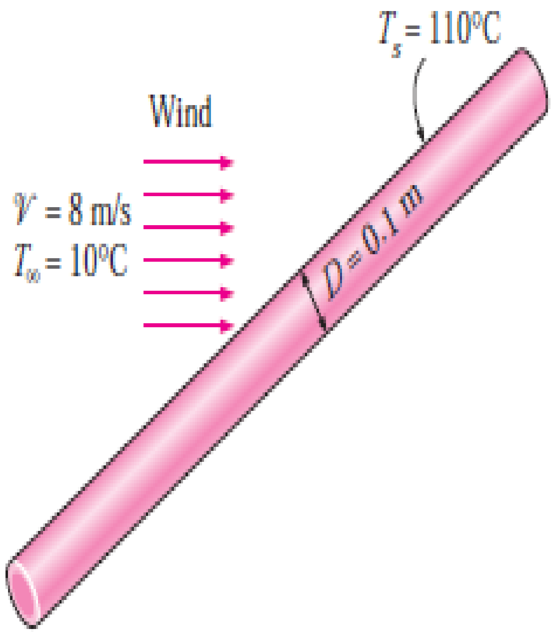
Cross section of the cylinder	Fluid	Range of Re	Nusselt number
Circle	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$\mathrm{Nu} = 0.989\mathrm{Re}^{0.330}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.911\mathrm{Re}^{0.385}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.683\mathrm{Re}^{0.466}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.193\mathrm{Re}^{0.618}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.027\mathrm{Re}^{0.805}\,\mathrm{Pr}^{1/3}$
Square	Gas	5000–100,000	$\mathrm{Nu} = 0.102\mathrm{Re}^{0.675}\,\mathrm{Pr}^{1/3}$
Square (tilted 45°)	Gas	5000–100,000	$\mathrm{Nu} = 0.246\mathrm{Re}^{0.588}\,\mathrm{Pr}^{1/3}$
Hexagon	Gas	5000–100,000	$\mathrm{Nu} = 0.153\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$
Hexagon (tilted 45°)	Gas	5000–19,500 19,500–100,000	$\mathrm{Nu} = 0.160\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.0385\mathrm{Re}^{0.782}\,\mathrm{Pr}^{1/3}$
Vertical plate	Gas	4000–15,000	$\mathrm{Nu} = 0.228\mathrm{Re}^{0.731}\,\mathrm{Pr}^{1/3}$
Ellipse	Gas	2500–15,000	$\mathrm{Nu} = 0.248\mathrm{Re}^{0.612}\,\mathrm{Pr}^{1/3}$

- Pls note that these relations are essentially for smooth surfaces.

Example

1. A long 10 cm diameter steam pipe whose external surface temperature is 110 °C passes through some open area that is not protected against the winds (Fig. below). Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.
2. A 25 cm diameter stainless steel ball (ρ = 8055 kg/m3, *Cp* 480 J/kg °C) is removed from the oven at a uniform temperature of 300 °C. The ball is then subjected to the flow of air at 1 atm pressure and 25 °C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200 °C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.



Flow in Pipes

- Liquids or gases flow in ducts or pipes for the purpose of cooling or heating. For example refrigeration and steam generation (boilers)

- To establish whether the flow is laminar or turbulent the Reynolds number is evaluated as follows

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\upsilon}$$

- Where V_m is the mean velocity, and $\nu = \mu/\rho$ is the kinematic viscousity of the fluid

Transition from laminar to turbulent depends on 1. ***Reynolds number***, 2. ***surface roughness***, 3. ***pipe vibrations*** and 4. ***fluctuations in the flow***.

- Flow is considered laminar for Re < 2300, fully turbulent for Re > 4000 and transitional in between.

Flow in Pipes

- For fluid flow in a pipe during heating or cooling, it is convenient to work with a *mean* or an *average temperature*.
- This is evaluated from the energy conservation principle and it yields:

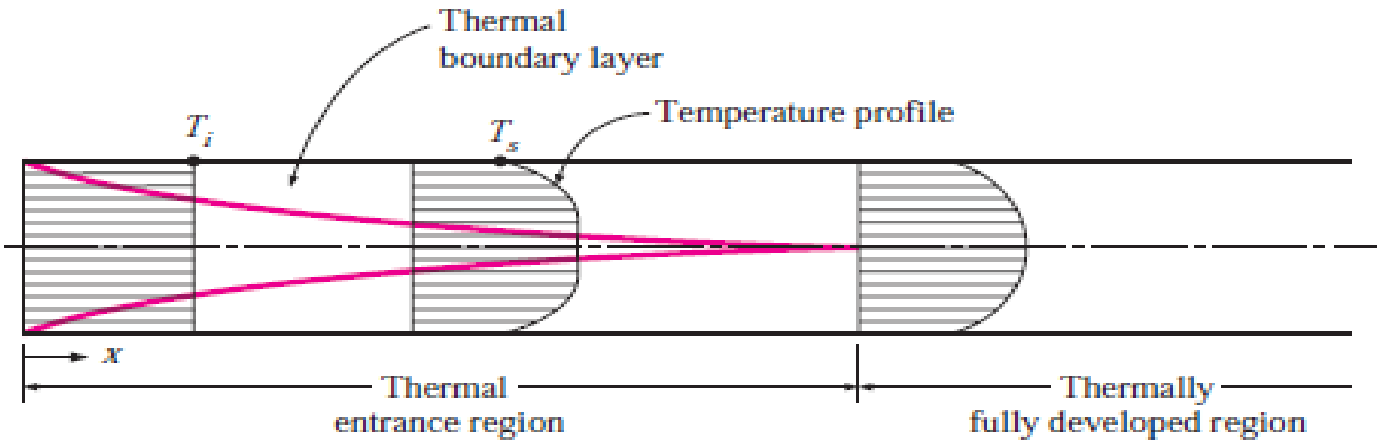
$$T_m = \frac{\int_{\dot{m}} C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T (\rho \mathcal{V} 2 \pi r \, dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2} \int_0^R T(r, x) \, \mathcal{V}(r, x) \, r \, dr$$

- The fluid properties for flow in a pipe are determined at the *bulk mean fluid temperature* as an average of mean temperature at inlet and exit

$$T_b = (T_{m,i} + T_{m,e})/2$$

Thermal Entrance Region

- Fluid at a uniform temp entering a pipe at a different surface temp will lead to convection heat transfer and the initiation of VBL and TBL.
- For the VBL revise your fluid mechanics. The thickness of the boundary layer increases along the direction of flow until it reaches the center of the tube and fills it.



The development of the thermal boundary layer in a tube (The fluid in the tube is being cooled)

- **Thermal entrance region**: the region of flow over which the thermal boundary develops and reaches the tube center.
- The length of the region is called **thermal entry length**. Flow in this region is called *thermally developing flow*.

Thermal Entrance Region

In the ***fully developed flow***, i.e., where the flow is fully developed both ***hydrodynamically*** and ***thermally***, the velocity profile and the dimensionless temperature remain unchanged.

- The mathematical relation is given as

Hydrodynamically fully developed: $\dfrac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r)$

Thermally fully developed: $\dfrac{\partial}{\partial x}\left[\dfrac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0$

- Deducing from the expression above, in a thermally fully developed region, the surface heat flux may be expressed as

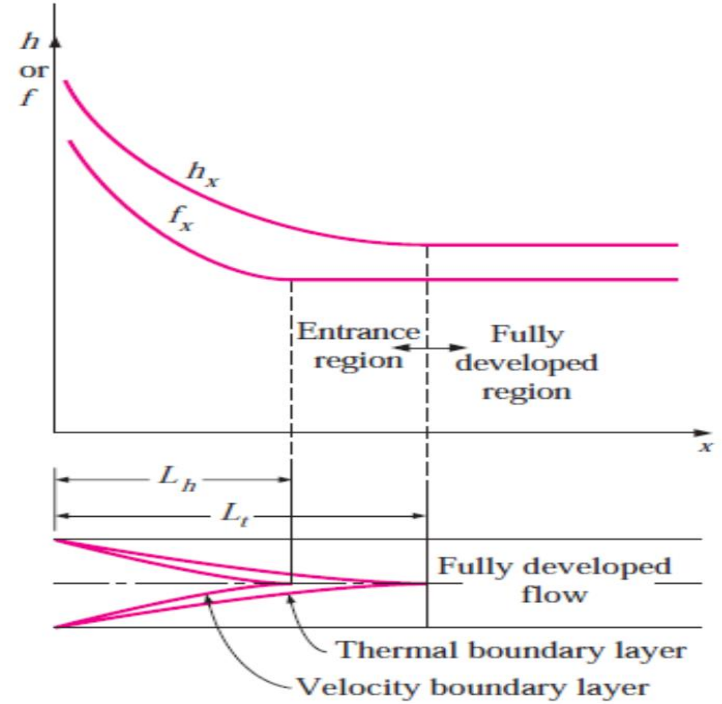
$$\dot{q}_s = h_x(T_s - T_m) = k\left.\frac{\partial T}{\partial r}\right|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)\big|_{r=R}}{T_s - T_m}$$

- In conclusion, the local convection coefficient, h_x, is constant (does not vary with x) in a thermally fully developed region , ∴, friction and convection coefficients remain constant in the fully developed region

Thermal Entrance Region

- **Prandtl Number:** is a measure of the growth of the VBL to the TBL in a laminar flow in a tube.
- So fluid flow with Pr ≈ 1 (e,g gases), the VBL and the TBL essentially coincides.
- But in a situation where Pr ≫ 1 (e.g oils), the growth of the VBL surpasses the TBL. As a result the hydrodynamic entry length is smaller than the thermal entry length.

NOTE: the effect of the *entrance region* is always to *increase* the *average friction factor* and *heat transfer coefficient* for the entire tube. (Fig beside)



- This enhancement can be significant for short tubes but negligible for long ones

Entry Lengths

- In *laminar flow*, the hydrodynamic and thermal entry lengths are given approximately as

$$L_{h,\,\text{laminar}} \approx 0.05\,\text{Re}\,D$$
$$L_{t,\,\text{laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\,\text{laminar}}$$

- In *turbulent flow*, the hydrodynamic and thermal entry lengths are about the same size and are independent of the Prandtl number because of intense mixing during random fluctuations .
- There dependence on the Reynolds number is also very weak and are thus estimated as

$$L_{h,\,\text{turbulent}} \approx L_{t,\,\text{turbulent}} \approx 10D$$

General Thermal Analysis

- Convection heat transfer may occur between a flowing fluid and the wall surface of a pipe, when the outer surface is either at

Constant surface temperature (T_s = constant), e.g. during a phase change process such as boiling and condensation

- Or

Constant surface heat flux ($\dot{q}_s$ = constant), e.g. when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

- Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad (\text{W/m}^2)$$

Constant Surface Heat Flux ($\dot{q}_s = constant$)

- In the case of **($\dot{q}_s = constant$)** heat flux can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i) \qquad \text{(W)}$$

- and the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

- After a few manipulations it is shown that in a fully developed region the temperature gradient is constant and independent of x

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant}$$

- So in a circular pipe, $p = 2\pi R \text{ and } \dot{m} = \rho V_m A_c = \rho V_m (\pi R^2)$, we have

$$\text{Circular tube:} \qquad \frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_m C_p R} = \text{constant}$$

In conclusion, for a fully developed flow, in a tube, subjected to constant surface heat flux, the ${}^{dT}/_{dx}$ is independent of x and thus the temperature profile does not change along the tube

Constant Surface Temperature ($T_s = constant$)

- Heat transfer rate is expressed as Newton's law of cooling

$$\dot{Q} = hA_s \Delta T_{ave} = hA_s(T_s - T_m)_{ave} \qquad \text{(W)}$$

- Where ΔT_{ave} may be taken as **arithmetic mean temperature difference**, however, this assumes a linear variation which is hardly the case in most practical problems.
- So a suitable alternative is a **logarithmic mean temp. difference**

$$\dot{Q} = hA_s \Delta T_{ln}$$

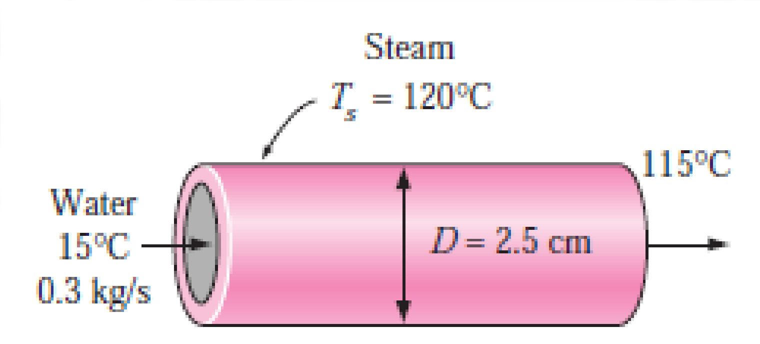
Where ΔT_{ln} is given as

$$\Delta T_{ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

- $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temp differences between the surface and the fluid at the inlet and the exit of the tube, respectively.
- It is advisable to use the logarithmic mean temperature difference for convection heat transfer when a tube is maintained at a constant surface temperature T_s

Example

3. Water enters a 2.5 cm internal diameter thin copper tube of a heat exchanger at 15 °C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120 °C. If the average heat transfer coefficient is 800 W/m^2 .°C, determine the length of the tube required in order to heat the water to 115 °C (Fig. below).



Laminar Flow in Tubes (Fully Developed Regions)

CONSTANT SURFACE HEAT FLUX, $\dot{q}_s$

- For a fully developed flow in a circular tube subjected to constant surface heat flux, the Nusselt number is expressed as

$$\text{Circular tube, laminar } (\dot{q}_s = \text{constant}): \qquad \text{Nu} = \frac{hD}{k} = 4.36$$

- ∴ for fully developed laminar flow in a circular tube subjected to $\dot{q}_s$ the Nu is constant and there is dependence on Re or Pr numbers.

CONSTANT SURFACE TEMPERATURE, T_s

- For a fully developed flow in a circular tube subjected to constant surface temperature , the Nusselt number is expressed as

$$\text{Circular tube, laminar } (T_s = \text{constant}): \qquad \text{Nu} = \frac{hD}{k} = 3.66$$

- The thermal conductivity, *k*, for the above relations should be evaluated at the bulk mean fluid temperature. Nu for fully developed laminar flow in other cross-sections are also available (P.880).

Laminar Flow in Tubes (Developing Entry Region)

CONSTANT SURFACE TEMPERATURE, T_s

- For a circular tube of length L subjected to constant surface temp , the average Nusselt number for the thermal entrance region is given as

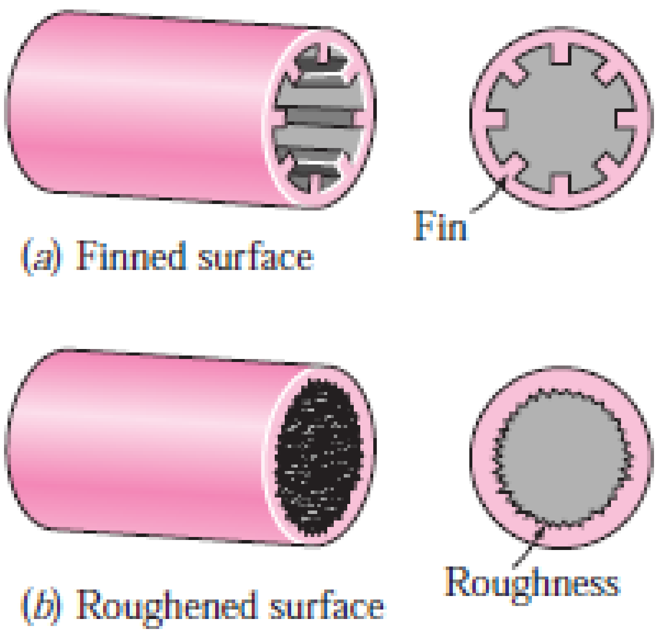
Entry region, laminar: $$\text{Nu} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

Turbulent Flow in Tubes

- For smooth tubes flow is usually turbulent at Re > 10,000. Turbulent flow is usually utilized in practice because of the higher heat transfer coefficient associated with it.
- Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.
- A couple of correlations are available in literature though (P.883-885)

Heat Transfer Enhancement

- Tube surfaces are often *roughened* , *corrugated,* or *finned* so as to enhance the convection heat transfer coefficient and thus increase the convection heat transfer rate.



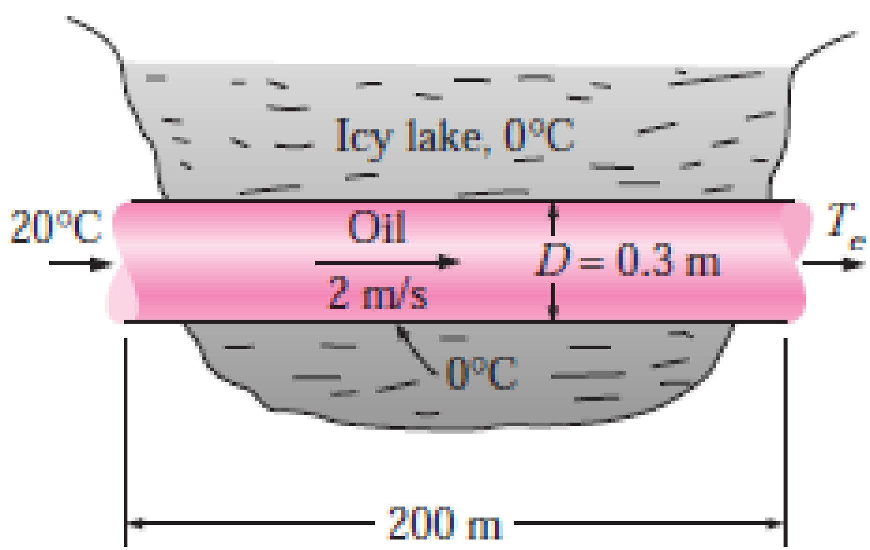
(*a*) Finned surface

(*b*) Roughened surface

- Roughening the surface also increases the friction factor and thus the power requirement for the pump or the fan.
- Convection heat transfer coefficient may also be increased by introducing pulsating flow by pulse generators, or
- inducing swirl by inserting a twisted tape into the tube, or
- by inducing secondary flows by coiling the tube

Example

4. Consider the flow of oil at 20C in a 30-cm-diameter pipeline at an average velocity of 2 m/s (Fig. below). A 200-m-long section of the pipeline passes through icy waters of a lake at 0C. Measurements indicate that the surface temperature of the pipe is very nearly 0C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

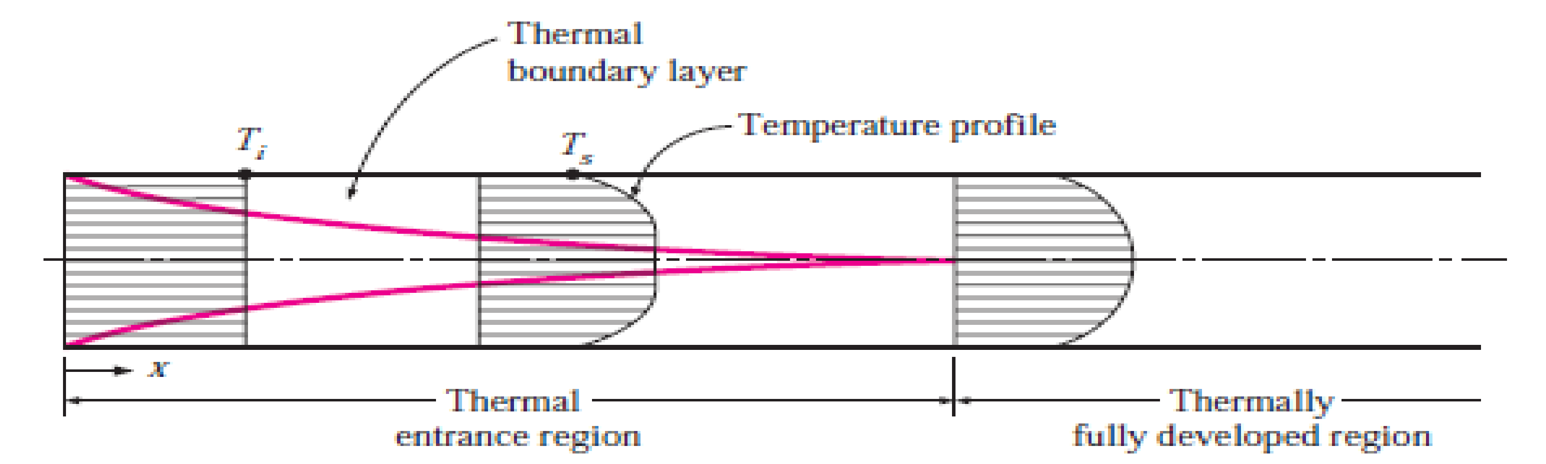


Assignment

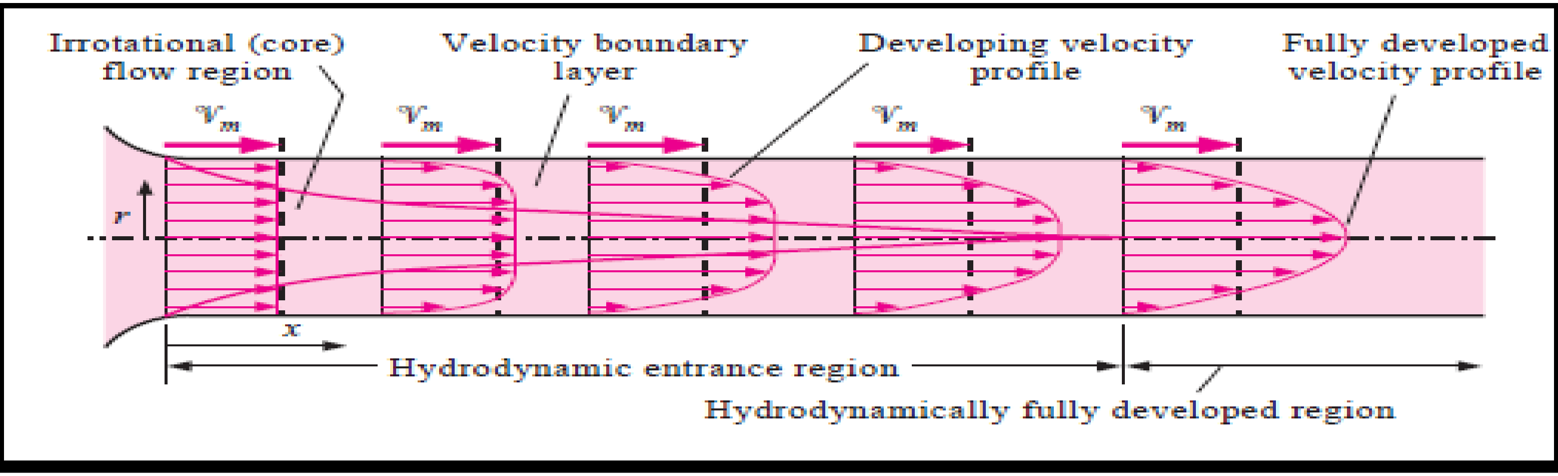
Fundamentals of thermal-fluid sciences by Cengel Y.A., Turner R.H. & Cimbala J.M. 3rd Edition

From the above textbook, PP 895-898, answer 1-2

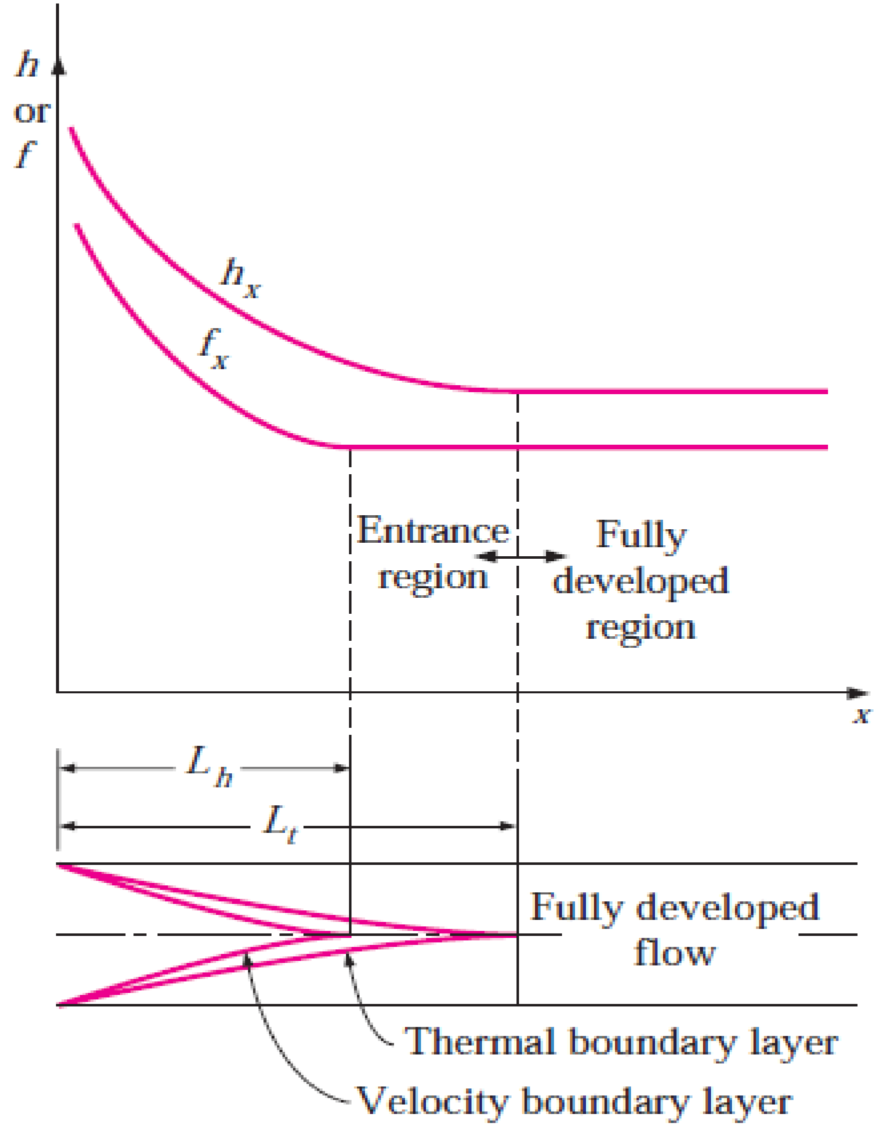
1. Question 19-32C to 19-37
2. Question 19-61C, 19-63C, 19-74, 19-75



The development of the thermal boundary layer in a tube (The fluid in the tube is being cooled)



The development of the velocity boundary layer in a tube. (The developed average velocity profile is parabolic in laminar flow, as shown, but somewhat flatter or fuller in turbulent flow



Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr > 1)