LANDMARK UNIVERSITY, OMU-ARAN

LECTURE NOTE 2
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ENGR. ALIYU, S.J.

PERFECT GASES, GAS LAWS, GAS PROCESSES.

The expansion and compression of gases, such as air and combustion gases, is a very important subject in the study of the operation of compressors, all types of reciprocating engines, gas turbines, and in pneumatic systems. We need to be able to predict how the volume, temperature and pressure of a gas inter-relate in a process in order to design systems in which gas is the working medium. The cycle of operations of your car engine is a good example of the practical application of this study. In this lecture, therefore, we look at the different ways in which a gas can be expanded and compressed, what defines the 'system', the significance of reversibility and the First Law of Thermodynamics. In order to be able to calculate changes in properties during a process, gas law expressions and equations are introduced.

Boyle's and Charles' laws

The gases we deal with are assumed to be '*perfect gases*', i.e. theoretically ideal gases which strictly follow Boyle's and Charles' laws. What are these laws? *Boyle's law*. This says that if the temperature of a gas is kept constant, the product of its pressure and its volume will always be the same.

Hence,

$P \, x \ V$ = constant

or,

$p_1V_1 = p_2V_2 = p_3V_3$, etc.

This means that if you have a quantity of gas and you change its pressure and therefore its volume, as long as the temperature is kept constant (this would require heating or cooling), you will always get the same answer if you multiply the pressure by the volume.

Charles' law. This says that if you keep the pressure of a gas constant, the value of its volume divided by its temperature will always be the same. Hence,

$\frac{V}{T} = constant$ or,

$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{V_3}{T_3}, etc.$

These laws can be remembered separately, but from an engineer's point of view they are better combined to give a single very useful expression.

$\frac{p_1V_1}{T_1} = \; constant$ or,

$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = \frac{p_3V_3}{T_3}, etc.$

It is important to remember that this expression is always valid, no matter what the process, and when doing calculations it should always be the first consideration.

Units

p = pressure (bar) or $\left(\frac{N}{m^2}\right)$

V = volume (m^3)

T = absolute temperature (K) (°C + 273)

EX. 1.

A perfect gas in an engine cylinder at the start of compression has a volume of 0.01m3, a temperature of 20°C and a pressure of 2 bar. The piston rises to compress the gas, and at top dead centre the volume is 0.004m3 and the pressure is 15 bar. Find the temperature. See Figure 2.2.1.

Key note

- There are some instances, as we shall see later, where the pressure must be in N/m2, but for the time being, bar can be used. 1 bar = 105 N/m2.
- Volume is always m^3.
- Temperature is always absolute, i.e. K. The best practice is to convert all temperatures to K immediately by adding 273 to your centigrade temperatures. There is an exception to this when you have a temperature difference, since this gives the same value in °C or K.

Solution

$p_1 = 2\ bar$

$p_2 = 15\ bar$

$V_1 = 0.01m^3$

$V_1 = 0.004m^3$

$T_1 = 20 + 273 = 293K$

$T_2 = ?$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{2\ x\ 0.01}{293} = \frac{15\ x\ 0.004}{T_2}$

$T_2 = \frac{15\ x\ 0.004\ x\ 293}{2\ x\ 0.01} = 879\ K$

The geometry of the cylinder and piston is fixed, so the only way in which the value of temperature (or pressure) at the end of compression can be different for the same initial conditions is if there are different rates of heat energy transfer across the cylinder walls, i.e. how much cooling there is of the engine.

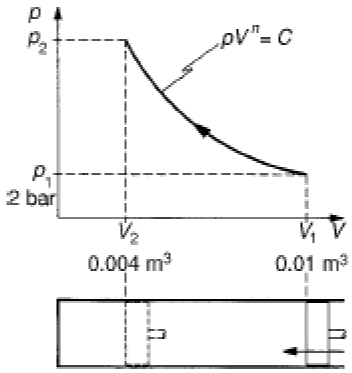

Figure 1 *p/V diagram.*
Example 1

Reversibility

We draw the *p*/*V* diagrams for our processes assuming that the processes are *reversible*. Put simply, this means that the process can be reversed so that no evidence exists that the process happened in the first place. The best analogy is of a frictionless pendulum swinging back and forth without loss of height. However, this is not what happens in practice, because of temperature differences, pressure differences, and turbulence within the fluid during the process, and because of friction, all of which are ‘irreversibilities’. The only way in which a reversible process could be achieved is to allow equilibrium to be reached after each of an infinite number of stages during the process, i.e. extremely slowly to let things settle down. This, unfortunately, would take an infinite time. Strictly speaking, the *p*/*V* diagrams should be dotted to show that we are dealing with irreversible processes, but for convenience solid lines are usually used. We assume that at the end states our gas is in equilibrium. The significance of the concept of reversibility becomes apparent in later studies of thermodynamics.

The first law of thermodynamics

This says that energy cannot be created or destroyed and the total quantity of energy available is constant. Heat energy and mechanical energy are convertible. As a crude illustration we can consider the closed process of work input to rotate a flywheel which is then brought to a stop by use of a friction brake. The kinetic energy of rotation of the flywheel is converted into heat energy at the brake. ‘If any system is taken through a closed thermodynamic process, the net work energy transfer and the net heat energy transfer are directly proportional’ A corollary of the first law says that a property exists such that a change in its value is equal to the difference between the net work transfer and the net heat transfer in a closed system. This property is the internal energy, *U*, of the fluid, which is the energy the fluid has before the process begins and which can change during the process.

Internal energy

Internal energy is the intrinsic energy of the fluid, i.e. the energy it contains because of the movement of its molecules. Joule’s law states that the internal energy of a gas depends only upon its temperature, and is independent of changes in pressure and volume. We can therefore assume that if the temperature of a gas increases, its internal energy increases and if the temperature falls, the value of internal energy falls. We will always be dealing with a change in internal energy, and we can show by applying the non-flow energy equation (see below, ‘The non-flow energy equation’) to a constant volume process that the change in internal energy is given by,

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

Where c_v is the specific heat of the gas at constant volume and *m* is the mass. This expression is true for all the processes which can be applied to a gas, and will be used later.

The non-flow energy equation.

This is a very important expression, which we use later in non-flow processes

$Q = W + (U2 - U1)$

In words, the heat energy supplied is equal to the work done plus the change in internal energy. This can be thought of as an expression of the first law with the internal energy change taken into account.

The system

To study thermodynamics properly, we must know what we are dealing with and where the boundaries are, so that our system is defined. The gas in an engine cylinder forms a *closed system* bounded by the cylinder walls and the piston head. The processes we have been looking at have occurred without mass flow of the gas across these boundaries.

- These are called *non-flow processes*.

On the other hand, if we move the boundary to encompass the complete engine so as to include the inlet and exhaust, there is a mass flow into and out of the system.

- This is called a *steady flow process*.

Steady flow processes also occur in gas turbines, boilers, nozzles and condensers, wherever there is an equal mass flow in and out across the boundary of the system. It is traditional to show the boundary of a system on a diagram by a broken line.

The characteristic gas equation

In the section on gas processes, we saw that

$\frac{PV}{T} = constant$

This is true for any gas, and we can use it for any process. If we take a specific volume of gas, i.e. the volume which 1 kg occupies, we give it the designation, *v*, and the result of our expression is a constant called the *gas constant*, *R*.

Putting in the units,

$$\frac{P\ x\ V}{T} = \frac{N}{m^2}\ x\ \frac{m^3}{kg}\ x\ \frac{1}{k} = \frac{N\ x\ m}{kgK} = \frac{J}{kgK} = R$$

Reverting to our use of *V* for non-specific volume, and using *m* for the corresponding mass, the equation can be arranged to give,

$$p.V = m.R.T$$

This is called the *characteristic gas equation*.

Each gas has a particular value of gas constant. Air, for instance, has a gas constant *R* = 287 J/kgK. The equation is often used to find the mass of a gas when other properties (*p*, *V* and *T*) are known, but can be transposed to find any of the variables. It must be used at only one point in the process. It is often necessary to use it during a gas process calculation, alongside the gas laws expressions.

Example 2

An air receiver contains 40 kg of air at 20 bar, temperature 15°C. 5 kg of air leaks from the receiver until the pressure is 15 bar and the temperature falls to 10°C. Find the volume of the receiver and the mass of air lost.

R = 287 J/kgK

$p.V = m.R.T$

Using the initial conditions to find the volume of the receiver,

$V = \frac{m.R.T}{P} = \frac{40\ x\ 287\ x\ (15+273)}{20\ x\ 10^5} = 1.65m^3$

Hence, after leakage,

$$m = \frac{P.V}{R.T} = \frac{15\ x\ 10^5\ x\ 1.65}{287\ x\ (10+273)} = 30.47kg$$

Mass used =40 -30.47 = 9.53kg.

Gauge pressure and absolute pressure

As the name implies, *gauge pressure* is the pressure recorded by a gauge attached to the tank or receiver. An empty tank would record a gauge pressure of zero, but in fact atmospheric pressure would be acting. The pressures used in thermodynamics are *absolute pressures*, i.e. gauge pressure + atmospheric pressure.

To change a gauge to an absolute pressure, add atmospheric pressure, which is 1.013 bar = 1.013 x 10^5 N/m^2 = 101.3 kN/m^2.

Problems 1

(1) An air bottle has a volume of 1m3. Find the mass of air in the bottle when the contents are at 20 bar and 15°C. *R* = 287 J/kgK.

(2) An air receiver contains 25 kg of air at a pressure of 2000 kN/m2, gauge, temperature 20°C. Find the volume of the receiver. *R* = 287 J/kgK. Atmospheric pressure = 1 bar.

(3) A starting-air receiver for a large diesel engine has an internal volume of 1.5m3. Calculate:

(a) The mass of air in the receiver when the contents are at 25 bar, 15°C;

(b) The starting-air pressure in the receiver if 2 kg of air are used and the temperature remains constant.

R = 287 J/kgK.

The gas processes

We have already seen that for any process we can use the combined Boyle's and Charles' laws expression,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

If we apply this to a constant pressure, constant volume or isothermal (constant temperature) process, all we need do is strike out the constant term before we start calculating. The use of other expressions will depend upon the particular process. We will consider the following non-flow processes, e.g. gas in an engine cylinder.

Constant pressure

The gas is held at constant pressure as the volume changes. This would require the addition or extraction of heat energy. For instance, if the piston is moved up the cylinder, the heat energy produced would need to be taken away if the pressure was to remain constant (Figure 2).

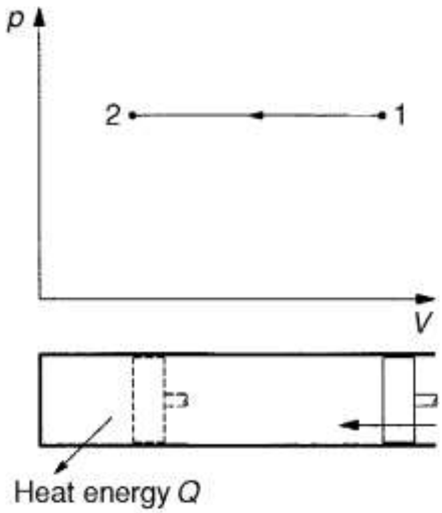

Figure 2 *Constant pressure process*

Constant volume

The volume remains constant, i.e. the piston is fixed. Clearly, the only process which can occur is heating or cooling of the gas (Figure 3).

- It is important to remember that these are the only two processes which are straight lines on the *p*/*V* diagram.

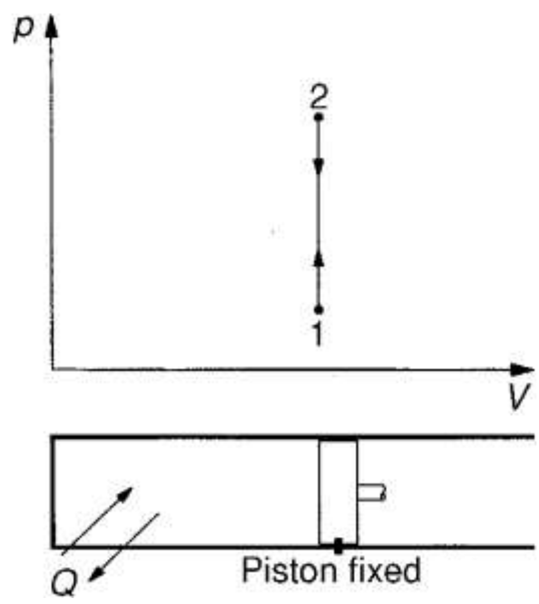

Figure 3 *Constant volume process*

Adiabatic compression and expansion

During an adiabatic process, no heat transfer occurs to or from the gasduring the process. This would require the perfect insulation of the cylinder, which is not possible, and it is worth noting that even the insulation itself will absorb some heat energy (Figure 4). The index of expansion, _, for a reversible adiabatic process is the ratio of the specific heats of the gas, i.e.,

$\gamma = \frac{c_p}{c_v}$

The equation of the curve is

$P.V^{\gamma} = constant$

$P_1V_1^{\gamma} = P_2V_2^{\gamma} = P_3V_3^{\gamma}, etc.$

- Note that a reversible adiabatic process is known as an isentropic process, i.e. constant entropy. Entropy is discussed later.

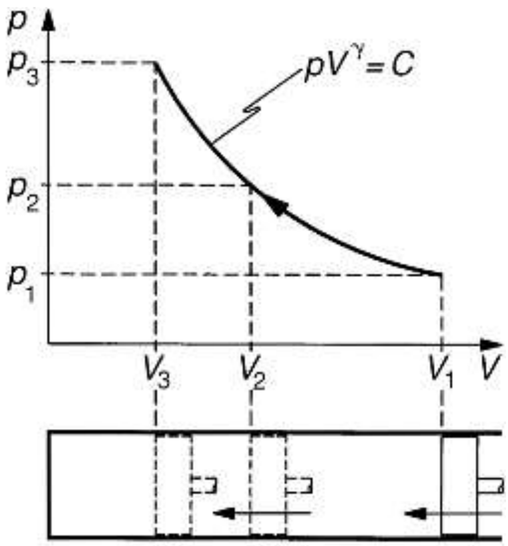

Figure 4 *Adiabatic process*

Polytropic expansion and compression

This is the practical process in which the temperature, pressure and volume of the gas all change. All gas processes in the real world are polytropic – think of the gas expanding and compressing in an engine cylinder (Figure 5).

The equation of the curve is

$P.V^{n} = constant$

$P_1V_1^{n} = P_2V_2^{n} = P_3V_3^{n}, etc$

where 'n' is the index of polytropic expansion or compression.

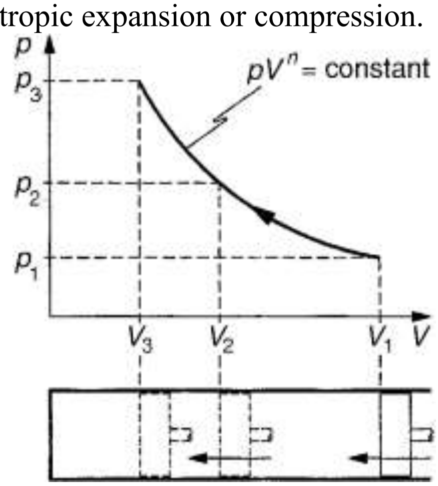

Figure 5 *Polytropic process*

Isothermal process

In this case, which is Boyle's law, the temperature of the gas remains constant during the process. Like the adiabatic process, this cannot be achieved in practice (Figure 6).

The equation of the curve is

$p.V = C$

$p_1V_1 = p_2V_2 = p_3V_3$, etc.

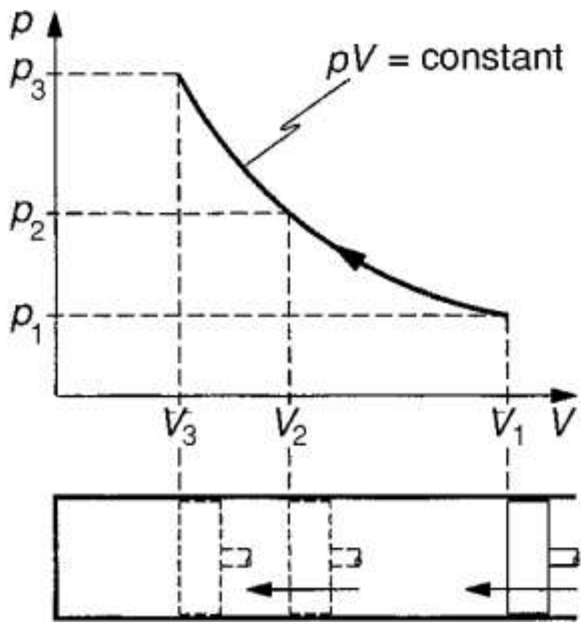

Figure 6 *Isothermal process*

By combining the equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ with $P_1V_1^n = P_2V_2^n$ we can produce another important expression,

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$ for a polytropic process.

This can be applied to the adiabatic process to give,

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{n}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Key Note.

- Before calculating, determine the type of process.
- Always sketch the *p*/*V* diagram for the process.
- Remember that polytropic, adiabatic and isothermal processes are all curves on the *p*/*V* diagram. A curve must therefore be labelled to show which it is.
- The adiabatic curve is steepest and, as the index lowers, so the *p*/*V* curve flattens (Fig.7).

- The three curves, i.e. polytropic, adiabatic and isothermal, are all of the general form $pV^n = C$, where in the isothermal case, $n = 1$, and in the adiabatic case, $n = \gamma$ .The value of n in the polytropic case lies between 1 and γ.
- Compression and expansion processes are calculated in exactly the same way, but remember that on the p/V diagram they must be indicated by the direction of the arrow.

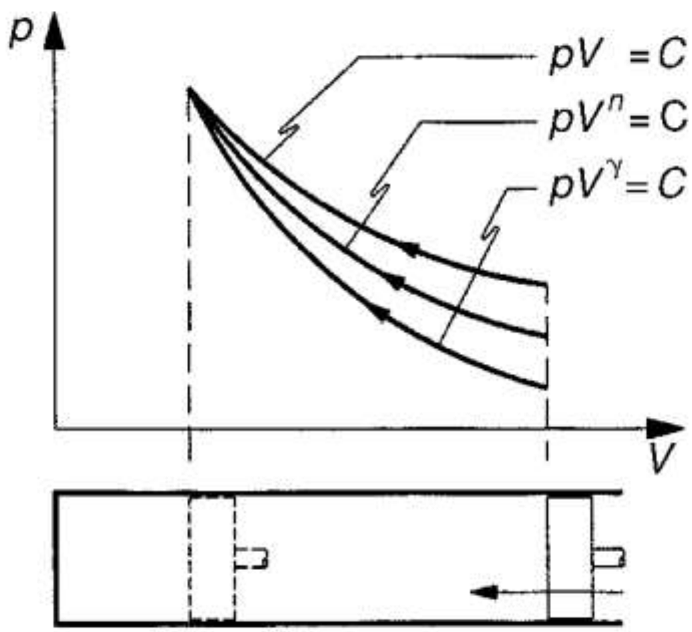

Figure 7 *Compression curves*

NOTE

In determining values of pressure, volume and temperature using the gas laws formulae, it is necessary to transpose equations which include powers. We have the formula,

$P_1 V_1^n = P_2 V_2^n$, this gives,

$p_1 = \frac{p_2 V_2^n}{V_1^n} = P_2 \left(\frac{V_2}{V_1}\right)^n$, and,

$V_1 = \left(\frac{P_2 V_2^n}{P_1}\right)^{\frac{1}{n}} = \sqrt[n]{\frac{P_2 V_2^n}{P_1}}$

Example 3

Gas of volume 0.02m^3 is cooled until its volume is 0.015m^3 while its pressure remains constant. If the initial temperature is 50°C, find the final temperature (Figure 8).

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, the pressure terms are omitted.

$\frac{0.02}{50+273} = \frac{0.015}{T_2}$

$T_2 = \frac{0.015\ x\ 323}{0.02} = 242.25\ K$

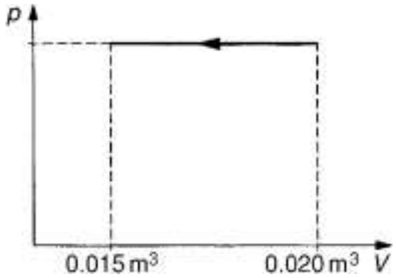

Figure 8 *Example 3*

Example 4
Air of volume 0.15m^3 is compressed adiabatically from a pressure of 2 bar, temperature 12°C, to a volume of 0.02m^3. If $\gamma = 1.4$, find the final pressure and temperature (Figure 9).

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$2\ x\ 0.15^{1.4} = P_2\ x\ 0.02^{1.4}$

$P_2 = 2\ x\ \frac{0.15^{1.4}}{0.02^{1.4}} = 33.58\ bar$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{2\ x\ 0.15}{12+273} = \frac{33.58\ x\ 0.02}{T_2}$

$T_2 = 638\ K$

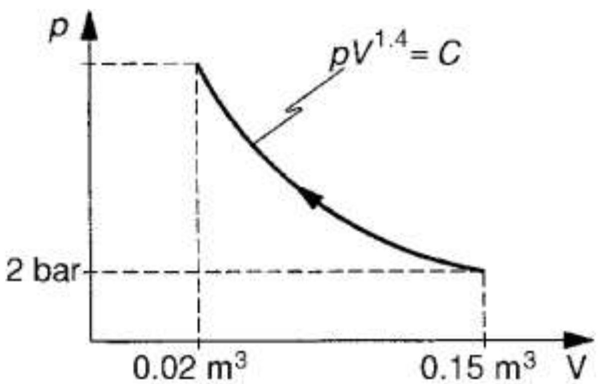

Figure 9 *Example 4*

Example 5
Gas at a temperature of 20°C, pressure 2 bar, is compressed to 10 bar. The compression is polytropic, n = 1.3. Find the final temperature (Figure 10). In this case we must use the relationship

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

$\frac{T_2}{273+20} = \left(\frac{10}{2}\right)^{\frac{1.3-1}{1.3}}$

$T_2 = 293\ x\ 5^{0.23}$

$T_2 = 424.3\ K$

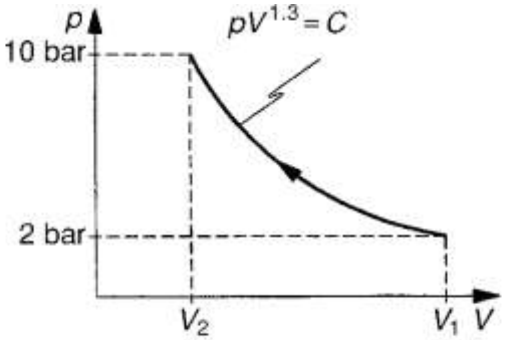

Figure 10 *Example 5*

NOTE

Using logs

It may be that properties are known, but the index of compression or expansion is not. Let us assume in a compression process we know *p*1 and V_1 and p_2 and V_2, and we want to find the equation of the curve.

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

Rearranging,

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

Taking logs of both sides,

$$log\left(\frac{P_1}{P_2}\right) = n.log\left(\frac{V_2}{V_1}\right)$$

$$n = \frac{log(P_1/P_2)}{log(V_2/V_1)}$$

Home Problems

(In each case sketch the *p*/*V* diagram.)

(1) A quantity of gas is heated from 15°C to 50°C at constant volume. If the initial pressure of the gas is 3 bar, find the new pressure.

(2) Gas of volume 0.5m^3 is heated at constant pressure until its volume is 0.9m^3. If the initial temperature of the gas is 40°C, find the final temperature.

(3) 2m^3 of gas is compressed polytropically (n = 1.35) from a pressure of 2 bar to a pressure of 25 bar. Calculate:

(a) final volume;

(b) final temperature if the initial temperature is 25°C.

(4) Gas at 20°C, volume 1m3, is compressed polytropically from a pressure of 2 bar to a pressure of 10 bar. If the index of compression is n = 1.25, find:

(a) final volume;

(b) final temperature.

(5) Gas of volume 0.2m^3, pressure 5 bar, temperature 30°C, is compressed until its volume is 0.04m^3. If the compression occurs in a perfectly insulated cylinder, calculate the final temperature and pressure. γ = 1.4.

READ MORE ON:

Work done and heat energy supplied