MCE415
Heat and Mass Transfer

Lecture 02: 18/09/2017

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Class: Monday (12 – 2 pm)
Venue: B13

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Forced Convection

- Physical mechanism of convection
- Nusselt number
- Thermal boundary layer
- Prandtl number
- Parallel flow over flat plates

Recommended textbook

- Fundamentals of Thermal-Fluid Sciences by Cengel Y.A., Turner R.H., & Cimbala J.M. 3rd edition

Conceptual Understanding

What is the objective of this processes?

Physical Mechanism of Convection Heat Transfer

- Convection heat transfer occurs between a surface and a fluid as a result of the presence of bulk motion in the fluid. In the absence of any bulk fluid motion then heat transfer is by conduction.
- Convection heat transfer is subdivided into ***forced*** or *natural* convection. Convection may also viewed as ***external*** or *internal*

Successive heat transfer enhancement by a blowing fan and replacement with water

- Convection heat transfer depends primarily on the three factors highlighted below:
  1. Fluid properties
  2. Solid surface
  3. Fluid flow type
- A breakdown of the factors is shown of the succeeding slide

Convection Heat Transfer

Fluid properties

1. Dynamic viscousity, μ
2. Thermal conductivity, k
3. Density, ρ
4. Specific heat capacity, c_p
5. Fluid velocity, V

Solid surface characteristics

1. Surface geometry
2. Surface roughness

Fluid Flow Type

1. Streamlined (Laminar)
2. Turbulent

Fig 1: List of variables that affect convection heat transfer

Physical Mechanism of Convection Heat Transfer

- Convection is the most complex of mechanism of heat transfer because of its dependence on many variables as shown in Fig 1.
- The complexity notwithstanding, the heat transfer rate is proportional to temperature difference and is expressed as Newton's law of cooling as:

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2) \qquad (1)$$

Or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W) \qquad (2)$$

Where,

- h = convection heat transfer coefficient, $W/m^2.C$
- A_s= heat transfer surface area, m^2
- T_s= temperature of the surface, °C
- T_∞= temperature of the fluid sufficiently far from the surface, °C

- **NOTE**: h depends on the several variables early mentioned and its determination is rather difficult.

Physical Mechanism of Convection Heat Transfer

- The motion of fluid over solid surfaces assumes zero velocity at the point of contact with the surface due to viscous effects. This phenomenon is referred to as a **no-slip condition (NSC)**.
- The no-slip condition leads to the development of the **velocity profile**, i.e. variation in the velocity of adjacent layers of fluid flow.
- The region adjacent to the wall where this viscous effect is significant is called the **boundary layer** (Fig 2).
- The fluid property responsible for NSC and the development of the BL is ***viscousity***.
- ***Zero-velocity*** at wall surface and ***surface drag*** are consequences of the NSC.

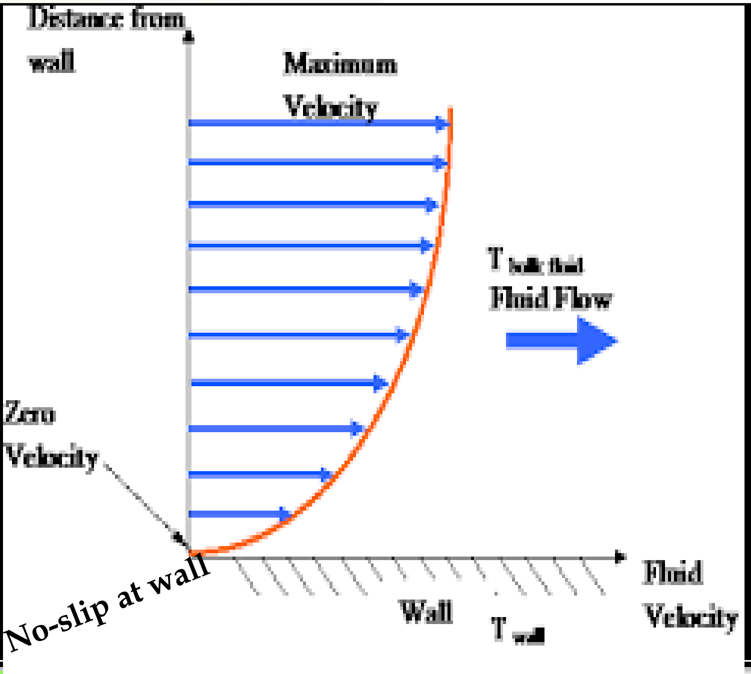

Fig 2: Velocity Profile of Boundary Layer

Physical Mechanism of Convection Heat Transfer

- As a result of the NSC, heat transfer from the surface to the adjacent fluid layer is by pure conduction.

WHY?

This may be expressed as (Eq 3)

$$\dot{q}_{conv} = \dot{q}_{cond} = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad ({}^{W}/{}_{m^2}) \qquad (3)$$

- ∴ equating (1) and (3) for the heat flux yields (Eq 4)

$$h = \frac{-k(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (W/m^2.°C) \qquad (4)$$

for the determination of the heat transfer coefficient when the temperature distribution within the fluid is known.

- The heat transfer coefficient, h, usually varies along the flow ($x-$) direction, ∴, it is determined by averaging of the *local* heat transfer coefficient over the entire surface area A_s or length L as (Eq 5).

$$h = \frac{1}{A_s}\int_{A_s} h_{local}\, dA_s \quad \text{and } h = \frac{1}{L}\int_0^L h_x\, dx \qquad (5)$$

Nusselt Number

- The **Nusselt number**, Nu, is a dimensionless quantity that measures the ratio of heat transfer by convection to conduction as expressed as (Eq 6)

$$Nu = \frac{hL_c}{k} \qquad (6)$$

- The Nusselt number represents the enhancement of heat transfer by convection relative to conduction. Thus the higher Nu the more effective the convection
- An $Nu = 1$ indicates pure conduction for the heat transfer across a the fluid layer.
- The enhancement of heat transfer by forced convection in daily life are varied and they include:

Thermal Boundary Layer

- Recall the development of the velocity boundary layer, which is defined as the region in which the fluid velocity varies from zero to $0.99V$.
- Similarly, a thermal boundary layer develops as shown in Fig 2.
- The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the **thermal boundary layer (TBL)**.
- The *thickness*, δ_t, of the TBL at any location along the surface is *the distance from the surface at which thee temperature difference* $T - T_s$ *equals* $0.99(T_\infty - T_s)$.

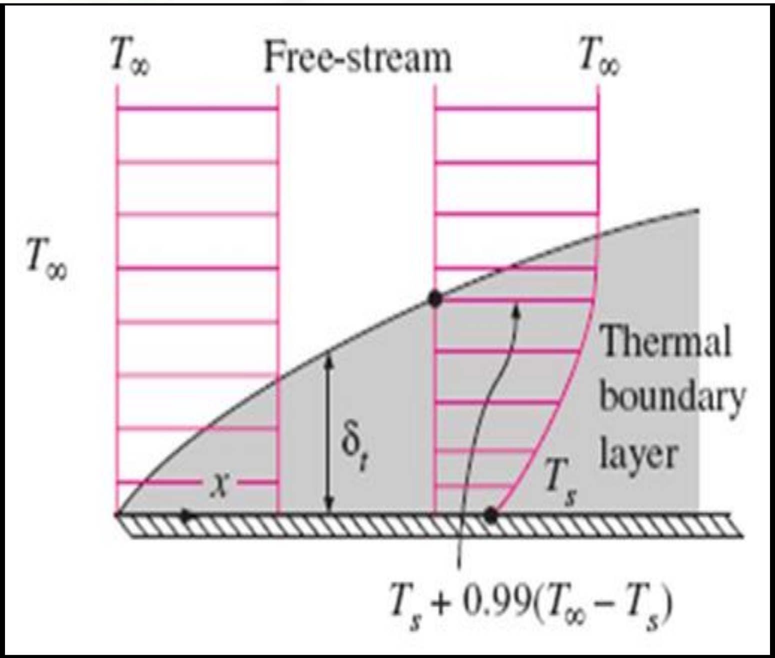

Fig 3: Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface)

Salient points on TBL & VBL

What would T be for a special case of $T_s = 0$? How does it relate to the velocity boundary layer, u?

- The *thickness*, δ_t, of the TBL increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream
- The convection heat transfer rate anywhere along the surface is directly elated to the temperature gradient at that location.
- Therefore the shape of the temperature profile in the TBL dictates the convection at transfer between the solid and the fluid flowing over
- In a flow over a heated (cooled) surface , both the VBL and TBL develops simultaneously.
- Note that fluid velocity influences the development of the temperature profile, therefore the development of the VBL relative to the TBL will have a strong effect on the convection heat transfer.

Prandtl Number

- The relative thickness of the velocity and the thermal boundary layer is defined by the dimensionless parameter **Prandlt number**, Pr, as (Eq 7)

$$\Pr = \frac{\textit{Molecular diffusivity of momentum}}{\textit{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \qquad (7)$$

- The Pr for gases is about 1. This indicates that both momentum and heat dissipate through the fluid at about the same rate.
- Heat diffuses quickly in liquid metals ($Pr \ll 1$) and very slowly in oils ($Pr \gg 1$), relative to momentum.

Consequently *the TBL is much thicker for liquid metals and much thinner for oils relative to the VBL*.

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004 - 0.030
Gases	0.7 – 1.0
Water	1.7 – 13.7
Light organic fluids	5 – 50
Oils	50 – 100,000
Glycerin	2,000 – 100,000

Parallel Flow Over Flat Plates

- Transition from laminar to turbulent regions in the velocity boundary layer during a flow over a flat plate is as shown in Fig 4.
- This transition depends on *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and *type of fluid* among other things. The transition is best characterized by Reynolds number at a distance x from the leading edge.

- The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

$$Re_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu} \qquad (8)$$

Note, Re varies along x direction reaching a $Re_L = VL/\nu$ at the end of the plate

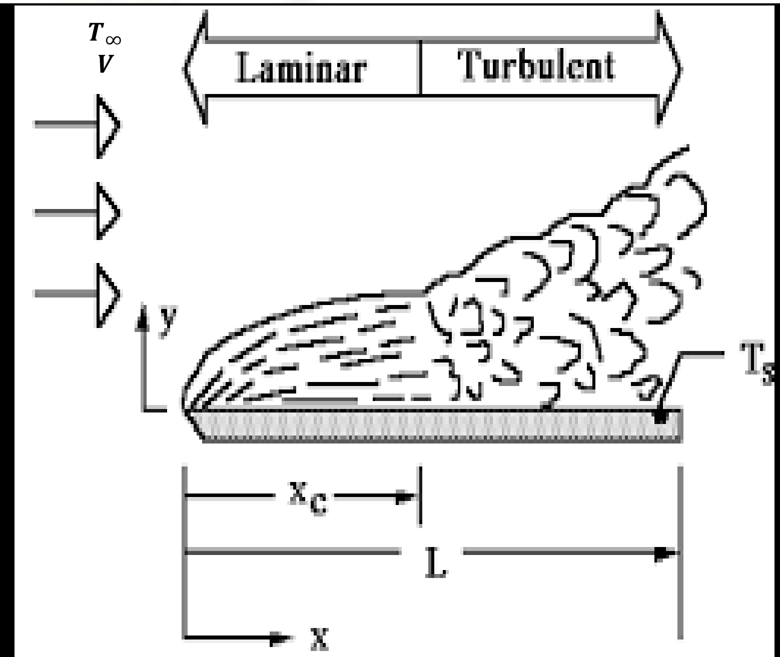

Fig 4: Laminar and turbulent regions of the boundary layer during flow over a flat plate

Parallel Flow Over Flat Plates

- To establish the critical point of transition between laminar to turbulent regions in engineering analysis, the critical Reynolds number is usually set at Eq 9

$$Re_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5 \qquad (9)$$

- Though the actual value of engineering critical Reynolds number for a flat plate may vary from 10^5 to 3×10^6, depending on the *surface roughness*, *turbulence level*, and *the variation of pressure* along the surface.

The local Nusselt number

- The *local* Nusselt number at a location x for laminar flow over a flat plate is expressed as Eq 10

$$Laminar: \quad Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{1/3} \qquad Pr > 0.6 \qquad (10)$$

- and the corresponding turbulent flow is Eq 11

$$Turbulent: \quad Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{1/3} \qquad 0.6 \leq Pr \leq 60 \qquad (11)$$

$$5 \times 10^5 \leq Re_x \leq 10^7$$

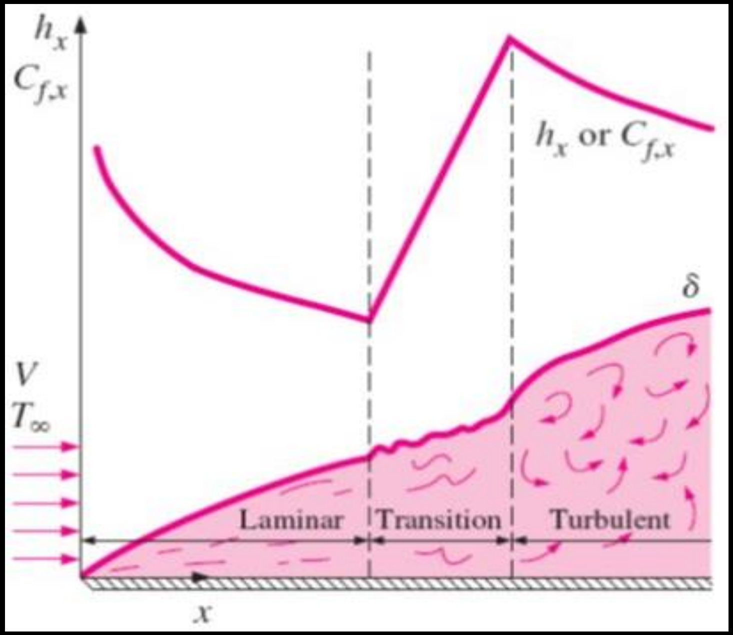

Fig 5: The variation of the local friction & heat transfer coefficients for flow over a flat plate

- Note the $h_x \propto Re_x^{0.5}$ and thus $\propto x^{-0.5}$ for laminar flow.
- $\therefore h_x \rightarrow \infty$ at leading edge ($x = 0$) & $\downarrow_{ses}$ by a factor of $x^{-0.5}$ in the flow direction.
- In Fig 5, the local $C_{f,x}$ & h_x reaches maximum when flow becomes fully turbulent and $\downarrow_{ses}$ by a factor of $x^{-0.2}$

The average Nusselt number

- The *average* Nusselt number over the entire is obtained putting Eqs 10 & 11 into Eq. 5 to yield Eq 12

$$\text{Laminar:} \quad Nu = \frac{hL}{k} = 0.664 Re_L^{0.5} Pr^{1/3} \qquad Re_L < 5 \times 10^5 \qquad (12)$$

- and the corresponding turbulent flow is Eq 13

$$\text{Turbulent:} \quad Nu = \frac{hL}{k} = 0.037 Re_x^{0.8} Pr^{1/3} \qquad 0.6 \le Pr \le 60 \qquad (13)$$

$$5 \times 10^5 \le Re_L \le 10^7$$

- Eq 12 represents the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate.
- Eq 13 is for the flow that is *turbulent* over the entire plate, or when the laminar flow region is too small relative to the turbulent flow.
- Where both laminar and turbulent flows contribute significantly, Eq. 14 is the average Nusselt number applicable

$$Nu = \frac{hL}{k} = (0.037 Re_L^{0.8} - 871) Pr^{1/3} \qquad 0.6 \le Pr \le 60 \qquad (14)$$

$$5 \times 10^5 \le Re_L \le 10^7$$

The Nusselt number

- A single correlation that applies to all fluids, including metal liquids have been proposed by Churchill and Ozoe (1973) as Eq 15

$$Nu_x = \frac{h_x x}{k} = \frac{0.3387 Pr^{1/3} Re_x^{0.5}}{\left[1+(0.0468/Pr)^{2/3}\right]^{1/4}} \qquad (15)$$

- These relations are for *isothermal* surfaces but could also be used for non-isothermal surfaces by assuming constant surface temperature at some average value.
- The assumptions for these relation includes
    1. Smooth surface, and
    2. Free stream region is turbulent free
- The effect of variable properties can be accounted for by evaluating properties at the film temperature

Flat Plate with Unheated Starting Length

- In cases where the starting section is unheated as shown in Fig 6, which means no heat transfer at $0 < x < \xi$
- This implies that the VBL starts to develop at the leading edge ($x = 0$), while the onset of the TBL is where heating starts ($x = \xi$).
- For flat plate with heated section maintained at a constant temperature ($T = T_s$ constant for $x > \xi$)
- Using integral solution methods, the local Nu respectively for laminar and turbulent is Eq 16 & 17 for $x > \xi$

$$Nu_x = \frac{Nu_{x(for\ \xi=0)}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} = \frac{0.332 Re_x^{0.5} Pr^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} \quad (16)$$

$$Nu_x = \frac{Nu_{x(for\ \xi=0)}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}} = \frac{0.0296 Re_x^{0.8} Pr^{1/3}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}} \quad (17)$$

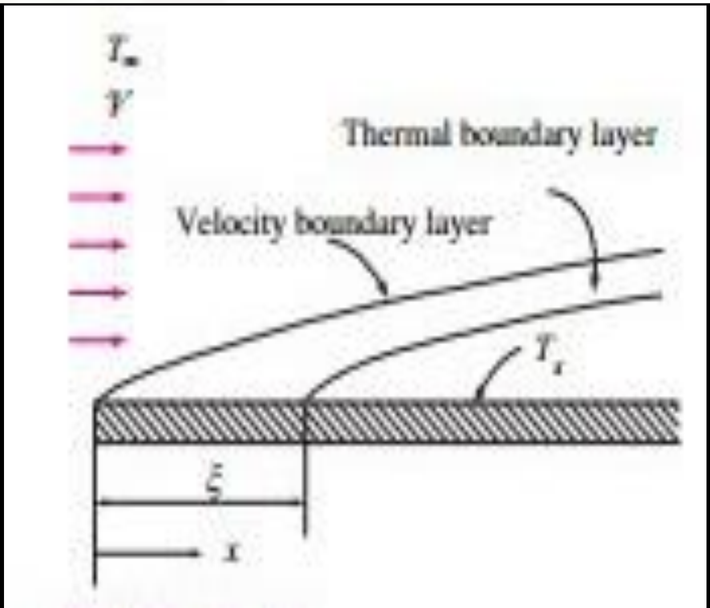

Fig 6: Flow over a flat plate with an unheated starting length

Flat Plate with Unheated Starting Length

- The determination of the average Nusselt number for the heated section of the plate is obtained by integrating Eqs 16 & 17. This can not be done analytically, therefore, the numerical integration yields Eqs 18 & 19 respectively for laminar and turbulent flow

Laminar: $$h = \frac{2\left[1-(\xi/x)^{3/4}\right]}{1-\xi/L} h_{x=L} \qquad (18)$$

Turbulent: $$h = \frac{5\left[1-(\xi/x)^{9/10}\right]}{4(1-\xi/L)} h_{x=L} \qquad (19)$$

- Eq 18 is the average heat transfer coefficient for the entire heated section of the plate when flow is laminar over the entire plate. Note that for ξ=0 it reduces to $h_L = 2h_{x=L}$, as expected.
- Eq 19 is the average convection coefficient for the case of turbulent flow over the entire plate or when the laminar region is small relative to the turbulent region.

Uniform Heat Flux

- When a flat plate is subjected to uniform heat flux instead of uniform temperature, the local Nusselt number is given as

Laminar: $$Nu_x = 0.453 Re_x^{0.5} Pr^{1/3} \quad (20)$$

Turbulent: $$Nu_x = 0.0308 Re_x^{0.8} Pr^{1/3} \quad (21)$$

- These relations give values that are 36% higher for laminar flow and 4% higher for turbulent flow relative to the isothermal case
- When the palte involves an unheated starting length, the relations in Eqs 16 & 17 can still be used provided that Eqs 20 & 21 are used for Nu_x respectievly.
- When heat flux $\dot{q}_s$ is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance x are determined as

$$\dot{Q} = \dot{q}_s A_s \quad (22)$$

$$\dot{q}_s = h_x[T_s(x) - T_\infty] = T_s(x) = T_\infty + \dot{q}_s/h_x \quad (23)$$

Example

1. Engine oil at 60 °C flows over the upper surface of a 5 m long flat plate whose temperature is 20 °C with a velocity of 2m/s. determine the rate of heat transfer pet unit width of the entire plate.
2. The local atmospheric pressure in Denver, Colorado (elevation 1610 m) is 83.4 kPa. Air at this pressure and 20 °C flows with a velocity of 8 m/s over a 1.5 m x 6 m flat plate whose temperature is 140 °C. Determine the rate of heat transfer from the plate it the air flows parallel to the (a) 6 m long side (b) the 1.4 m side.
3. Repeat example 2a, using MATLAB, plot the rate of heat transfer against velocity of air form 1 – 17 m/s. Discuss the result.

Assignment

Fundamentals of thermal-fluid sciences by Cengel Y.A., Turner R.H. & Cimbala J.M. 3rd Edition

From the above textbook, PP 892-893, answer 1-2

1. Question 19-1C to 19-6C
2. Question 19-12C, 19-16 to 19-17

Forced convection