DISCTRETE CASH FLOW: END-OF-PERIOD SINGLE CASH FLOW

1. **Compounding:** is calculating an equivalent amount of money in the future given some amount of money in the present $F = P(1+i)^N$ where F, P, i and N are future amount, present amount, interest rate and number of interest periods respectively**.** $(1+i)^N$ is the **single payment compound amount factor.**

Compound Interest Formula

Year/period	**Capital at time, t (Beginning Balance)**	**Interest from t to t+1 (Interest Earned)**	**Capital growth at t+1 (Ending Balance)**
0			p
1	p	ip	$p+ip$
2	$p+ip$ or $p(1+i)$	$ip(1+i)$	$p(1+i)+ip(1+i) = p(1+i)^2$
3	$p(1+i)^2$	$ip(1+i)^2$	$p(1+i)^2 + ip(1+i)^2 = p(1+i)^3$
4	$p(1+i)^3$	$ip(1+i)^3$	$p(1+i)^3 + ip(1+i)^3 = p(1+i)^4$
.			
.			
N	$p(1+i)^{N-1}$	$ip(1+i)^{N-1}$	$p(1+i)^{N-1} + ip(1+i)^{N-1} = p(1+i)^N$

Example: What is the capital growth of N100 invested at interest rate $i = 3\%$ per year for three years.

Solution:

Year	**Capital at time, t(Beginning Balance)**	**Interest from t to t+1(Interest Earned)**	**Capital growth at t+1 (Ending Balance)**
0			N100
1	N100	N0.03(100)=N3	N103
2	N103	N0.03(103)=N3.09	N106.09
3	N106.09	N0.03(106.09)=N3.18	N109.27

Or

Capital growth = $p(1+i)^N = N100(1+0.03)^3 = N109.27$ Note that the value of the **compound amount factor** could be read from compound interest tables. For instance, the future equivalent amount, F given the present amount, P may be computed as below;

Future amount, F= $P(1+i)^N = P(F/P,i,N)$. Note that $(1+i)^N = (F/P,i,N)$ is the compound amount factor which can be read from the compound interest table.

Find F, given P = N100, i = 5% and N = 5 years. **Answer:** Open to the 5% page of the interest tables and look for the value corresponding to year, N = 5 under the F/P column. The value from the table is 1.276. Hence, Future amount, F= $P(1+i)^N = P(F/P,i,N) = N100(1.276) = N127.60$. Note that this future amount can be calculated without tables e.g. $F = P(1+i)^N = 100(1+0.05)^5 = N100(1.276) = N127.60$

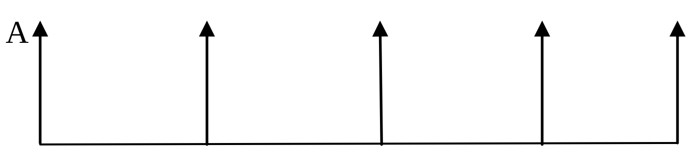
2. **<u>Discounting:</u>** is calculating an equivalent amount of money in the present amount given some amount of money in the future $P = \frac{F}{(1+i)^N} = F(1+i)^{-N}$. Note that $(1+i)^{-N} = (P/F,i,N)$ is the **single payment present worth factor.**

Find P, given F = N1000, i = 6%, N = 8 years. You can use the interest formula or the tables. **Answer:** $P = \frac{F}{(1+i)^N} = F(1+i)^{-N} = N1000(1+0.06)^{-8} = N627.40$ **or** open to the 6% page of the interest table, under the P/F column, read the value corresponding to year N = 8. The value $(P/F,i,N) = 0.6274$, therefore, P = N1000(0.6274) = N627.40.

Solving for time (interest periods) and interest rate:

a. Given F = 20, N = 5, and P = 10 find interest rate, i.
   Solution: $F = P(1+i)^N = 20 = 10(1+i)^5 \quad \therefore i = \sqrt[5]{2} - 1 = 0.1487 = 14.87\%$
b. Find N, given P = N3000, F = N6000 and i = 12%
   Solution: $F = P(1+i)^N = 6000 = 3000(1+0.12)^N = 3000(1.12)^N$
   $\log(6000/3000) = N\log(1.12)$
   $N = \left(\frac{\log 2}{\log 1.12}\right) = 6.1165$ time units

3. **<u>Uniform Series (Annuity) Equal Payment Series:</u>**



Given A, i and N find F, where A

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A,i,N).$$

The factor in the bracket i.e. $\left[\frac{(1+i)^N - 1}{i}\right]$ is known as the Uniform Series Compound Amount Factor or Equal Payment Series Compound Amount Factor.

Example: A sum of N10, 000 is invested at the end of each period for 15 periods. What is the amount in the fund after the 15th payment has been made if the interest rate, i is 10%.

Solution: $F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$

The equal payment, A= N10,000, i=10%, N=15

Hence, $F = 10{,}000\left[\frac{(1+0.1)^{15} - 1}{0.1}\right] = 10{,}000(31.7725) = N317{,}725$

Alternatively, $F = A(F/A, i, N) = 10{,}000(F/A, 10\%, 15)$. Open to 10% page of interest table under F/A pick the value that corresponds with the 15 time periods.

4. **<u>Sinking Fund:</u>** When a future amount is given with interest rate and time periods, equivalent uniform series can be calculated. Given F, i and N, calculate A

Solution: $A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$. The factor in the bracket i.e. $\left[\frac{i}{(1+i)^N - 1}\right] = (A/F, i, N)$ is known as the **Sinking Fund Factor.**

Example: How much must be invested at the end of each time period for 15 time periods, such that at the end of the 15th payment we would have N20, 000. Interest rate, i = 10%.

Solution: $A = 20{,}000\left[\frac{0.1}{(1+0.1)^{15} - 1}\right] = 20{,}000(0.0315) = N629.48$

Alternatively, $A = 20{,}000(A/F, 10\%, 15\,periods)$ and the value read from compound interest table.

5. **<u>Capital Recovery:</u>** Given P, i and N find A (equivalent uniform series).

$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$. The factor in bracket $\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = (A/P, i, N)$ is known as the **Capital Recovery Factor.**

Example: An equipment costs N50, 000 and will be used for 5 years. It will have no salvage value at the end of 5 years. Find the equivalent uniform series if interest rate, i = 10%.

Solution: $A = 50000\left[\frac{0.1(1+0.1)^5}{(1+0.1)^5 - 1}\right] = N13{,}190$

Alternatively, $A = 50{,}000(A/P, 10\%, 5)$. The capital recovery factor can be read from the compound interest table.

6. Uniform Series Present Worth Factor:

Find P, the equivalent present amount given, annuity (A), interest rate (i) and interest periods N.

$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$. The factor in the bracket is known as the **Uniform Series Present Worth Factor.**

Example: How must be invested today in order to yield returns of N2500 at the end of each and every year for 8 years at i = 10%.

Solution: A = N2500, i = 10% and N = 8 years

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = 2500\left[\frac{(1.1)^8 - 1}{0.1(1.1)^8}\right] = N13{,}337$$

Alternatively, $P = 2500(A/P, 10\%, 8)$. The uniform series present worth factor can be read from the compound interest table.