CHAPTER ONE

Review on Strength of Materials

1a. Definition: Strength of materials is the study of the behaviour of structural and machine members under the action of external loads, taking into the internal forces created and the resulting deformations into consideration**.**

b. Classification of materials

Materials are classified into the following categories: (i) Elastic, (ii) Elasto-plastic, (iii) Plastic, (iv) Ductile and Brittle.

i. Elastic material is that which undergoes deformation when subjected to external force, up to a certain limit such that deformation disappears on removal of the force.

ii. Elasto-plastic material undergoes deformation when subjected to external force such that deformation partially disappears on the removal of the force. That is a permanent deformation remains even after removal of the force.

iii. Plastic material undergoes deformation when subjected to external force such that the deformation does not disappear at all; even after the force is removed.

iv. Ductile material allows itself to be drawn out by tension to a smaller section.

v. Brittle material is that which does not deform when subjected to external force before it fails by rupture. Brittleness is the lack or absence of ductility.

c. Elasticity

This can be defined as the property of a material to return to its original shape or size after deformation, when the external force has been removed. A material is said to be ***elastic*** if it returns to its original, unloaded dimensions when load is removed. A particular form of elasticity which applies to a large range of engineering materials, at least over part of their load range, produces deformations which are proportional to the loads producing them. Since loads are proportional to the stresses they produce and deformations are proportional to the strains, this also implies that, whilst materials are elastic, stress is proportional to strain. ***Hooke's law,*** in its simplest form, therefore states that

$$\text{Stress } (\boldsymbol{\sigma}) \propto \text{ strain } (\varepsilon)$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

Whilst a material is elastic the deformation produced by any load will be ***completely*** recovered when the load is removed; there is no permanent deformation. A material which has a uniform structure throughout without any flaws or discontinuities is termed a ***homogeneous*** material. ***Non-homogeneous*** or ***inhomogeneous*** materials such as concrete and poor-quality cast iron will thus have a structure which varies from point to point depending on its constituents and the presence of casting flaws or impurities. If a material exhibits uniform properties throughout in all directions it is said to be ***isotropic;*** conversely one which does not exhibit this uniform behaviour is said to be ***nonisotropic*** or ***anisotropic.*** An ***orthotropic*** material is one which has different properties in different planes. A typical example of such a material is wood, although some composites which contain systematically orientated "inhomogeneities" may also be considered to fall into this category.

1.1 Simple Stress and Strain

In any engineering structure or mechanism, components are subjected to external forces arising from the service conditions or environment in which the component works. If the component or member is in equilibrium, the resultant of the external forces will be zero but, nevertheless, they together place a load on the member which tends to deform that member and which must be reacted by internal forces which are set up within the material. If a cylindrical bar is subjected to a direct pull or push along its axis; then it is said to be subjected to ***tension*** or ***compression.*** Typical examples of tension are the forces present in towing ropes or lifting hoists, whilst compression occurs in the legs of your chair as you sit on it or in the support pillars of buildings.

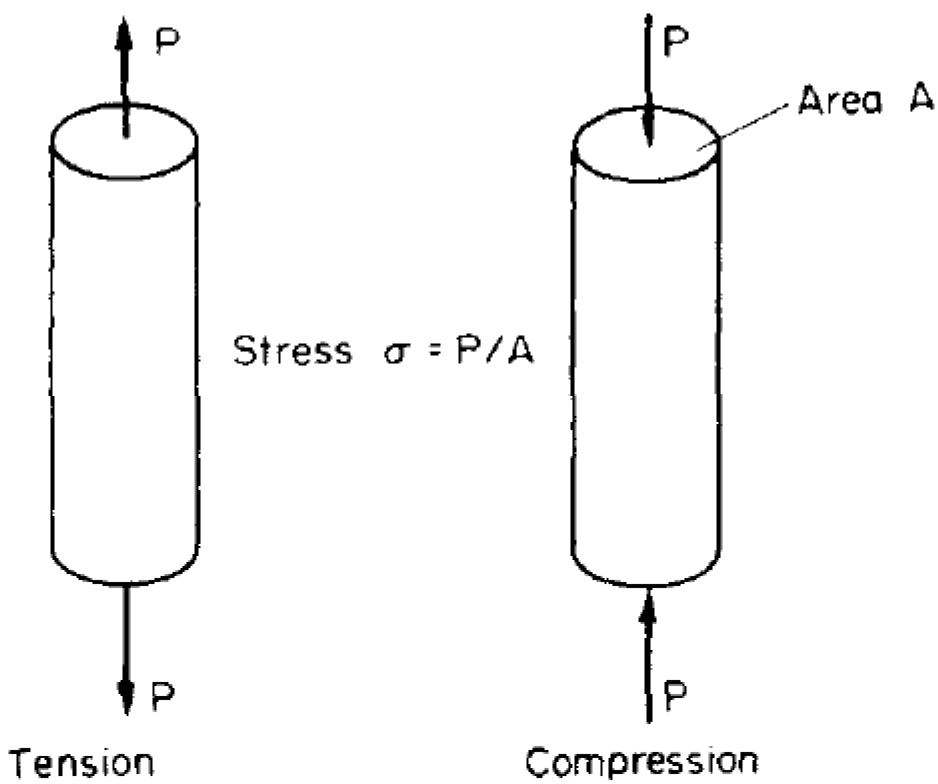

Fig. 1 (a) Tensile stress (b) Compressive stress

1.2 Types of direct stress.

There are a number of different ways in which load can be applied to a member. Typical loading types are:

(a) ***Static*** or dead loads, i.e. non-fluctuating loads, generally caused by gravity effects.

(b) ***Live*** loads, as produced by, for example, lorries crossing a bridge.

(c) ***Impact*** or shock loads caused by sudden blows.
(d) ***Fatigue, fluctuating*** or ***alternating*** loads, the magnitude and sign of the load changing with time.

1.3 Direct or normal stress (σ)

It has been noted above that external force applied to a body in equilibrium is reacted by internal forces set up within the material. If, therefore, a bar is subjected to a uniform tension or compression, i.e. a direct force, which is uniformly or equally applied across the cross section, then the internal forces set up are also distributed uniformly and the bar is said to be subjected to a uniform ***direct or normal stress,*** the stress being defined as

$$\textbf{Stress } (\sigma) = \frac{\textbf{load}}{\textbf{area}} = \frac{\textbf{P}}{\textbf{A}}$$

Stress σ may thus be compressive or tensile depending on the nature of the load and will be measured in units of newtons per square metre (N/m ^{2}) or multiples of this. In some cases the loading situation is such that the stress will vary across any given section, and in such cases the stress at any point is given by the limiting value of $\delta P/\delta A$ as δA tends to zero.

1.4 Direct strain (ε)

If a bar is subjected to a direct load, and hence a stress, the bar will change in length. If the bar has an original length **L** and changes in length by an amount δL**,** the ***strain*** produced is defined as follows:

$$\textbf{Strain } (\varepsilon) = \frac{\textbf{change in length}}{\textbf{original length}} = \frac{\delta L}{L}$$

Strain is thus a measure of the deformation of the material and is non-dimensional, i.e. it has no units; it is simply a ratio of two quantities with the same unit.

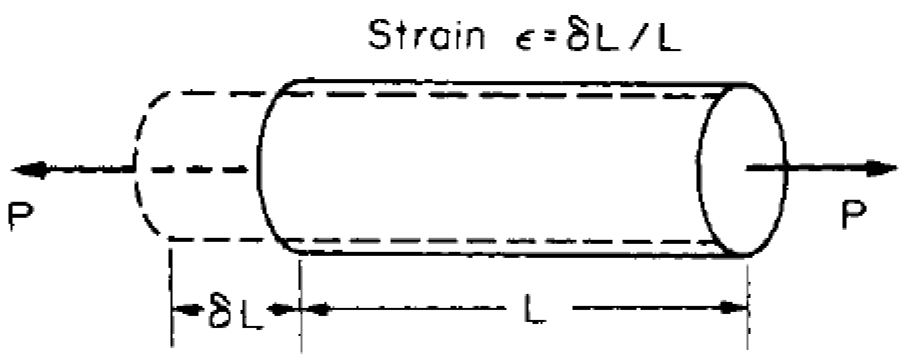

Fig. 2 longitudinal strain in a cylindrical rod

Modulus of elasticity, $E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$

$$= \frac{P}{A} / \frac{\delta L}{L} = \frac{PL}{A\delta L}$$

Young's modulus E is generally assumed to be the same in tension or compression and for most engineering materials has a high numerical value. Typically, E = 200 **x** 10^9 N/m^2 for In most common engineering applications strains do not often exceed 0.003 or 0.3 %.

1.5 Poisson's ratio

Consider a rectangular bar subjected to a tensile load. Under the action of this load the bar will increase in length by an amount δ*L* giving a longitudinal strain in the bar of

$$\varepsilon_L = \frac{\delta L}{L}$$

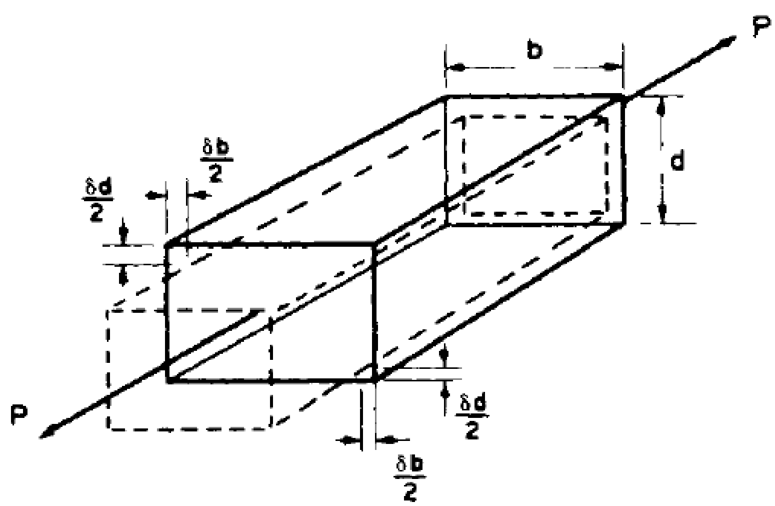

Fig. 3 Lateral strain

The bar will also exhibit, however, a ***reduction*** in dimensions laterally, i.e. its breadth and depth will both reduce. The associated lateral strains will both be equal, will be of opposite sense to the longitudinal strain, and will be given by

$$\varepsilon_{lat.} = -\frac{\delta b}{b} = \frac{\delta d}{d}$$

Provided the load on the material is retained within the elastic range the ratio of the lateral

$$\textbf{Poisson's ratio } (\nu) = \frac{lateral\ strain}{longitudinal\ strain} = \frac{(-\frac{\delta d}{d})}{\delta L/L}$$

The negative sign of the lateral strain is normally ignored to leave Poisson's ratio simply as a ratio of strain magnitudes. It must be remembered, however, that the longitudinal strain induces a lateral strain of opposite sign, e.g. tensile longitudinal strain induces compressive lateral strain. For most engineering materials the value of v lies between *0.25* and 0.33.

$$\textbf{Since longitudinal strain} = \frac{\textit{longitudinal stress}}{\text{Young's modulus}} = \frac{\sigma}{E}$$

$$\textbf{Lateral strain} = \nu \frac{\sigma}{E}$$

1.6 Application of Poisson's ratio to a two-dimensional stress system

A two-dimensional stress system is one in which all the stresses lie within one plane such as the X-Y plane. It will be seen that if a material is subjected to a tensile stress σ on one axis producing a strain σ/E and hence an extension on that axis, it will be subjected simultaneously to a lateral strain of ν times σ/E on any axis at right angles. This lateral strain will be compressive and will result in a compression or reduction of length on this axis.
Consider, therefore, an element of material subjected to two stresses at right angles to each other and let both stresses, σ_x and σ_y, be considered tensile.

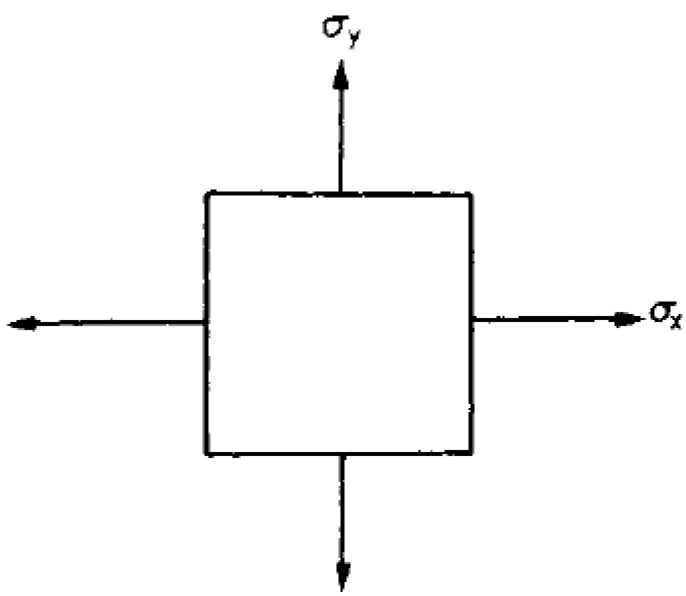

Fig. 4 Simple two dimensional system of direct stresses

The following strains will be produced
(a) in the X direction resulting from $\sigma_x = \sigma_x /E$,
(b) in the Y direction resulting from $\sigma_y = \sigma_y /E$.
(c) in the X direction resulting from σ_y, $= - v(\sigma_y /E)$
(d) in the Y direction resulting from $\sigma_x = - v(\sigma_x /E)$

strains (c) and (d) being the so-called ***Poisson's ratio strain,*** opposite in sign to the applied strains, i.e. compressive. The total strain in the X direction will therefore be given by:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

and the total strain in the Y direction will be:

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

If any stress is, in fact, compressive its value must be substituted in the above equations together with a negative sign following the normal sign convention.

1.7 Shear stress

Consider a block or portion of material being subjected to a set of **equal** and opposite forces **Q.** (Such a system could be realised in a bicycle brake block when contacted with the wheel.) There is then a tendency for one layer of the material to slide over another. If this failure is restricted, then a ***shear stress T*** is set up, defined as follows:

$$\textbf{Shear stress} = \frac{\textit{Shear load}}{\textit{arearesisting the shear}} = \frac{\sigma}{A}$$

This shear stress will always be ***tangential*** to the area on which it acts; direct stresses, however, are always ***normal*** to the area on which they act.

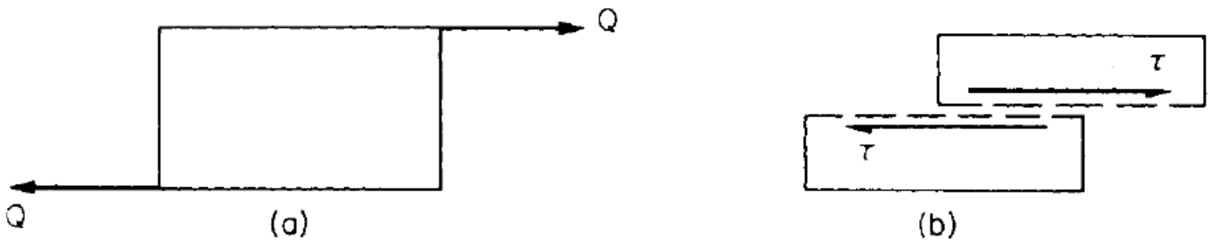

FIG. 5 Shear force and resulting shear stress system showing typical form of failure by relative sliding of planes.

1.8 Shear strain

If one again considers the block of Fig. to be a bicycle brake block it is clear that the rectangular shape of the block will not be retained as the brake is applied and the shear forces introduced. The block will in fact change shape or "strain" into the form shown in Fig .
The angle of deformation *y* is then termed the ***shear strain.*** Shear strain is measured in radians and hence is non-dimensional, i.e. it has no units.

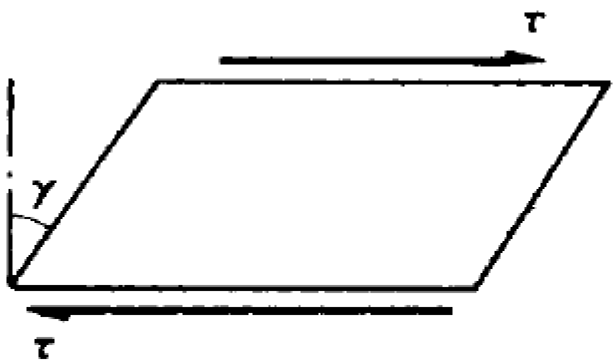

Fig. 6 Deformation (shear strain) produced by shear stresses.

1.9 Modulus of rigidity

For materials within the elastic range the shear strain is proportional to the shear stress producing it, i.e.

$$\frac{\textbf{Shear stress}}{\textbf{Shearstrain}} = \frac{\tau}{\gamma} = \textbf{constant} = \textbf{G}$$

The constant G is termed the ***modulus of rigidity*** or ***shear modulus*** and is directly comparable to the modulus of elasticity used in the direct stress application. The term ***modulus*** thus implies a ratio of stress to strain in each case.

1.10 Double shear

Consider a simple riveted lap. When load is applied to the plates the rivet is subjected to shear forces tending to shear it on one plane as indicated. In the butt joint with two cover plates of Fig. 7 b, however, each rivet is subjected to possible shearing on two faces, i.e. ***double shear*.** In such cases twice the area of metal is resisting the applied forces so that the shear stress set up is given by

$$\textbf{Shear stress } \tau \textbf{ (in double shear)} = \frac{P}{2A}$$

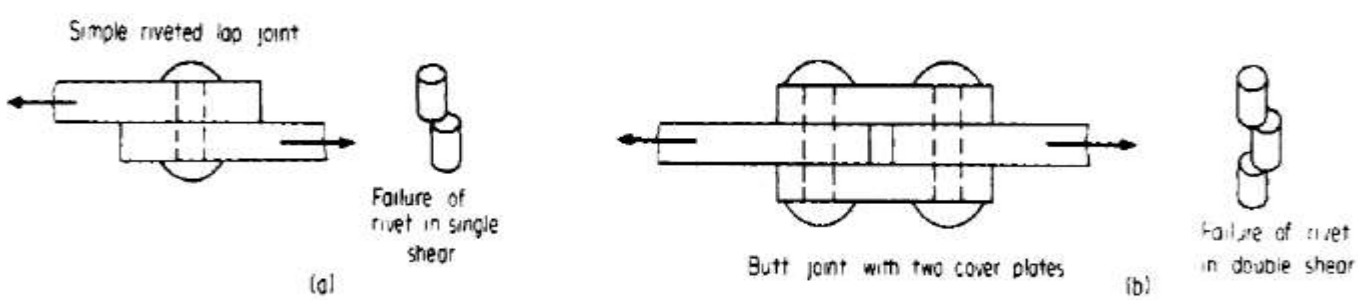

Fig. 7 (a) Single shear. (b) Double shear

1.11 Temperature stresses

When the temperature of a component is increased or decreased the material respectively expands or contracts. If this expansion or contraction is not resisted in any way then the processes take place free of stress. If, however, the changes in dimensions are restricted then stresses termed ***temperature stresses*** will be set up within the material. Consider a bar of material with a linear coefficient of expansion *a*. Let the original length ***of*** the bar be ***L*** and let the temperature ***increase*** be **t.** If the bar is free to expand the change in length would be given by

$$\Delta L = L\propto t$$

And the new length

$$L' = L + L\propto t = L(1+\propto t)$$

If this extension were totally prevented, then a compressive stress would be set up equal to that produced when a bar of length $L(1+\propto t)$ is compressed through a distance of $L\propto t$. In this case the bar experiences a compressive strain

$$\varepsilon = \frac{\Delta L}{L} = \frac{L\alpha t}{L(1+\alpha t)}$$

In most cases αt is very small compared with unity so that

$$\varepsilon = \frac{L\alpha t}{L} = \alpha t$$

$$\text{But} \quad \frac{\sigma}{\varepsilon} = E$$

$$\text{Stress } \sigma = E\varepsilon = E\propto t$$

Tutorials - 1

1. A 25mm squarecross-section bar of length 300mm carries an axial compressive load of 50kN. Determine the stress **set** up ip the bar and its change of length when the load is applied. For the bar material E = 200 GN/m2. [80 MN/m2; 0.12mm.]

2. A steel tube, **25** mm outside diameter and 12mm inside diameter, cames an axial tensile load of 40 kN. What will be the stress in the bar? What further increase in load is possible if the stress in the bar is limited **to** 225 MN/m^2? [l06 MN/m^2; 45 kN.]

3 Define the terms shear ***stress*** and ***shear strain,*** illustrating your answer by means of a simple sketch. Two circular bars, one of brass and the other of steel, are **to** be loaded by a shear load of 30 kN. Determine the **necessary** diameter of the **bars** (a) in single shear, (b) in double shear, if the shear stress in the two materials must not exceed ***50*** MN/m2 and 100 MN/m^2 respectively. [27.6, 19.5, 19.5, 13.8mm.]

4. Two fork end pieces are **to** be joined together by a single steel pin of 25mm diameter and they are required **to** transmit **50** kN. Determine the minimum cross-sectional area of material required in one branch of either fork if the stress in the fork material is not **to** exceed 180 MN/m2. What will be the maximum shear stress in the pin? [1.39 **x** 10^{-4} m^2; 50.9MN/m^2.].