LECTURE NOTE

MCE 326

ENGR. ALIYU, S.J

Fluid Flow—Basic Concepts

INTRODUCTION

In this course the flow of ideal fluids will be discussed. The main attempt in this section is to visualise flow fields. A flow field is a region in which the flow is defined at all points at any instant of time. The means to that is to define the velocities at all the points at different times. It should be noted that the velocity at a point is the velocity of the fluid particle that occupies that point. In order to obtain a complete picture of the flow the fluid motion should be described mathematically. Just like the topography of a region is visualised using the contour map, the flow can be visualised using the velocity at all points at a given time or the velocity of a given particle at different times. It is then possible to also define the potential causing the flow. Application of a shear force on an element or particle of a fluid will cause continuous deformation of the element. Such continuing deformation will lead to the displacement of the fluid element from its location and this results in fluid flow. The fluid element acted on by the force may move along a steady regular path or randomly changing path depending on the factors controlling the flow. The velocity may also remain constant with time or may vary randomly. In some cases the velocity may vary randomly with time but the variation will be about a mean value . It may also vary completely randomly as in the atmosphere. The study of the velocity of various particles in the flow and the instantaneous flow pattern of the flow field is called flow kinematics or hydrodynamics. Such a study is generally limited to ideal fluids, fluids which are incompressible and inviscid. In real fluid shows, beyond a certain distance from the surfaces, the flow behaves very much like ideal fluid. Hence these studies are applicable in real fluid flow also with some limitations.

LAGRANGIAN AND EULARIAN METHODS OF STUDY OF FLUID FLOW

In the Lagrangian method a single particle is followed over the flow field, the co-ordinate system following the particle. The flow description is particle based and not space based. A moving coordinate system has to be used. This is equivalent to the observer moving with the particle to study the flow of the particle. This method is more involved mathematically and is used mainly in special cases. In the Eularian method, the description of flow is on fixed coordinate system based and the description of the velocity etc. are with reference to location and time *i.e.*, $V = V(x, y, z, t)$ and not with reference to a particular particle. Such an analysis provides a picture of various parameters at all locations in the flow field at different instants of time. This method provides an easier visualisation of the flow field and is popularly used in fluid flow studies. However the final description of a given flow will be the same by both the methods.

BASIC SCIENTIFIC LAWS USED IN THE ANALYSIS OF FLUID FLOW

(i) Law of conservation of mass: This law when applied to a control volume states that the net mass flow through the volume will equal the mass stored or removed from the volume. Under conditions of steady flow this will mean that the mass leaving the control volume should be equal to the mass entering the volume. The determination of flow velocity for a specified mass flow rate and flow area is based on the continuity equation derived on the basis of this law.

(ii) Newton's laws of motion: These are basic to any force analysis under various conditions of flow. The resultant force is calculated using the condition that it equals the rate of change of momentum. The reaction on surfaces are calculated on the basis of these laws. Momentum equation for flow is derived based on these laws.

(iii) Law of conservation of energy: Considering a control volume the law can be stated as "the energy flow into the volume will equal the energy flow out of the volume under steady conditions". This also leads to the situation that the total energy of a fluid element in a steady flow field is conserved. This is the basis for the derivation of Euler and Bernoulli equations for fluid flow.

(iv) Thermodynamic laws: are applied in the study of flow of compressible fluids.

FLOW OF IDEAL / INVISCID AND REAL FLUIDS

Ideal fluid is nonviscous and incompressible. Shear force between the boundary surface and fluid or between the fluid layers is absent and only pressure forces and body forces are controlling. Real fluids have viscosity and surface shear forces are involved during flow. However the flow after a short distance from the surface is not affected by the viscous effects and approximates to ideal fluid flow. The results of ideal fluid flow analysis are found applicable in the study of flow of real fluids when viscosity values are small.

STEADY AND UNSTEADY FLOW

In order to study the flow pattern it is necessary to classify the various types of flow. The classification will depend upon the constancy or variability of the velocity with time. In the next three sections, these are described. In steady flow the property values at a location in the flow are constant and the values do not vary with time. The velocity or pressure at a point remains constant with time. These can be expressed as $V = V(x, y, z)$, $P = P(x, y, z)$ etc. In steady flow a picture of the flow field recorded at different times will be identical. In the case of unsteady flow, the properties vary with time or $V = V(x, y, z, t)$, $P = P(x, y, z, t)$ where t is time. In unsteady flow the appearance of the flow field will vary with time and will be constantly changing. In turbulent flow the velocity at any point fluctuates around a mean value, but the mean value at a point over a period of time is constant. For practical purposes turbulent flow is considered as steady flow as long as the mean value of properties do not vary with time.

COMPRESSIBLE AND INCOMPRESSIBLE FLOW

If the density of the flowing fluid is the same all over the flow field at all times, then such flow is called incompressible flow. Flow of liquids can be considered as incompressible even if the density varies a little due to temperature difference between locations. Low velocity flow of gases with small changes in pressure and temperature can also be considered as incompressible flow. Flow through fans and blowers is considered incompressible as long as the density variation is below 5%. If the density varies with location, the flow is called compressible flow. In this chapter the study is mainly on incompressible flow.

LAMINAR AND TURBULENT FLOW

If the flow is smooth and if the layers in the flow do not mix macroscopically then the flow is called laminar flow. For example a dye injected at a point in laminar flow will travel along a continuous smooth line without generally mixing with the main body of the fluid. Momentum, heat and mass transfer between layers will be at molecular level of pure diffusion. In laminar flow layers will glide over each other without mixing. In turbulent flow fluid layers mix macroscopically and the velocity/temperature/mass concentration at any point is found to vary with reference to a mean value over a time period. For example $u = \overline{u} + u'$ where u is the velocity at an instant at a location and $\overline{u}$ is the average velocity over a period of time at that location and u' is the fluctuating component. This causes higher rate of momentum/heat/mass transfer. A dye injected into such a flow will not flow along a smooth line but will mix with the main stream within a short distance. The difference between the flows can be distinguished by observing the smoke coming out of an incense stick. The smoke in still air will be found to rise along a vertical line without mixing. This is the laminar region. At a distance which will depend on flow conditions the smoke will be found to mix with the air as the flow becomes turbulent. Laminar flow will prevail when viscous forces are larger than inertia forces. Turbulence will begin where inertia forces begin to increase and become higher than viscous forces.

CONCEPTS OF UNIFORM FLOW, REVERSIBLE FLOW AND THREE DIMENSIONAL FLOW

If the velocity value at all points in a flow field is the same, then the flow is defined as uniform flow. The velocity in the flow is independent of location. Certain flows may be approximated as uniform flow for the purpose of analysis, though ideally the flow may not be uniform. If there are no pressure or head losses in the fluid due to frictional forces to be overcome by loss of kinetic energy (being converted to heat), the flow becomes reversible. The fluid can be restored to its original condition without additional work input. For a flow to be reversible, no surface or fluid friction should exist. The flow in a venturi (at low velocities) can be considered as reversible and the pressures upstream and downstream of the venturi will be the same in such a case. The flow becomes irreversible if there are pressure or head losses. If the components of the velocity in a flow field exist only in one direction it is called one dimensional flow and $V = V(x)$. Denoting the velocity components in x, y and z directions as u, v and w, in one dimensional flow two of the components of velocity will be zero. In two dimensional flow one of the components will be zero or $V = V(x, y)$. In three dimensional flow all the three components will exist and $V = V(x, y, z)$. This describes the general steady flow situation. Depending on the relative values of u, v and w approximations can be made in the analysis. In unsteady flow $V = V(x, y, z, t)$.

VELOCITY AND ACCELERATION COMPONENTS

The components of velocity can be designated as

$$u = \frac{dx}{dt}, v = \frac{dy}{dt} \text{ and } w = \frac{dz}{dt}$$

where t is the time and dx, dy, dz are the displacements in the directions x, y, z.

In general as $u = u(x, y, z, t)$, $v = v(x, y, z, t)$ and $w = w(x, y, z, t)$

Defining acceleration components as

$$a_x = \frac{du}{dt}, a_y = \frac{dv}{dt} \text{ and } a_z = \frac{dw}{dt}, \text{ as } u = u(x,y,z,t)$$

$$a_x = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Similarly,

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

And

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial v}{\partial t}$$

The first three terms in each case is known as convective acceleration terms, because these represent the convective act of moving from one position to another. The last term is known as local accleration term, because the flow at a point is changing with time. Under steady flow conditions, only the convective acceleration terms will exist.

CONTINUITY EQUATION FOR FLOW—CARTESIAN CO-ORDINATES

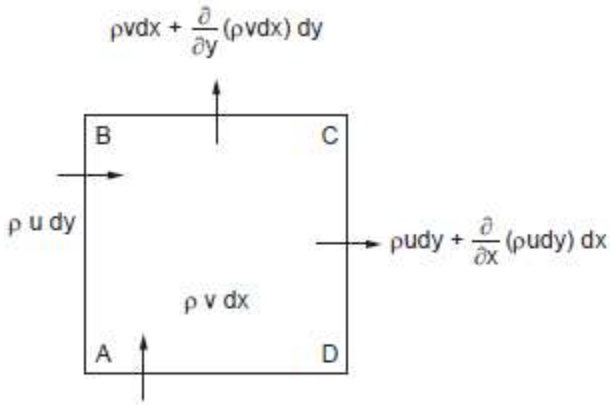

Figure. 1 Derivation of continuity equation

Consider an element of size *dx*, *dy*, *dz* in the flow as shown in Fig. 1.

Applying the law of conservation of mass, for a given time interval, The net mass flow into the element through all the surfaces

= The change in mass in the element.

First considering the $y - z$ face, perpendicular to the *x* direction and located at *x*, the flow through face during time *dt* is given by

$\rho u\, dy\, dz\, dt$ (1)

The flow through the $y - z$ face at $x + dx$ is given by

$\rho udydzdt + \frac{d}{dx}(\rho\ \ u\, dy\, dz\, dt)\, dx$ (2)

The net mass flow in the *x* direction is the difference between the quantities given by (1) and (2) and is equal to

$\frac{d}{dx}\ (\rho u)\ dx\ dy\ dz\ dt$ (3)

Similarly the net mass through the faces $z - x$ and $x - y$ in y and z directions respectively are given by

$\frac{d}{dx}\ (\rho v)\ dx\ dy\ dz\ dt$ (4)

$\frac{d}{dx}\ (\rho w)\ dx\ dy\ dz\ dt$ (5)

The change in the mass in the control volume equals the rate of change of density × volume × time or

$\frac{d}{dx}\ \ dx\ dy\ dz\ dt$ (6)

The sum of these quantities should equal zero, cancelling common terms *dx dy dz dt*

$$\frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dw} = \frac{d\rho}{dt} \quad (7)$$

This is the general equation. For steady flow this reduces to

$$\frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dw} = 0 \quad (8)$$

For incompressible flow this becomes

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad (9)$$

Whether a flow is steady can be checked using this equation when the velocity components are specified. For two dimensional steady incompressible flow, the equation reduces to

$$\frac{du}{dx} + \frac{dv}{dy} = 0 \quad (10)$$

For one dimensional flow with varying area, the first term of the general equation alone need be considered. For steady flow

$\frac{d(\rho u\, dy\, dz)}{dx} = 0$ as $dy\, dz = dA$. Integrating ρuA = constant. Or

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \quad (11)$$

This equation is used to calculate the area, or velocity in one dimensional varying area flow, like flow in a nozzle or venturi.

IRROTATIONAL FLOW AND CONDITION FOR SUCH FLOWS

Irrotational flow may be described as flow in which each element of the moving fluid suffers no net rotation from one instant to the next with respect to a given frame of reference. In flow along a curved path fluid elements will deform. **If the axes of the element rotate equally towards or away from** each **other, then the flow will be irrotational**. This means that as long as the algebraic average rotation is zero, the flow is irrotational. The idea is illustrated in Fig. 2.

Irrotational
flow : $\Delta\alpha = \Delta\beta$

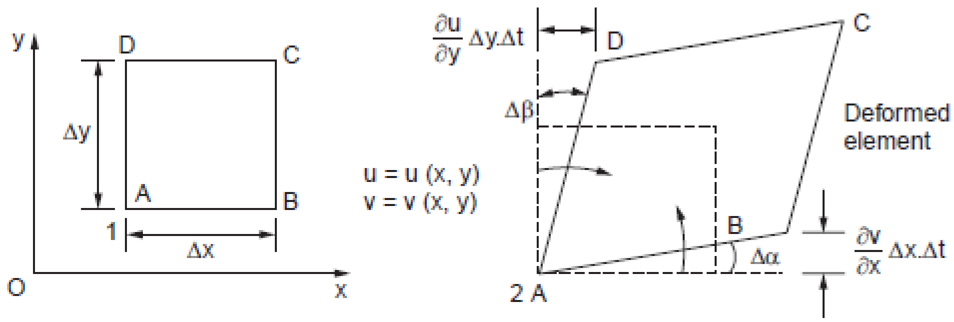

Figure 2 Rotation in Flow

An element is shown moving from point 1 to point 2 along a curved path in the flow field.

At 1 the undeformed element is shown. As it moves to location 2 the element is deformed. The angle of rotation of x axis is given by (dv/dy). $\Delta y.\Delta t$. The angle of rotation of y axis is given by (du/dy). $\Delta y.\Delta t$. (It is assumed that $\Delta x = \Delta y$. For irrotational flow, the angle of rotation of the axes towards each other or away from each other should be equal *i.e.*, the condition to be satisfied for irrotational flow is,

$$\frac{dv}{dx} = \frac{du}{dy} \; or \; \frac{dv}{dx} - \frac{du}{dy} = 0 \qquad (12)$$

Another significance of irrotational flow is that it is defined by a potential function $\phi$for the flow described in para 5.15.
In case there is rotation, then the rotation is given by (with respect to the Z axis in the case of two dimensional flow along x and y)
$\omega_z = (1/2)\,(dv/dx - du/dy)$ (13)
and $\omega_z = 0$ for irrotational flow.

CONCEPTS OF CIRCULATION AND VORTICITY

Considering a closed path in a flow field as shown in Fig.3, circulation is defined as the line integral of velocity about this closed path. The symbol used is Γ

$$.\Gamma = \oint_L u\,ds = \oint_L u\cos\beta dL$$

where dL is the length on the closed curve, u is the velocity at the location and β is the angle between the velocity vector and the length dL. The closed path may cut across several stream lines and at each point the direction of the velocity is obtained from the stream line, as its tangent at that point.

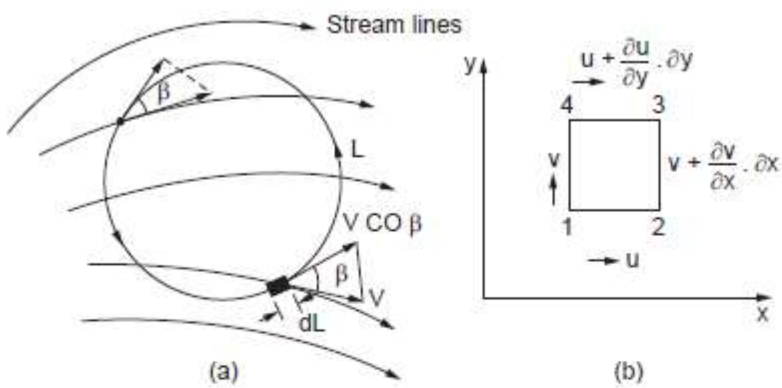

Figure. 3. Circulation in flow

The integration can be performed over an element as shown in Fig. 3 (*b*).
In the cartesian co-ordinate if an element dx. dy is considered, then the circulation can be calculated as detailed below:
Consider the element 1234 in Fig. 5.3*b*. Starting at 1 and proceeding counter clockwise,

$d\Gamma = u\,dx + [v+(\partial v/\partial x)dx]dy - [u+(\partial u/\partial y).dy]\,dx - vdy$

$= [\partial v/\partial x - \partial u/\partial y]dxdy$ (14)

Vorticity is defined as circulation per unit area. i.e.,
Vorticity = circulation per unit area, here area is $dx\,dy$, so

$$Vorticity = \frac{d\Gamma}{dxdy} = \frac{dv}{dx} - \frac{du}{dy} \qquad (15)$$

For irrotational flow, vorticity and circulation are both zero. In polar coordinates

$$Vorticity = \frac{dv_\theta}{dr} - \frac{1}{r}\frac{dv_r}{d\theta} + \frac{v_\theta}{r}$$

STREAM LINES, STREAM TUBE, PATH LINES, STREAK LINES AND TIME LINES

The analytical description of flow velocity is geometrically depicted through the concept of stream lines. The velocity vector is a function of both position and time. If at a fixed instant of time a curve is drawn so that it is tangent everywhere to the velocity vectors at these locations then the curve is called a stream line. Thus stream line shows the mean direction of a number of particles in the flow at the same instant of time. **Stream lines are a series of curves drawn tangent to the mean velocity vectors of a number of particles in the flow. Since stream lines are tangent to the velocity vector at every point in the flow field, there can be no flow across a stream line.**

A bundle of neighbouring stream lines may be imagined to form a passage through which the fluid flows. Such a passage is called a stream tube. Since the stream tube is bounded on all sides by stream lines, there can be no flow across the surface. Flow can be only through the ends. A stream tube is shown diagrammatically in Figure 4. Under steady flow condition, the flow through a stream tube will be constant along the length.

Path line is the trace of the path of a single particle over a period of time. Path line shows the direction of the velocity of a particle at successive instants of time. In steady flow path lines and stream lines will be identical.

Streak lines provide an instantaneous picture of the particles, which have passed through a given point like the injection point of a dye in a flow. In steady flow these lines will also coincide with stream lines. Path lines and streak lines are shown in Figure 4.

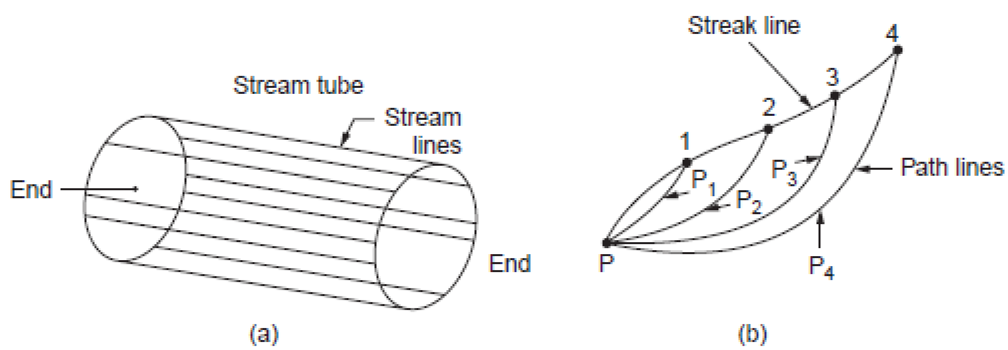

Figure 4 Stream tube, Path lines and Streak lines

Particles P_1, P_2, P_3, P_4, starting from point P at successive times pass along path lines shown. At the instant of time considered the positions of the particles are at 1, 2, 3 and 4. A line joining these points is the streak line.

If a number of adjacent fluid particles in a flow field are marked at a given instant, they form a line at that instant. This line is called time line. Subsequent observations of the line may provide information about the flow field. For example the deformation of a fluid under shear force can be studied using time lines.

CONCEPT OF STREAM LINE

In a flow field if a continuous line can be drawn such that the tangent at every point on the line gives the direction of the velocity of flow at that point, such a line is defined as a stream line. In steady flow any particle entering the flow on the line will travel only along this line. This leads to visualisation of a stream line in laminar flow as the path of a dye injected into the flow. **There can be no flow across the stream line**, as the velocity perpendicular to the stream line is zero at all points. The flow along the stream line can be considered as one dimensional flow, though the stream line may be curved as there is no component of velocity in the other directions. Stream lines define the flow paths of streams in the flow. The flow entering between two stream lines will always flow between the lines. The lines serve as boundaries for the stream. In the cartesian co-ordinate system, along the stream line in two dimensional flow it can be shown that

$$\frac{dx}{u} = \frac{dy}{v} \; or \; vdx - udy = 0 \qquad (16)$$

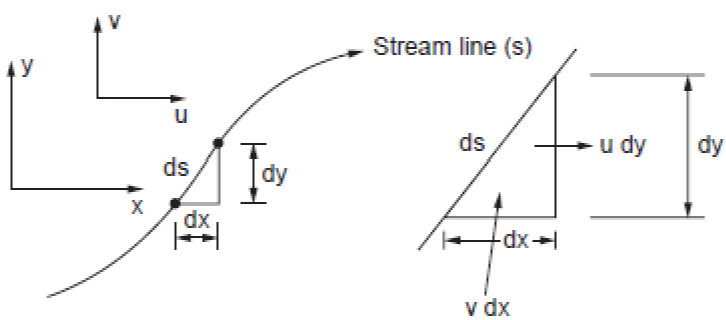

Figure 5. Velocity components along a stream line

Referring to Fig. 5. considering the velocity at a point and taking the distance *ds* and considering its *x* and *y* components as *dx* and *dy*, and noting that the net flow across *ds* is zero, the flow along *y* direction = *dx v* the flow along *x* direction = *dy u*

These two quantities should be equal for the condition that the flow across *ds* is zero, thus proving the equation (16).

In the next para, it is shown that stream lines in a flow can be described by a stream function having distinct values along each stream line.