LECTURE NOTE ON MCE 326.
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Flow in Closed Conduits (Pipes)

A). PARAMETERS INVOLVED IN THE STUDY OF FLOW THROUGH CLOSED CONDUITS.

In this course the determination of drop in pressure in pipe flow systems due to friction is attempted.

INTRODUCTION.

Fluids are conveyed (transported) through closed conduits in numerous industrial processes. It is found necessary to design the pipe system to carry a specified quantity of fluid between specified locations with minimum pressure loss. It is also necessary to consider the initial cost of the piping system. The flow may be laminar with fluid flowing in an orderly way, with layers not mixing macroscopically. The momentum transfer and consequent shear induced is at the molecular level by pure diffusion. Such flow is encountered with very viscous fluids. Blood flow through the arteries and veins is generally laminar. Laminar condition prevails up-to a certain velocity in fluids flowing in pipes. The flow turns turbulent under certain conditions with macroscopic mixing of fluid layers in the flow. At any location the velocity varies about a mean value. Air flow and water flow in pipes are generally turbulent. **The flow is controlled by (*i*) pressure gradient (*ii*) the pipe diameter or hydraulic mean diameter (*iii*) the fluid properties like viscosity and density and (*iv*) the pipe roughness.** The velocity distribution in the flow and the state of the flow namely laminar or turbulent also influence the design. Pressure drop for a given flow rate through a duct for a specified fluid is the main quantity to be calculated. The inverse-namely the quantity flow for a specified pressure drop is to be also worked out on occasions. The basic laws involved in the study of incompressible flow are (*i*) Law of conservation of mass and (*ii*) Newton's laws of motion. Besides these laws, modified Bernoulli equation is applicable in these flows.

B). BOUNDARY LAYER CONCEPT IN THE STUDY OF FLUID FLOW.

When fluids flow over surfaces, the molecules near the surface are brought to rest due to the viscosity of the fluid. The adjacent layers also slow down, but to a lower and lower extent. This slowing down is found limited to a thin layer near the surface. The fluid beyond this layer is not affected by the presence of the surface. The fluid layer near the surface in which there is a general slowing down is defined as boundary layer. The velocity of flow in this layer increases from zero at the surface to free stream velocity at the edge of the boundary layer. The development of the boundary layer in flow over a flat plate and the velocity distribution in the layer are shown in Fig.1. Pressure drop in fluid flow is to overcome the viscous shear force which depends on the velocity gradient at the surface. Velocity gradient exists only in the boundary layer. The study thus involves mainly the study of the boundary layer. The boundary conditions are (*i*) at the wall surface, (zero thickness) the velocity is zero. (*ii*) at full thickness the velocity equals the free stream velocity (*iii*) The velocity gradient is zero at the full thickness. Use of the concept is that the main analysis can be limited to this layer.

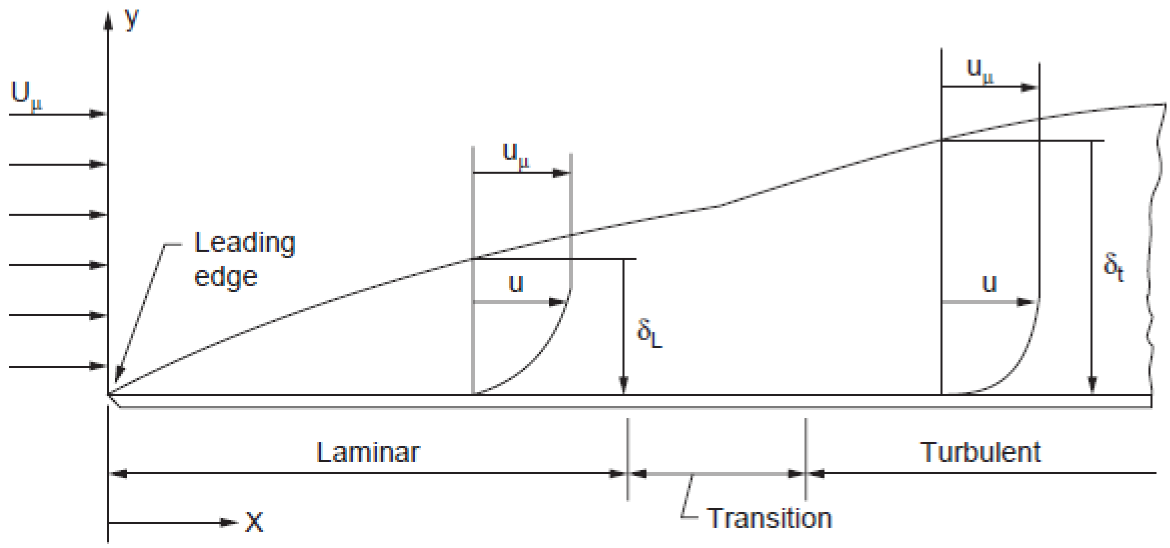
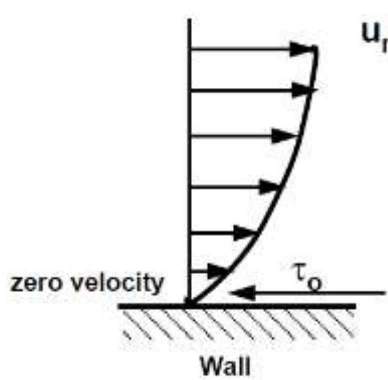


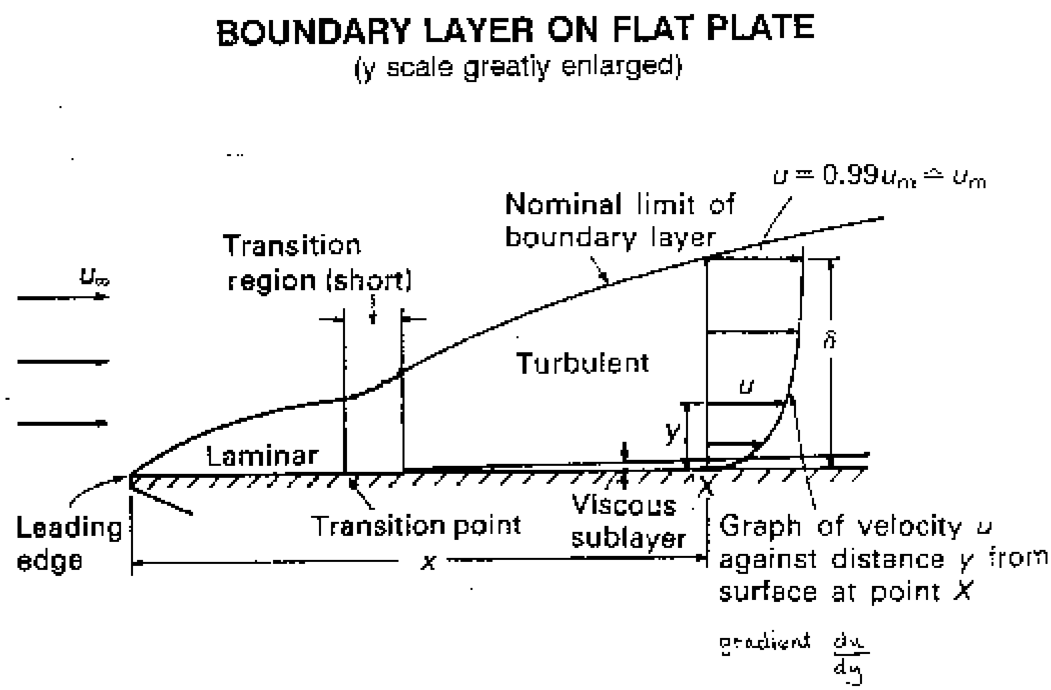
Figure. 1 Boundary Layer Development (flat-plate)

C). BOUNDARY LAYER DEVELOPMENT OVER A FLAT PLATE.

The situation when a uniform flow meets with a plane surface parallel to the flow is shown in Fig.1. At the plane of entry (leading edge) the velocity is uniform and equals free stream velocity. Beyond this point, the fluid near the surface comes to rest and adjacent layers are retarded to a larger and larger depth as the flow proceeds. The thickness of the boundary layer increases due to the continuous retardation of flow. The flow initially is laminar. There is no intermingling of layers. Momentum transfer is at the molecular level, mainly by diffusion. The viscous forces predominate over inertia forces. Small disturbances are damped out. Beyond a certain distance, the flow in the boundary layer becomes turbulent with macroscopic mixing of layers. Inertia forces become predominant. This change occurs at a value of Reynolds number (given Re = ux/v, where v is the kinematic viscosity) of about 5×10^5 in the case of flow over flat plates. Reynolds number is the ratio of inertia and viscous forces. In the turbulent region momentum transfer and consequently the shear forces increase at a more rapid rate. When a fluid flows over a stationary surface, e.g. the bed of a river, or the wall of a pipe, the fluid touching the surface is brought to rest by the shear stress τ_o at the wall. The velocity increases from the wall to a maximum in the main stream of the flow.



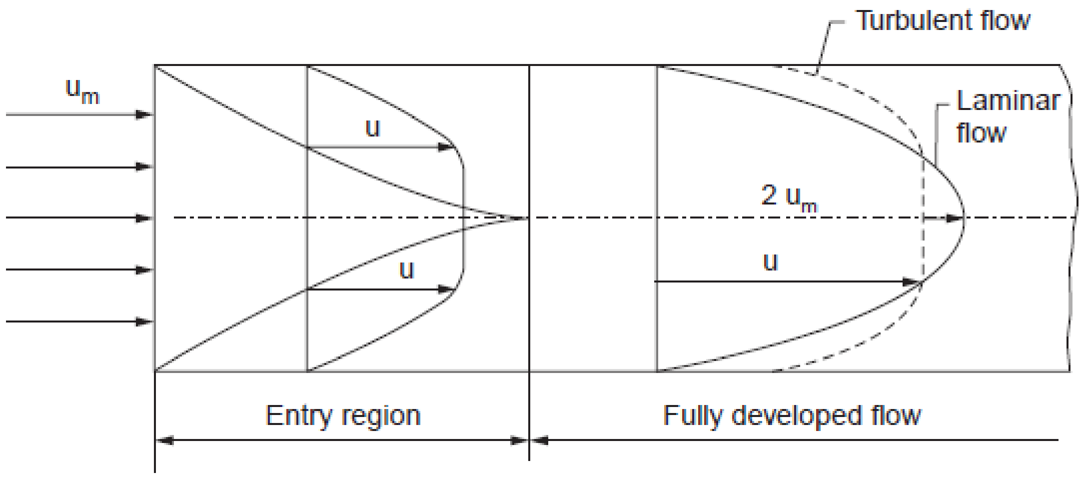
Looking at this two-dimensionally we get the above velocity profile from the wall to the centre of the flow. This profile doesn't just exit, it must build up gradually from the point where the fluid starts to flow past the surface - e.g. when it enters a pipe. If we consider a flat plate in the middle of a fluid, we will look at the build up of the velocity profile as the fluid moves over the plate. Upstream the velocity profile is uniform, (free stream flow) a long way downstream we have the velocity profile we have talked about above. This is the known as **fully developed flow**. But how do we get to that state? This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**. The stages of the formation of the boundary layer are shown in the figure below:



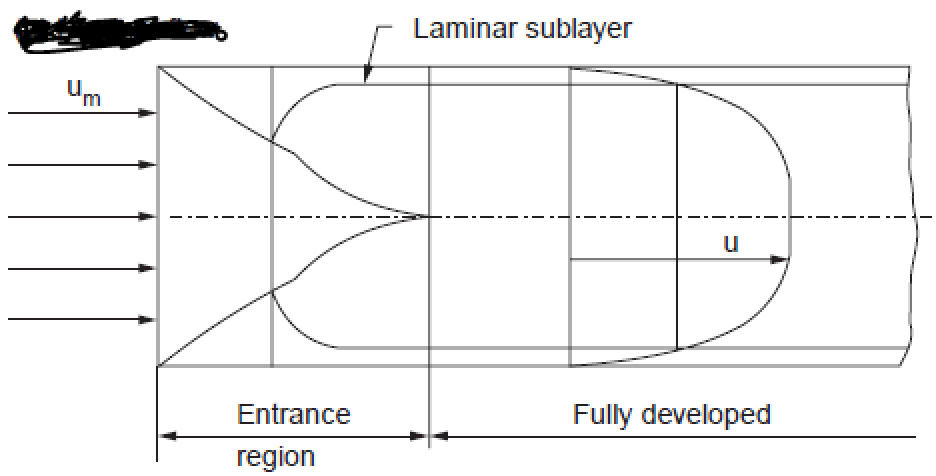
We define the thickness of this boundary layer as the distance from the wall to the point where the velocity is 99% of the "free stream" velocity, the velocity in the middle of the pipe or river. boundary layer thickness, δ = distance from wall to point where $u = 0.99\ u_{mainstream}$ .The value of δ will increase with distance from the point where the fluid first starts to pass over the boundary - the flat plate in our example. It increases to a maximum in fully developed flow. Correspondingly, the drag force D on the fluid due to shear stress τ_o at the wall increases from zero at the start of the plate to a maximum in the fully developed flow region where it remains constant. We can calculate the magnitude of the drag force by using the momentum equation. But this complex and not necessary for this course. Our interest in the boundary layer is that its presence greatly affects the flow through or round an object. So here we will examine some of the phenomena associated with the boundary layer and discuss why these occur.

D) DEVELOPMENT OF BOUNDARY LAYER IN CLOSED CONDUITS (PIPES).

In this case the boundary layer develops all over the circumference. The initial development of the boundary layer is similar to that over the flat plate. At some distance from the entrance, the boundary layers merge and further changes in velocity distribution becomes impossible. The velocity profile beyond this point remains unchanged. The distance up to this point is known as entry length. It is about 0.04 Re × *D.* The flow beyond is said to be fully developed. The velocity profiles in the entry region and fully developed region are shown in Fig. 2*a*. The laminar or turbulent nature of the flow was first investigated by Osborn Reynolds in honour of whom the dimensionless ratio of inertia to viscous forces is named. The flow was observed to be laminar till a Reynolds number value of about 2300. The Reynolds number is calculated on the basis of diameter (*ud*/*v*). In pipe flow it is not a function of length. As long as the diameter



a. Laminar flow.



b. Turbulent flow.

Figure 2 Boundary layer development (pipe flow)

is constant, the Reynolds number depends on the velocity for a given flow. Hence the value of velocity determines the nature of flow in pipes for a given fluid. The value of the flow Reynolds number is decided by the diameter and the velocity and hence it is decided at the entry itself. The development of boundary layer in the turbulent range is shown in Fig.2*b*. In this case, there is a very short length in which the flow is laminar. This length, *x*, can be calculated using the relation $ux/v = 2000$. After this length the flow in the boundary layer turns turbulent. A very thin laminar sub-layer near the wall in which the velocity gradient is linear is present all through. After some length the boundary layers merge and the flow becomes fully developed. The entry length in turbulent flow is about 10 to 60 times the diameter. The velocity profile in the fully developed flow remains constant and is generally more flat compared to laminar flow in which it is parabolic.

E). FEATURES OF LAMINAR AND TURBULENT FLOWS.

In laminar region the flow is smooth and regular. The fluid layers do not mix macroscopically (more than a molecule at a time). If a dye is injected into the flow, the dye will travel along a straight line. Laminar flow will be maintained till the value of Reynolds number is less than of the critical value (2300 in conduits and 5×10^5 in flow over plates). In this region the viscous forces are able to damp out any disturbance. The friction factor, f for pipe flow defined as $4\tau_s / (\rho u^2/2g_o)$ is obtainable as $f = 64/\mathrm{Re}$ where τ_s is the wall shear stress, u is the average velocity and Re is the Reynolds number. In the case of flow through pipes, the average velocity is used to calculate Reynolds number. The dye path is shown in Fig.3.

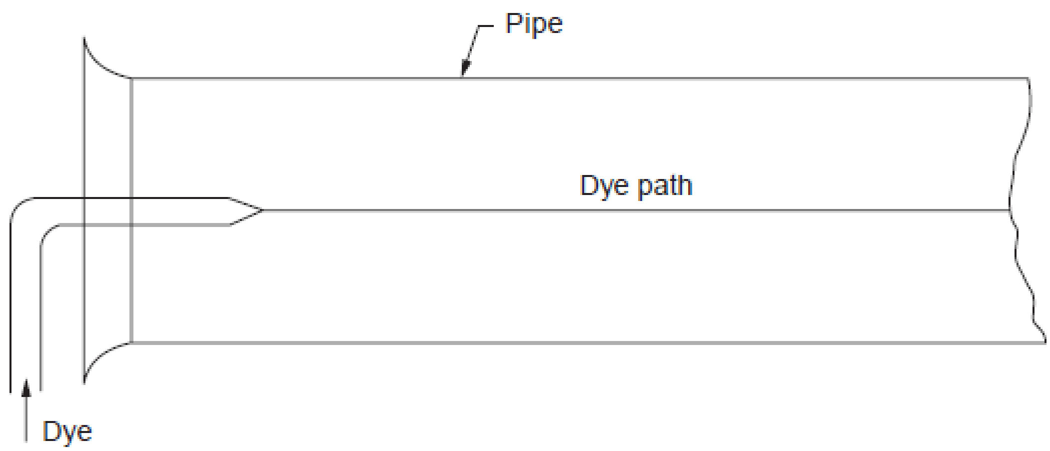


Figure 3 Reynolds Experiment.

In turbulent flow there is considerable mixing between layers. A dye injected into the flow will quickly mix with the fluid. Most of the air and water flow in conduits will be turbulent. Turbulence leads to higher frictional losses leading to higher pressure drop. The friction factor is given by the following empirical relations.

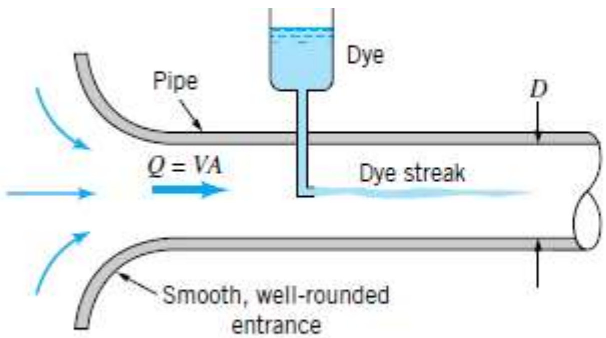
$$f = 0.316/\mathrm{Re}^{0.25} \qquad \text{for Re} < 2 \times 10^4 \qquad (1)$$

$$f = 0.186/\mathrm{Re}^{0.2} \qquad \text{for Re} > 2 \times 10^4 \qquad (2)$$

These expressions apply for smooth pipes. In rough pipes, the flow may turn turbulent below the critical Reynolds number itself. The friction factor in rough pipe of diameter D, with a roughness height of ε, is given by

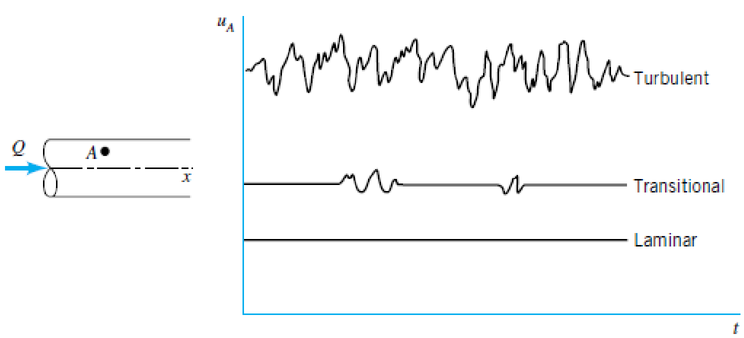
$$f = 1.325/[ln\{(\varepsilon/3.7D) + 5.74/Re^{0.9}\}]^2 \qquad (3)$$

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. **Osborne Reynolds** 11842–19122, a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus as shown in Fig. 3*a*. If water runs through a pipe of diameter D with an average velocity V, the following characteristics are observed by injecting neutrally buoyant dye as shown. For “small enough flow-rates” the dye streak (a streakline) will remain as a well-defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water. For a somewhat larger “intermediate flowrate” the dye streak fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak. On the other hand, for “large enough flow-rates” the dye streak almost immediately becomes blurred and spreads across the entire pipe in a random fashion. These three characteristics, denoted as *laminar*, *transitional*, and *turbulent* flow, respectively, are illustrated in Fig.3*b*. The curves shown in Fig.3c. represent the *x* component of the velocity as a function of time at a point *A* in the flow. The random fluctuations of the turbulent flow (with the associated particle mixing) are what disperse the dye throughout the pipe and cause the blurred appearance illustrated in Fig. 3*b*. For laminar flow in a pipe there is only one component



(*a*) Experiment to illustrate type of flow.

(*b*) Typical dye streaks.



(3c). Time dependence of fluid velocity at a point.

of velocity, For turbulent flow the predominant component of velocity is also along the pipe, but it is unsteady 1random2 and accompanied by random components normal to the pipe axis, Such motion in a typical flow occurs too fast for our eyes to follow. Slow motion pictures of the flow can more clearly reveal the irregular, random, turbulent nature of the flow. We should not label dimensional quantities as being “large” or “small,” such as “small enough flow-rates” in the preceding paragraphs. Rather, the appropriate dimensionless quantity should be identified and the “small” or “large” character attached to it. A quantity is “large” or “small” only relative to a reference quantity. The ratio of those quantities results in a dimensionless quantity. For pipe flow the most important dimensionless parameter is the Reynolds number, Re—the ratio of the inertia to viscous effects in the flow. Hence, in the previous paragraph the term flow-rate should be replaced by Reynolds number, where *V* is the average velocity in the pipe. That is, the flow in a pipe is laminar, transitional, or turbulent provided the Reynolds number is “small enough,” “intermediate,” or “large enough.” It is not only the fluid velocity that determines the character of the flow—its density, viscosity, and the pipe size are of equal importance. These parameters combine to produce the Reynolds number. The distinction between laminar and turbulent pipe flow and its dependence on an appropriate dimensionless quantity was first pointed out by Osborne Reynolds in 1883. The Reynolds number ranges for which laminar, transitional, or turbulent pipe flows are obtained cannot be precisely given. The actual transition from laminar to turbulent flow may take place at various Reynolds numbers, depending on how much the flow is disturbed by vibrations of the pipe, roughness of the entrance region, and the like. For general engineering purposes 1i.e., without undue precautions to eliminate such disturbances2, the following values are appropriate: The flow in a round pipe is laminar if the Reynolds number is less than approximately 2100. The flow in a round pipe is turbulent if the Reynolds number is greater than approximately 4000. For Reynolds numbers between these two limits, the flow may switch between laminar and turbulent conditions in an apparently random fashion (transitional flow).

F). HYDRAULICALLY “ROUGH” AND “SMOOTH” PIPES.

In turbulent flow, a thin layer near the surface is found to be laminar. As no fluid can flow up from the surface causing mixing, the laminar nature of flow near the surface is an acceptable assumption. The thickness of the layer δ_l is estimated as

$$\delta_l = 32.8\upsilon/u\sqrt{f} \qquad (4)$$

If the roughness height is ε and if $\delta_l > 6\varepsilon$, then the pipe is considered as hydraulically smooth. Any disturbance caused by the roughness is within the laminar layer and is smoothed out by the viscous forces. So the pipe is hydraulically smooth. If $\delta_l < 6\varepsilon$, then the pipe is said to be hydraulically rough. The disturbance now extends beyond the laminar layer. Here the inertial forces are predominant. So the disturbance due to the roughness cannot be damped out. Hence the pipe is hydraulically rough. It may be noted that the relative value of the roughness determines whether the surface is hydraulically rough or smooth.

Surface roughness effect

Despite its thinness, the laminar sub-layer can play a vital role in the friction characteristics of the surface. This is particularly relevant when defining pipe friction. In **turbulent** flow if the height of the roughness of a pipe is greater than the thickness of the laminar sub-layer then this increases the amount of turbulence and energy losses in the flow. If the height of roughness is less than the thickness of the laminar sub-layer the pipe is said to be smooth and it has little effect on the boundary layer. In **laminar** flow the height of roughness has very little effect.

G). CONCEPT OF “HYDRAULIC DIAMETER”: (D_h).

The frictional force is observed to depend on the area of contact between the fluid and the surface. For flow in pipes the surface area is not a direct function of the flow. The flow is a direct function of the sectional area which is proportional to the square of a length parameter. The surface area is proportional to the perimeter. So for a given section, the hydraulic diameter which determines the flow characteristics is defined by equation 5and is used in the calculation of Reynolds number.

$$D_h = 4A/P \qquad (5)$$

Where D_h is the hydraulic diameter, A is the area of flow and P is the perimeter of the section. This definition is applicable for any cross section. For circular section $D_h = D$, as the equals ($4\pi D^2/4\pi D$). For flow through ducts the length parameter in Reynolds number is the hydraulic diameter. *i.e.*,

$$\mathrm{Re} = D_h \times u/v \qquad (6)$$

Example 1 In model testing, similarity in flow through pipes will exist if Reynolds numbers are equal. Discuss how the factors can be adjusted to obtain equal Reynolds numbers. Reynolds number is defined as $Re = uD\rho/\mu$. For two different flows

$$\frac{u_1 D_1 \rho_1}{\mu_1} = \frac{u_2 D_2 \rho_2}{\mu_2} \ or \ \frac{u_1 D_1}{v_1} = \frac{u_2 D_2}{v_2}$$

As the kinematic viscosities $v1$ and $v2$ are fluid properties and cannot be changed easily (except by changing the temperature) the situation is achieved by manipulating u_2D_2 and $u_1\ D_1$

$$\frac{v_2}{v_1} = \frac{u_2 D_2}{u_1 D_1} \qquad (A)$$

this condition should be satisfied for flow similarity in ducts. Reynolds number will increase directly as the velocity, diameter and density. It will vary inversely with the dynamic viscosity of the fluid. Reynolds number can be expressed also by **Re = G.D**/μ where G is the mass velocity in kg /m^{2}s. So Reynolds number in a given pipe and fluid can be increased by increasing mass velocity. For example if flow similarity between water and air is to be achieved at 20 °C then (using v values in eqn. A)

$$\frac{1.006^{-6}}{15.06^{-6}} = \frac{velocity\ of\ water \times diameter\ in\ water\ flow}{velocity\ of\ air \times diameter\ of\ air\ flow}$$

If diameters are the same, the air velocity should be about 15 times the velocity of water for flow similarity. If velocities should be the same, the diameter should be 15 times that for water. For experiments generally both are altered by smaller ratios to keep $u \times D$ constant.

H). VELOCITY VARIATION WITH RADIUS FOR FULLY DEVELOPED LAMINAR FLOW IN PIPES.

In pipe flow, the velocity at the wall is zero due to viscosity and the value increases as the centre is approached. The variation if established will provide the flow rate as well as an average velocity. Consider an annular element of fluid in the flow as shown in Fig. 4*a*. The dimensions are: inside radius = r; outside radius = $r + d$, length = dx.

Surface area = $2\pi r dx$

Assuming steady fully developed flow, and using the relationship for force balance, the velocity being a function of radius only.

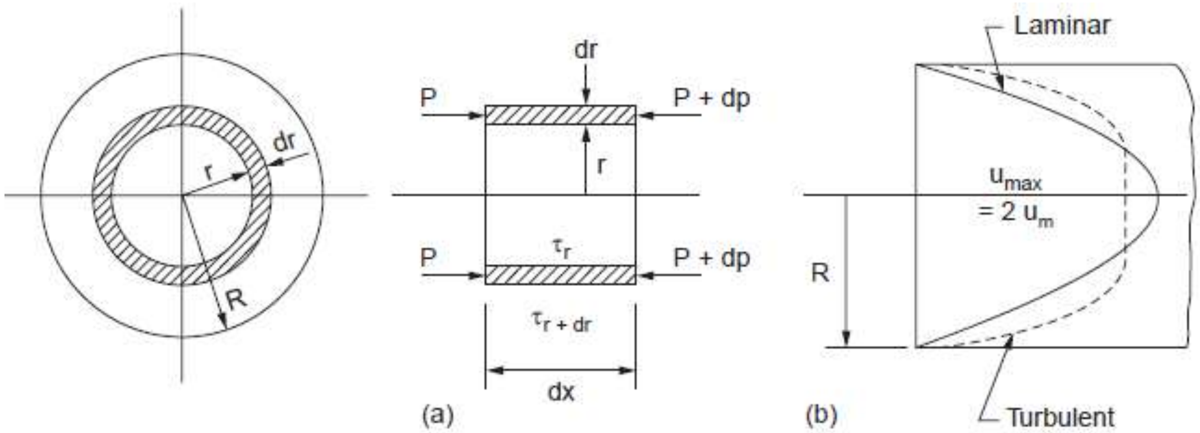


Figure 4

Net pressure force = $dp\ 2\pi r dr$

Net shear force = $\frac{d}{dr}\left(\mu\frac{du}{dr}2\pi dx\right)dr$, equating the forces and reordering,

$$\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\mu}\frac{dp}{dx}r$$

Integrating

$$r\frac{du}{dr} = \frac{1}{\mu}\frac{dp}{dx}\frac{r^2}{2} + C, \text{ at r = 0, } \therefore \text{ C = 0}$$

Integrating again and after simplification,

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{r^2}{4} + B$$

at r = R, u = 0 (at the wall)

$\therefore$ $$B = \frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}$$

$\therefore$ $$u = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}\left[1-\left(\frac{r}{R}\right)^2\right] \tag{7}$$

The velocity is maximum at r = 0,

$\therefore$ $$u_{max} = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4} \tag{8}$$

At a given radius, dividing 7and 8, we get 9, which represents parabolic distribution.

∴ $$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2 \qquad (9)$$

If the average velocity is u_{mean} then the flow is given by Q = $\pi R^2 u_{mean}$ (A)
The flow Q is also given by the integration of small annular flow streams as in the element concerned,

$$Q = \int_0^R 2\pi u r dr \text{ but u} = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Substituting and integrating between the limits 0 to *R*, and using equation *A*

$$Q = \frac{\pi R^2}{u} u_{max} = \pi R2 u_{mean} \ \therefore 2u_{mean} = u_{max}$$

The average velocity is half of the maximum velocity,

∴ $$\frac{u}{u_{max}} \ 2\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (10)$$

In turbulent flow the velocity profile is generally represented by the equation,

∴ $$\frac{u}{u_{max}} = \left(r - \frac{r}{R}\right)^{(1/n)}$$, where n varies with Reynolds number.

The average velocity is 0.79 u_{max} for n = 6 and 0.87 u_{max} for n = 10.

I). DARCY–WEISBACH EQUATION FOR CALCULATING PRESSURE DROP.

In the design of piping systems the choice falls between the selection of diameter and the pressure drop. The selection of a larger diameter leads to higher initial cost. But the pressure drop is lower in such a case which leads to lower operating cost. So in the process of design of piping systems it becomes necessary to investigate the pressure drop for various diameters of pipe for a given flow rate. Another factor which affects the pressure drop is the pipe roughness. It is easily seen that the pressure drop will depend directly upon the length and inversely upon the diameter. The velocity will also be a factor and in this case the pressure drop will depend in the square of the velocity (refer Bernoulli equation).
Hence we can say that,

$$\Delta p \propto \frac{LV^2}{2D} \qquad (11)$$

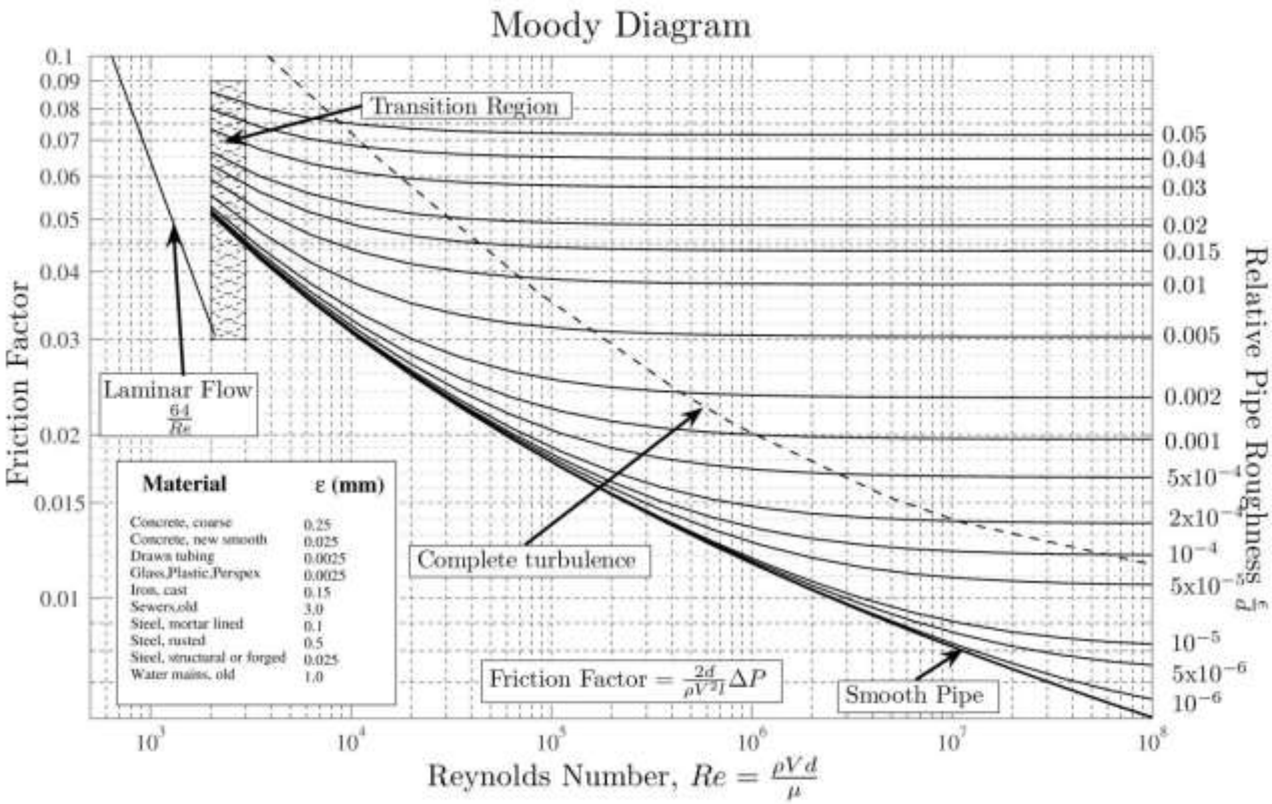
The proportionality constant is found to depend on other factors. In the process of such determination Darcy defined or friction factor *f* as;

$$f = 4\tau_0/(\rho u_m^2/2g_0) \qquad (12)$$

This quantity is dimensionless which may be checked. Extensive investigations have been made to determine the factors influencing the friction factor. It is established that in laminar flow f depends only on the Reynolds number and it is given by;

$$f = \frac{64}{R} \qquad (13)$$

In the turbulent region the friction factor is found to depend on Reynolds number for smooth pipes and both on Reynolds number and roughness for rough pipes. Some empirical equations are given in section E and also under discussions on turbulent flow. The value of friction factor with Reynolds number with roughness as parameter is available in Moody diagram, given below.



Using the definition of Darcy friction factor and conditions of equilibrium, expression for pressure drop in pipes is derived in this section. Consider an elemental length L in the pipe. The pressures at sections 1 and 2 are P_1 and P_2.

The other force involved on the element is the wall shear, τ_0, Net pressure force in the element is $(P_1 - P_2)$, Net shear force in the element is $\tau_0 \pi DL$ Force balance for equilibrium yields;

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_0 \pi DL \qquad (14)$$

From the definition friction factor,

$$f = 4\tau_0/(\rho u_m^2/2g_0)$$

$$\tau_0 = \frac{f\rho u_m^2}{8g_0}$$

Substituting and letting (P_1–P_2) to be ΔP.

$$\Delta P \times \frac{\pi D^2}{4} = \frac{f\rho u_m^2}{8g_0} \times \pi DL$$

This reduces to;

$$\Delta P = \frac{fL\rho u_m^2}{28g_0 D} \qquad 15$$

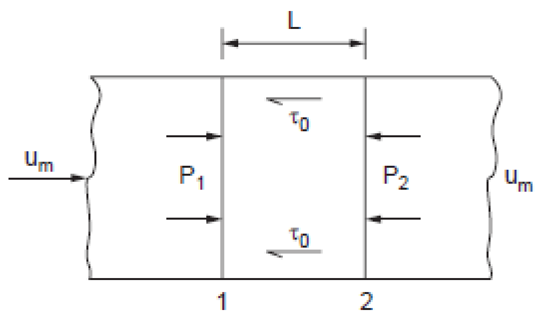


Figure 5

This equation known as Darcy-Weisbach equation and is generally applicable in most of the pipe flow problems. As mentioned earlier, the value of f is to be obtained either from equations or from Moody diagram. The diameter for circular tubes will be the hydraulic diameter D_h defined earlier in the text. It is found desirable to express the pressure drop as head of the flowing fluid. In this case as

$$h = \frac{P}{\gamma} = \frac{\mathrm{Pg_0}}{\rho g}$$

$$\Delta h = h_f = \frac{fLu_m^2}{2gD} \qquad (16)$$

The velocity term can be replaced in terms of volume flow and the equation obtained is found useful in designs as Q is generally specified in designs.

$$u_m = \frac{4Q}{\pi D^2}, u_m^2 = \frac{16Q^2}{\pi^2 D^4}$$

Substituting in (16), we get;

$$h_f = \frac{8fLQ^2}{\pi^2 g D^5} \qquad 17$$

It is found that $h_f = \frac{Q^2}{D^5}$

Another coefficient of friction C_f is defined as $C_f = f/4$

In this case; $$h_f = \frac{1}{\mu}\frac{dp}{dL}\frac{R^2}{4}\left[1-\left(\frac{r}{R}\right)^2\right] \qquad 18$$

Now a days equation 15 are more popularly used as value of f is easily available.

J). **HAGEN–POISEUILLE EQUATION FOR FRICTION DROP.**

In the case of laminar flow in pipes another equation is available for the calculation of pressure drop. The equation is derived in this section. Refer to section (H) equation (7)

$$u = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$\frac{dp}{dL}$ can be approximated to $\Delta P/L$ as the pressure drop is uniform along the length L under steady laminar flow;

Using eqn (8), $$u_{max} = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4} = 2u_m$$

$$\therefore \quad -\frac{dp}{dL} = \frac{8u_m\mu}{R^2}$$

$\therefore$ $-\frac{dp}{dL} = \frac{8u_m\mu}{R^2} = \frac{32u_m\mu}{D^2}$, Substituting for $-\frac{dp}{dL}$ as $\frac{\Delta p}{L}$

$$\Delta p = \frac{32u_mL}{D^2} \qquad 19$$

This can also be expressed in terms of volume flow rate Q as

$$Q = \frac{\pi D^2}{4} \times u_m$$

$\therefore$ $u_m = \frac{4Q}{\pi D^2}$, substituting, $\Delta p = 128\mu LQ/\pi D^4$ 20

Converting ΔP as head of fluid,

$$h_f = \frac{32\upsilon u_m L g_0}{gD^2} \qquad 21$$

This equation is known as Hagen-Poiseuille equation. g_0 is the force conversion factor having a value of unity in the SI system of unit. Also $(\mu/\rho) = \nu$. Equations 19, 20 and 21 are applicable for laminar flow only whereas Darcy-Weisbach equation (16) is applicable for all flows.

Example 2 *Using the Darcy-Weisbach equation and the Hagen Poiseuille equation obtain an expression for friction factor f, in terms of Reynolds number in laminar region.*

The equation are

$$h_f = \frac{fLu_m^2}{2gD} \text{ and } h_f = \frac{32\upsilon u_m L}{gD^2}$$

equating and simplifying a very useful relationship is obtained, namely

$$f = \frac{2\times 32\upsilon}{u_m D} = \frac{64}{Re}, \text{ as } \left(\frac{u_m D}{\upsilon}\right) = \text{Re}$$

In the laminar flow region the friction factor can be determined directly in terms of Reynolds number.

K). SIGNIFICANCE OF REYNOLDS NUMBER IN PIPE FLOW.

Reynolds number is the ratio of inertia force to viscous force. The inertia force is proportional to the mass flow and velocity *i.e.,(ρu.u*). The viscous force is proportional to μ(*du*/*dy*) or μ*u*/*D*, dividing

$$\frac{inertia\, force}{viscous\, force} = \frac{\rho u u D}{\mu u} = \frac{uD}{\upsilon}$$

Viscous force tends to keep the layers moving smoothly one over the other. Inertia forces tend to move the particles away from the layer. When viscous force are sufficiently high so that any disturbance is smoothed down, laminar flow prevails in pipes. When velocity increases, inertia forces increase and particles are pushed upwards out of the smoother path. As long as Reynolds number is below 2,300, laminar flow prevails in pipes. The friction factor in flow is also found to be a function of Reynolds number (in laminar flow, f = 64/Re).

The Reynolds number, Re, can be described as a ratio of inertial to viscous forces and note that if Re is large the diffusive terms (viscous force terms) will be small; hence, flow behavior will be dominated by the inertial forces (and possibly also pressure and body forces). A more precise way to obtain this characterization is to note that the inertial forces are associated with accelerations and thus come from $F_{inertial}$ = ma. Now observe that

$$m = \rho L^3, \text{ and } a \sim L/T^2 \sim U/T.$$

From this it follows that

$$ma \sim \frac{\rho L^3 U}{T} \sim \rho U^2 L^2 \sim inertia\ force.$$

On the other hand, we can estimate the viscous forces based on Newton’s law of viscosity:

$$\text{viscous force} = \tau A \simeq \mu \frac{du}{dy} A \sim \mu \frac{U}{L}. L^2 \sim \mu UL.$$

From this it follows that

$$\frac{inertia\, force}{viscous\, force} \sim \frac{\rho U^2 L^2}{\mu UL} = \frac{\rho UL}{\mu} = Re.$$

Boundary layers in pipes

As flow enters a pipe the boundary layer will initially be of the laminar form. This will change depending on the ration of inertial and viscous forces; i.e. whether we have laminar (viscous forces high) or turbulent flow (inertial forces high). From earlier we saw how we could calculate whether a particular flow in a pipe is laminar or turbulent using the Reynolds number.

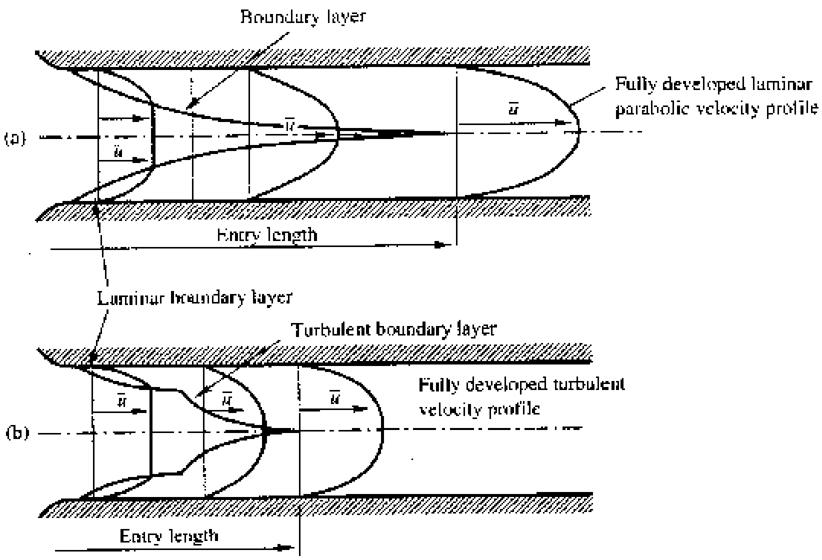
$Re = \frac{\rho u d}{\mu}$

(ρ = density, u = velocity, μ = viscosity, d = pipe diameter)

Laminar flow: Re < 2000

Transitional flow: 2000 < Re < 4000

Turbulent flow: Re > 4000



If we only have laminar flow the profile is parabolic – as proved in earlier lectures – as only the first part of the boundary layer growth diagram is used. So we get the top diagram in the above figure.

If turbulent (or transitional), both the laminar and the turbulent (transitional) zones of the boundary layer growth diagram are used. The growth of the velocity profile is thus like the bottom diagram in the above figure. Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.) The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the **entry length**.

Laminar flow entry length $\approx$120 $\times$ diameter

Turbulent flow entry length $\approx$ 60 $\times$ diameter

Example 3. *Lubricating Oil at a velocity of 1 m/s (average) flows through a pipe of 100 mm ID. Determine* ***whether the flow is laminar or turbulent.*** *Also* ***determine the friction factor*** *and the pressure drop over 10 m length. What should be the velocity for the flow to turn turbulent? Density = 930 kg/m^3 . Dynamic viscosity μ= 0.1 Ns/m^2 (as N/m^2 is call Pascal, μ can be also expressed as Pa.s).*

$Re = \frac{uD\rho}{\mu} = \frac{1\times0.1\times930}{1\times0.1}$ =930, so the flow is laminar;

Friction factor, **f** = 64/930 = **0.06882**

$h_f = \frac{fLu_m^2}{2gD}$ = (64/930) × 10 × 1^2/(2 × 9.81 × 0.1) = **0.351 m head of oil.**

or ΔP = 0.351 × 0.93 × 9810 = **3200 N/m²**

At transition Re = 2000 (can be taken as 2300 also) Using (15) (Hagen-Poiseuille eqn.)

$\Delta P = \frac{32\times\mu\times u_m\times L}{D^2} = \frac{32\times0.1\times1\times10}{0.1^2}$ = **3200 N/m².** (same as by the other equation)

To determine velocity on critical condition

2300 = 4 m × 0.1 × 930/0.1

∴ **u_m= 2.47 m/s.**

L). VELOCITY DISTRIBUTION AND FRICTION FACTOR FOR TURBULENT FLOW IN PIPES.

The velocity profile and relation between the mean and maximum velocity are different in the two types of flow. In laminar flow the velocity profile is parabolic and the mean velocity is half of the maximum velocity. Such a relation is more complex in turbulent flow. For example one such available relation is given by;

$$\frac{u_m}{u_{max}} = \frac{1}{1+1.33\sqrt{f}} \qquad 22$$

The friction factor f is a complex function of Reynolds number. A sample velocity variation is given in equation (37).

$$u = \left(1 + 1.33\sqrt{f}\right)u_m - 2.04\sqrt{f}u_m log(R/(R-1)) \qquad 23$$

For higher values of f the velocity variation will be well rounded at the centre compared to low values of f. A new reference velocity called shear velocity is defined as below.

$$u^* = \sqrt{\frac{\tau_0 g_0}{\rho}} \qquad 24$$

Several other correlation using the reference velocity are listed below.

$$\frac{u}{u^*} = 5.75\ log\frac{Ru^*}{\nu} + 5.5 \qquad 25$$

$$\frac{u}{u^*} = 5.75 ln\frac{(R-r)}{\varepsilon} + 8.5 \qquad 26$$

Where ε is the roughness dimension. The mean velocity u_m is obtained for smooth and rough pipes as

$$\frac{u_m}{u^*} = 5.75\ log\frac{(R-r)u^*}{\nu} + 7.5 \qquad 27$$

and

$$\frac{u_m}{u^*} = 5.75\ log\frac{R}{\varepsilon} + 4.75 \qquad 28$$

The laminar sub-layer thickness is used for defining smooth pipe. The thickness of this layer is given by;

$$\delta_t = 11.6\,\nu/u^* \qquad 29$$

In the case of turbulent show the wall shear force is given by the following equation.

$$\tau_0 = \frac{f}{4} \times \frac{\rho u_m^2}{2} \qquad 30$$

Similar to velocity profile, several correlations are available for friction factor. These correlations together with correlations for velocity profile are useful in numerical methods of solution. The friction factor for very smooth pipes can be calculated by assuming one seventh power law leading to,

$$f = 0.316/\mathrm{Re}^{0.25} \quad \text{for } \mathrm{Re} < 2 \times 10^4 \qquad (31)$$

For all ranges either of the following relations can be used

$$f = 0.0032 + (0.221/\mathrm{Re}^{0.237}) \qquad (32)$$

$$1/\sqrt{f} = 1.8 \log \mathrm{Re} - 1.5186 \qquad 33$$

For rough pipes of radius R

$$1/\sqrt{f} = 2log\frac{R}{\varepsilon} + 1.74 \qquad 34$$

Charts connecting f, Re and ε/D are also available and can be used without appreciable error. As in laminar flow the frictional loss of head is given by

$$h_f = {fLu_m^2}\Big/{2gD_h} \qquad 35$$

Also

$$h_f = \frac{8fL}{g\pi^2}\frac{Q^2}{D^5} \qquad 36$$

The value of f is to be determined using the approximate relations or the chart.

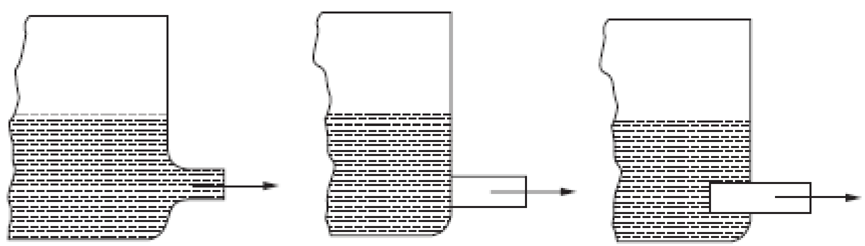
M). MINOR LOSSES IN PIPE FLOW.

Additional frictional losses occur at pipe entry, valves and fittings, sudden decrease or increase in flow area or where direction of flow changes. The frictional losses other than pipe friction are called minor losses. In a pipe system design, it is necessary to take into account all such losses. These losses are generally expressed as $h_f = Cu_m^2/2g$

Where C is constant, the value of which will depend on the situation and is called the loss coefficient. The expression is applicable both for laminar and turbulent flows.

(*i*) Loss of head at entrance: At the entrance from the reservoir into the pipe, losses take place due to the turbulence created downstream of the entrance. Three types of entrances are known.

(a) Bell mouthed: This is a smooth entrance and turbulence is suppressed to a great extent and $C = 0.04$ for this situation.
(b) Square edged entrance: Though it is desirable to provide a bell mouthed entrance it will not be always practicable. Square edged entrance is used more popularly. The loss coefficient, $C = 0.5$ in this case.
(c) Reentrant inlet: The pipe may sometimes protrude from the wall into the liquid. Such an arrangement is called reentrant inlet. The loss coefficient in this case is about 0.8.



Bell mouthed Square edge Re-entrant

Figure 6 Types of entrance

(***ii***) **Loss of head at submerged discharge:** When a pipe with submerged outlet discharges into a liquid which is still (not moving) whole of the dynamic head $u^2/2g$ will be lost. The loss coefficient is 1.0. The discharge from reaction turbines into the tail race water is an example. The loss is reduced by providing a diverging pipe to reduce the exit velocity.
(***iii***) **Sudden contraction:** When the pipe section is suddenly reduced, loss coefficient depends on the diameter ratio. The value is 0.33 for $D_2/D_1 = 0.5$. The values are generally available in a tabular statement connecting D_2/D_1 and loss coefficient. Gradual contraction will reduce the loss. For gradual contraction it varies with the angle of the transition section from 0.05 to 0.08 for angles of 10° to 60°.
(***iv***) **Sudden expansion:** Here the sudden expansion creates pockets of eddying turbulence leading to losses. The loss of head h_f is given by

$$\text{Loss of head} = (u_1 - u_2)^2 / 2g. \qquad (37)$$

Where u_1 and u_2 are the velocities in the smaller and larger sections. Gradual expansion will reduce the losses.
(***v***) **Valves and fittings:** Losses in flow through valves and fittings is expressed in terms of an equivalent length of straight pipe. For gate valves $L = 8D$, and for globe valves it is 340 D. For 90° bends it is about 30 D.

N). EXPRESSION FOR THE LOSS OF HEAD AT SUDDEN EXPANSION IN PIPE FLOW.

The situation is shown in Fig 7.

Using Bernoulli equation and denoting the ideal pressure at section 2 as P_2 (without losses), datum remaining unaltered,

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{u_1^2}{2g} - \frac{u_2^2}{2g} \text{ or } \frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{u_1^2}{2} - \frac{u_2^2}{2} \qquad (1)$$

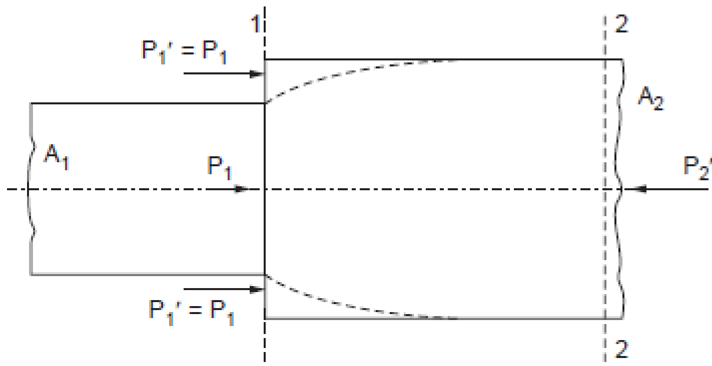


Figure 7 Sudden Expansion

Applying conservation of momentum principle to the fluid between section 1 and 2, and denoting the actual pressure at section 2 as P_2, The pressure forces are (here the pressure on the annular section of fluid at 1 is assumed as P_1)

$$\left(P_1A_1 - P_2'A_2\right)$$

The change in momentum is given by

$$(\rho A_2u_2u_2 - \rho A_1u_1u_1)$$

noting $A_1\ u_1 = A_2\ u_2$, replacing $A_1\ u_1$ by $A_2\ u_2$ and equating the net forces on the element to the momentum change,

$$P_1A_1 - P_2'A_2 = \rho A_2u_2^2 - \rho A_2u_2u_1$$

Dividing by ρ and A_2 all-through

$$\frac{P_1}{\rho} - \frac{P_2'}{\rho} = u_2^2 - u_1u_2 \text{ or } \frac{P_2'}{\rho} = \frac{P_1}{\rho} - (u_2^2 - u_1u_2) \qquad (2)$$

Subtracting on either side of equations 1 and 2 (ideal and real)

$$\frac{P_2 - P_2'}{\rho} = \frac{P_1}{\rho} + \frac{u_1^2}{2} - \frac{u_2^2}{2} - \frac{P_1}{\rho} - (u_2^2 - u_1u_2)$$

$$\therefore \qquad \frac{P_2 - P_2'}{\rho} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + (u_2^2 - u_1u_2)$$

Multiplying both sides by 2

$$\frac{2(P_2 - P_2')}{\rho} = u_1^2 - u_2^2 + 2u_2^2 - 2u_1u_2 = (u_1 - u_2)^2$$

Dividing the both sides by g and simplifying

$$\frac{P_2 - P_2'}{\rho g} = \frac{(u_2 - u_{12})^2}{2g} \text{ but, } \frac{P_2 - P_2'}{\rho g} = h_f \text{ (head loss),}$$

$\therefore \qquad h_f = \frac{(u_2 - u_{12})^2}{2g}$

O). LOSSES IN ELBOWS, BENDS AND OTHER PIPE FITTINGS.

Fittings like valves, elbows etc. introduce frictional losses either by obstruction or due to secondary flows. The losses may be accounted for by a term equivalent length which will depend on the type of fitting or in terms of ($u^2/2g$) or dynamic head. In the case of bends, the loss is due to the variation of centrifugal force along different stream lines which causes secondary flows. In large bends fitting curved vanes will reduce the loss. The loss will vary with radius of the bend. Globe valves are poorer compared to gate valves with regard to pressure drop.