



LANDMARK UNIVERSITY, OMU-ARAN

COLLEGE: COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT: MECHANICAL ENGINEERING

PROGRAMME: MECHANICAL ENGINEERING

Course

Course code: MCE 538

Course title: AUTO SYSTEM AND VEHICLE DYNAMICS

Course Units: 3 UNITS.

Course status: OPTIONAL.

LECTURE NOTE 1

ENGR. ALIYU, S. J

MCE 538 Auto System and Vehicle Dynamics (3 Units)

Friction forces in Automobile systems; Drag and propelling forces; Effect of body shape on vehicles. Production, assembly line and power systems control techniques. Principles of automation in mechanized systems. Application of thermal, pneumatic, hydraulic and fluidic systems to automatic control in plant processes and machinery.

LONGITUDINAL DYNAMICS

Introduction

The primary purpose of a vehicle is transportation, which requires longitudinal dynamics. This section is organised with a group of functions in each section as follows:

- Steady State Function
- Functions over longer events
- Functions in shorter events
- Control functions

Most of the functions in “Control functions”, but not all, could be parts of “Functions in shorter events”. However, they are collected in one own section, since they are special in that they partly rely on software algorithms.

There are a lot of propulsion related functions, originating from the attribute Driving dynamics. Examples of such, not covered in this lecture are:

- Off-road accessibility: Ability to pass obstacles of different kind, such as uneven ground, extreme up- and down-hills, mud depth, snow depth, etc.
- Shift quality: Quick and smooth automatic/automated gear shifts
- Shunt & shuffle: Absence from oscillation for quick pedal apply, especially accelerator pedal.

Steady State Functions

Functions as top speed and grade-ability are relevant without defining a certain time period. For such functions it is suitable to observe the vehicle in steady state, i.e. independent of time. Those functions are therefore called steady state functions, in this lecture.

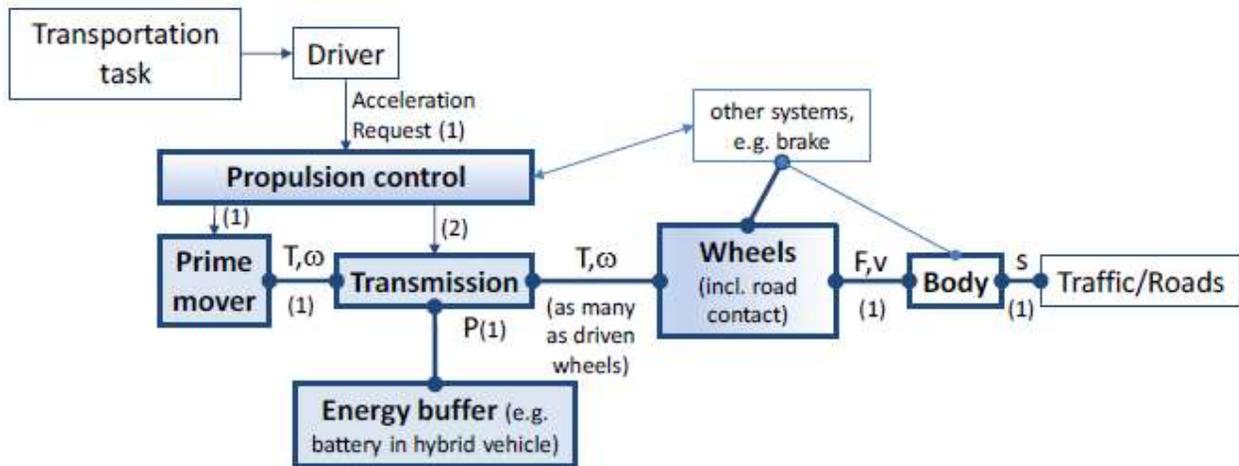
The main subsystems that influences here are the propulsion system and the (Friction) Brake system.

Propulsion System

A generalisation of the propulsion system is given in Figure 1, along with a specific example of a conventional type. The component “Energy buffer” refers to an energy storage that can “buffer” energy during vehicle operation. This means that an energy buffer can not only be emptied (during propulsion), but also refilled by regenerating energy from the vehicle during deceleration. A fuel tank is an energy storage, but not an energy buffer. Also, a battery which can only be charged from the grid, and not from regenerating deceleration energy, is not an Energy buffer.

Note that the approach in Figure 1 is one-dimensional: we consider neither the differential between left and right wheel on the driven axle nor distribution between axles. Instead, we sum up the torques at all wheels and assume same rotational speed.

Generalized propulsion system:



Conventional propulsion system:

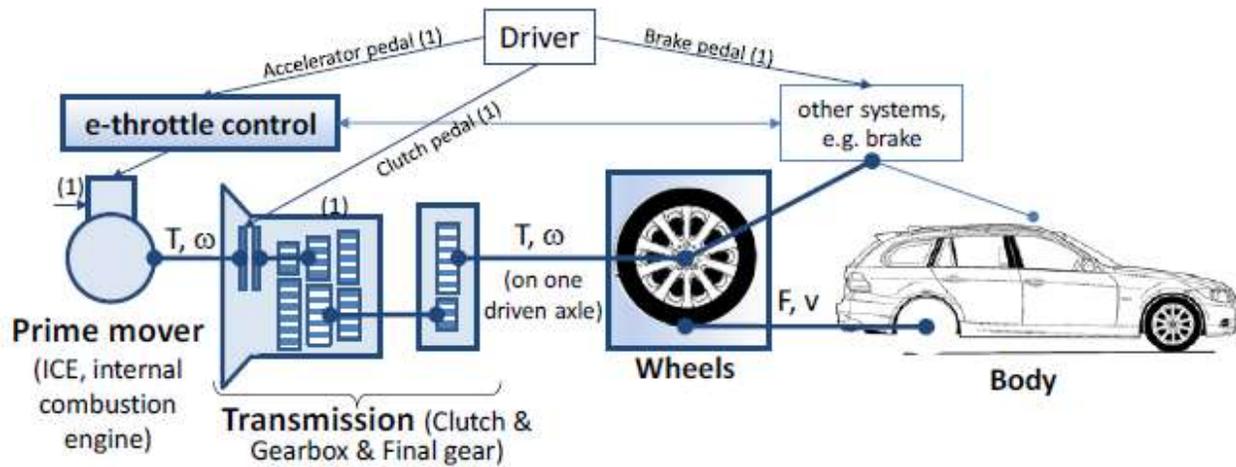


Figure 1: Propulsion system

Prime movers

The conversion of stored energy to power occurs in the prime mover, see Figure 1. Details of the conversion processes and transmission of power to the tyres are not covered in this compendium. Some basic background is still necessary to describe the longitudinal performance of the vehicle. The main information that is required is a description of the torque applied to the wheels over time and/or as a function of speed. Sketches of how the maximum torque varies with speed for different prime movers (internal combustion engine (ICE), electric motor or similar) are shown in Figure 2 and Figure 3. The torque speed characteristics vary dramatically between electric and internal combustion engines. Also, gasoline and diesel engines characteristics vary.

The curve for electric motors in Figure 3-2 shows that the main speciality, compared to ICEs, is that their operation range is nearly symmetrical for negative speeds and torques. However, the curve should be taken as very approximate, since electric motors can work at higher torque for short periods of time. The strong time duration dependency makes electric motors very different to ICEs from a vehicle dynamics point of view. Other properties that makes them special are quick and accurate response, well known actual torque and that it is much more realistic to divide them into several smaller motors, which can operate on different wheels/axles.

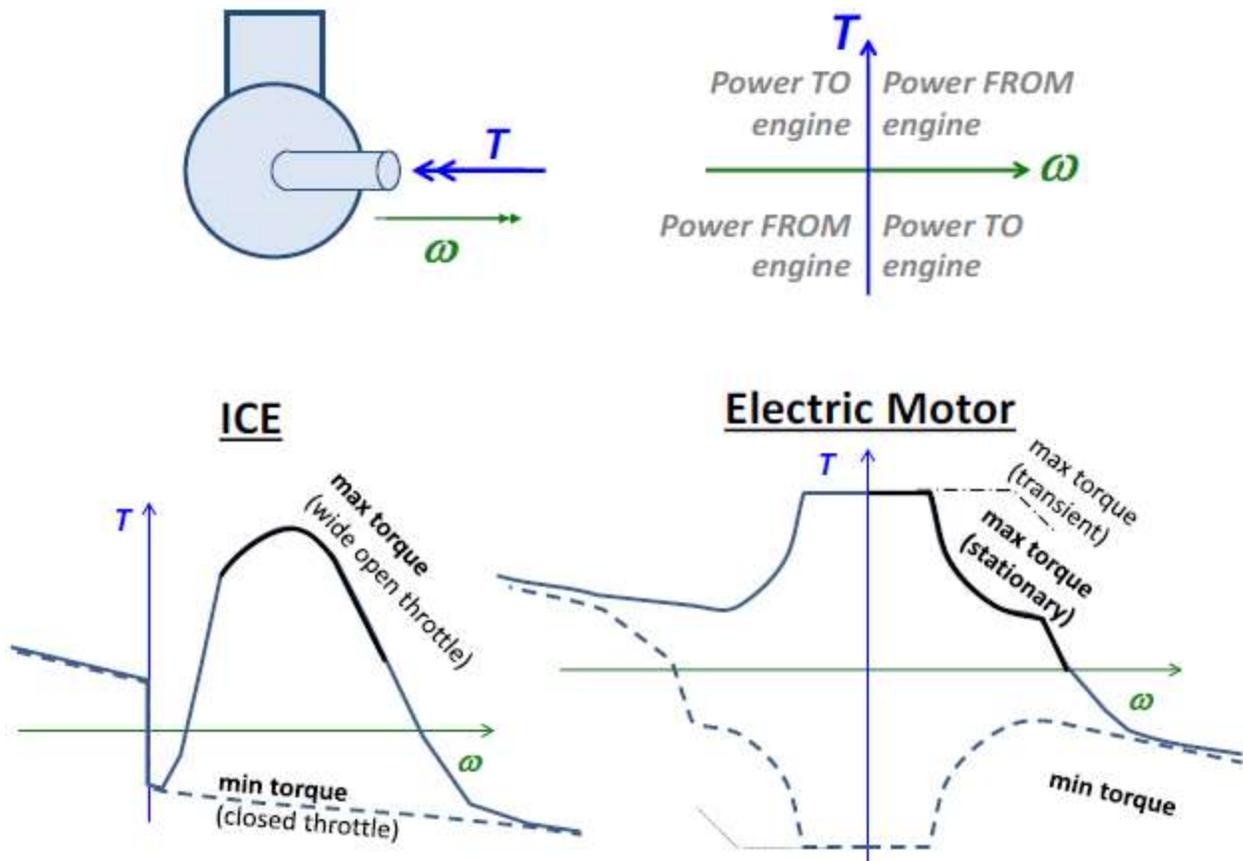


Figure 2: Torque Characteristics of Prime Movers.

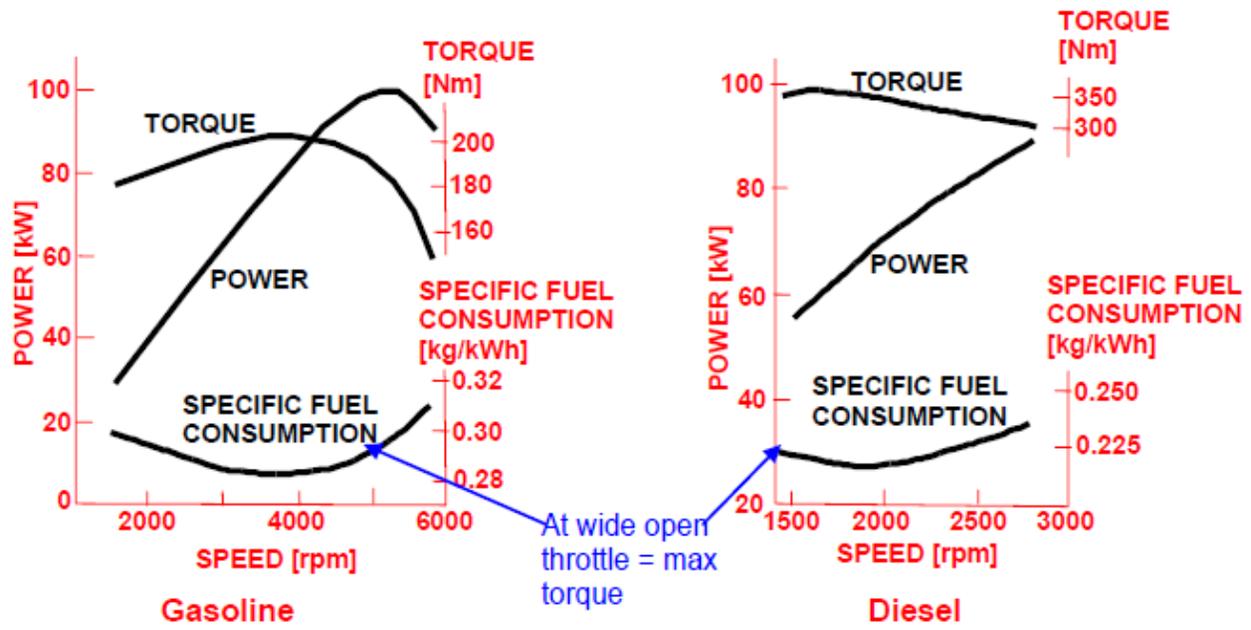


Figure 3: Engine Characteristics for Gasoline and Diesel Engines. Typical values for passenger car.

LATERAL DYNAMICS

Introduction

The lateral motion of a vehicle is needed to follow the roads' curves and select route in intersections as well as to laterally avoid obstacles that appear. The vehicle needs to be steerable. With some simplification, one can say that lateral dynamics is about how steerable the vehicle is for different given longitudinal speeds. Vehicle steering is studied mainly through the vehicle degrees of freedom: yaw rotation and lateral translation.

A vehicle can be steered in different ways:

- Applying steering angles on, at least one, road wheel. Normally both of front wheels are steered.
- Applying longitudinal forces on road wheels. Either unsymmetrical between left and right side of vehicle, e.g. one sided braking, or deliberately use up much friction longitudinally on one axle in a curve, so that that axle loses lateral force.
- Articulated steering, where the axles are fixed mounted on the vehicle but the vehicle itself can bend.

The section is organised with a group of functions in each section as follows:

- Low speed manoeuvrability
- Steady state cornering at high speed
- Stationary oscillating steering
- Transient handling
- Lateral Control Functions

Most of the functions in "Lateral Control Functions", but not all, could be parts of "Transient handling". However, they are collected in one own section, since they are special in that they partly rely on software algorithms.

The lateral dynamics of vehicles is often experienced as the most challenging for the new automotive engineer. Longitudinal dynamics is essentially motion in one plane and rectilinear. Vertical dynamics may be 3 dimensional, but normally the displacements are small and in this compendium the vertical dynamics is mainly studied in one plane as rectilinear. However, lateral dynamics involves analysis of motion in the vehicle coordinate system which introduces curvilinear motion since the coordinate system is rotating as the vehicle yaws.

The turning manoeuvres of vehicles encompass two concepts. Handling is the driver's perception of the vehicle's response to the steering input. Cornering is usually used to describe the physical response (open-loop) of the vehicle independent of how it influences the driver.

Steady state cornering roll-over

When going in curves, the vehicle will have roll angles of typically some degrees. At that level, the roll is a comfort issue. However, there are manoeuvres which can cause the vehicle to roll-over, which basically means that it rolls at least 90 degrees. So, this is an actual accident event. Roll-over can be seen as a special event, but if sorting into the chapters of this compendium it probably fits best in present chapter, about lateral dynamics. One can categorize roll-overs in e.g. 3 different types:

- **Tripped roll-over.** This is when the car skids sideways and hits an edge, which causes the roll-over. It can be an uprising edge, e.g. pavement or refuge. It can be the opposite, a ditch or loose gravel outside road. In both these cases, it is strong lateral forces on the wheels on one side of the vehicle that causes the roll-over. Tripped roll-over can also be when the vehicle is exposed to large one-sided vertical wheel forces, e.g. by running over a one-sided bump. A third variant of tripped roll-over is when the vehicle is hit by another vehicle so hard that it rolls over.
- **Un-tripped roll-over** or on-road roll-overs. These happen on the road and triggered by high tyre lateral forces. This is why they require high road friction. For sedan passenger cars these event are almost impossible, since road friction seldom is higher than approximately 1. For SUVs, un-tripped roll-overs can however occur but require dry asphalt roads, where friction is around 1. For trucks, un-tripped roll-over, can happen already at very moderate friction, like 0.4, due to their high CoG in relation to track width. Within un-tripped roll-overs, one can differ between:
 - o **Steady state roll-over.** If lateral acceleration is slowly increased, e.g. as running with into a hairpin curve or a highway exit, the vehicle can slowly lift off the inner wheels and roll-over. This is the only case of roll-over for which an analysis model is given in this compendium.
 - o **Transient roll-over.** This is when complex manoeuvres, like double lane changes or sinusoidal steering, are made at high lateral accelerations. This can trigger roll eigenmodes, which can be amplified due to unlucky timing between the turns. Analysis models can be used as a start, but it is required that load transfer is modelled carefully and includes wheel lifts, suspension end-stops and bump stops.

Roll over threshold definitions

An overall requirement on a vehicle is that the vehicle should not roll-over for certain manoeuvres. Heavy trucks will be possible to roll-over on high-mu conditions. The requirement for those are based on some manoeuvres which not utilize the full road friction. For passenger cars, it is often the intended design that they should be impossible to roll-over, even at high mu. Any requirement need a definition of what exactly roll-over is, i.e. a Roll over threshold definition. Candidates for Roll over threshold definition are:

- **One wheel** lift from ground
- All wheels on **one side** lift from ground
- Vehicle **COG** passes its **highest point**

Note that:

- It is the 3rd threshold which really is the limit, but other can still be useful in requirement setting. To use the 3rd for requirement setting makes the verification much more complex, of course in real vehicles but also in simulation.
- The 1st is not a very serious situation for a conventional vehicle with 4 wheels. However, for a 3-wheeled vehicle, such as small “tuc-tucs” or a 3-wheel moped, it is still a relevant threshold.
- The 2nd threshold is probably the most useful threshold for two-tracked vehicles, because it defines a condition from which real roll-over is an obvious risk, and still it is relatively easy to test and simulate. For 3-wheeled vehicle, 2nd and 3rd threshold generally coincide.

In the following, 4-wheeled vehicles will be assumed. The 2nd threshold will be used.

Static Stability Factor, SSF

One very simple measure of the vehicles tendency to roll-over is the Static Stability Factor, SSF. It is proposed by NHTSA, http://www.nhtsa.gov/cars/rules/rulings/roll_resistance/, and it is simply defined as:

$$SSF = \frac{\text{Half TrackWidth}}{\text{HeightOfCoG}} = \frac{w/2}{h};$$

A requirement which requires $SSF > \text{number}$ cannot be directly interpreted in terms of certain manoeuvre and certain roll-over threshold. It is not a *performance based requirement*, but a *design based requirement*. However, one of many possible performance based *interpretations* is that the vehicle shall not roll-over for steady-state cornering on level ground with a certain friction coefficient, using one-sided wheel lift as threshold. Since the requirement is not truly performance based, each interpretation will also stipulate a certain verification method; here it would be theoretical verification using a rigid suspension model. Such model and threshold is shown in Figure 4.

view from rear, when turning left

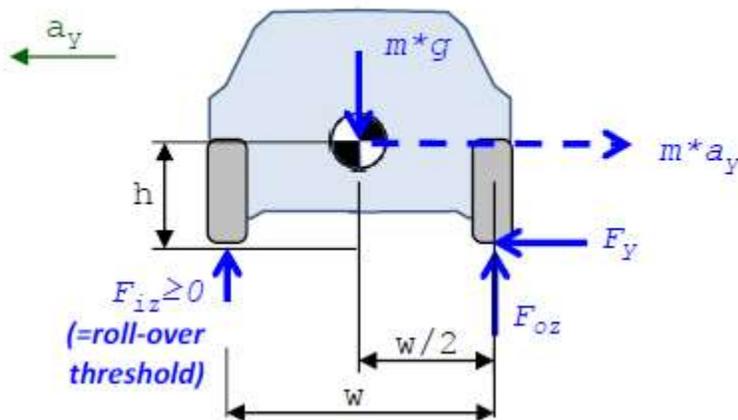


Figure 4: Model for verification of requirement based on Static Stability Factor, SSF. The derivation of the SSF based requirement looks as follows:

$$\left. \begin{array}{l} \text{Model:} \\ \left. \begin{array}{l} F_{iz} \cdot w + m \cdot a_y \cdot h = m \cdot g \cdot \frac{w}{2}; \\ F_{iz} + F_{oz} = m \cdot g; \\ m \cdot a_y = F_y = \mu \cdot (F_{iz} + F_{oz}); \end{array} \right\} \Rightarrow F_{iz} = m \cdot g \cdot \left(\frac{1}{2} - \frac{h \cdot \mu}{w} \right); \\ \text{Requirement:} \\ \left. \begin{array}{l} F_{iz} \geq 0; \end{array} \right\} \Rightarrow \frac{1}{2} > \frac{h \cdot \mu}{w} \Rightarrow \frac{w}{2 \cdot h} = SSF > \mu; \end{array} \right\}$$

1

Maximum road friction, μ , is typically 1, which is why $SSF > \mu = 1$ would be a reasonable. However, typical values of SSF for passenger vehicles are between 0.95 and 1.5. For heavy trucks, it can be much lower, maybe 0.3 to 0.5, much depending on how the load is placed. There

are objections to use SSF as a measure, because SSF ignores suspension compliance, handling characteristics, electronic stability control, vehicle shape and structure.

Steady-state cornering roll-over

Model with fore/aft symmetry

The model in Figure 4 and Equation [1] assumes fore/aft symmetry. One can derive this **requirement** on design: $\frac{w}{2.h} > \mu$, and this interpretation of **performance** (a_y) limitation due to roll-over: $\frac{a_y}{g} < \frac{w}{2.h}$. In the following, we will elaborate with 4 additional effects and derive how they affects this: $\frac{a_y}{g} < \frac{w}{2.h} > \mu$. The 4 effects are each connected to one measure, which is marked in Figure 5.

- The tyre will take the vertical load on its **outer edge** in a roll-over situation. This suggests a change of performance and requirement to: $\frac{a_y}{g} < \frac{w + w_{tyre}}{2.h} > \mu$. This effect is accentuated when low tyre profile and/or high inflation pressure. This effect **decreases** the risk for roll-over.
- Due to suspension and tyre **lateral deformation**, the body will translate laterally outwards, relative to the tyre. This could motivate $\frac{a_y}{g} < \frac{w - Def_y}{2.h} > \mu$. This effect **increases** the risk for roll-over.
- Due to suspension linkage and compliances, the **body will roll**. Since the CG height above roll axis, Δh , normally is positive, this could motivate $\frac{a_y}{g} < \frac{w - \Delta h \cdot \phi_x}{2.h} > \mu$. This effect **increases** the risk for roll-over.
- Due to suspension linkage and compliances, the body will also heave. This requires a suspension model with pivot points per wheel, as opposed to roll-centre per axle, to be taken into account. The heave is normally positive. This could motivate $\frac{a_y}{g} < \frac{w}{2.(h+z)} > \mu$. The effect is sometimes called “jacking” and it **increases** the risk for roll-over.
- Road leaning left/right (road banking), or driving with one side on a different level (e.g. outside road or on pavement) also influence the roll-over performance.

View from rear, when turning left

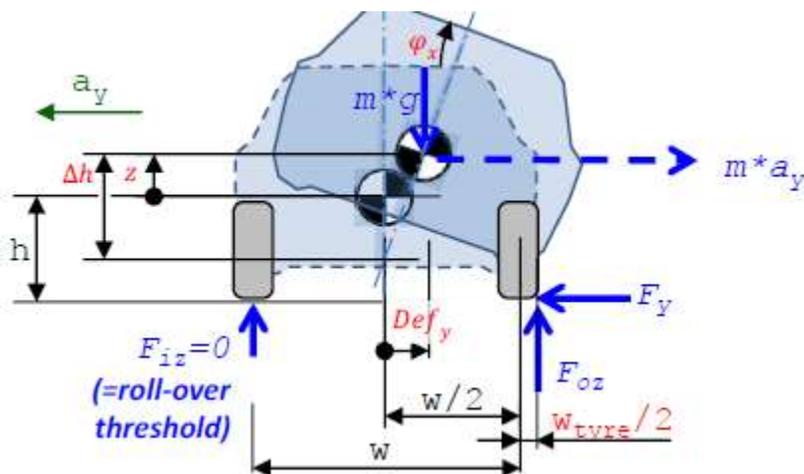


Figure 5: Steady-state roll-over model, with fore/aft symmetry. The measures w_{tyre} , Def_y , Δh , φ_x and z mark effects additional to what is covered with a simple SSF approach.

VERTICAL DYNAMICS

Introduction

The vertical dynamics are needed since vehicles are operated on real roads, and real roads are not perfectly smooth. Also, vehicle can be operated off-road, where the ground unevenness is even larger.

The irregularities of the road can be categorized. A **transient** disturbance, such as a pothole or object on the road, can be represented as a step input. Undulating surfaces like grooves across the road may be a type of sinusoidal or other **stationary** (or periodic) input. More natural input like the random surface texture of the road may be a **random** noise distribution. In all cases, the same mechanical system must react when the vehicle travels over the road at varying speeds including doing manoeuvres in longitudinal and lateral directions.

This section is organised with a group of functions in each section as follows:

- Suspension System
- Stationary oscillations theory
- Road models
- One-dimensional vehicle models
- Ride comfort
- Fatigue life
- Road grip
- Two dimensional
- Transient vertical dynamics
- Other excitation sources

This section is, organised so that all theory (knowledge) comes first and the vehicle functions comes after. In Figure 6 shows the 3 main functions, Ride comfort, Fatigue life and Road grip. It is supposed to explain the importance of the vehicle's dynamic structure. The vehicle's dynamic structure calls for a pretty extensive theory base.

Models in this section focus the disturbance from vertical irregularities from the road, i.e. only the vertical forces on the tyre from the road and **not** the forces in road plane. This enables the use of simple models which are independent of exact wheel and axle suspension, such as pivot axes and roll centres. Only the wheel stiffness rate (effective stiffness) and wheel damping rate (effective damping), has to be given. This has the benefit that the chapter becomes relatively independent of previous chapters, but it has the drawback that the presented models are **not** really suitable for studies of steep road irregularities (which have longitudinal components) and sudden changes in wheel torque or tyre side forces.

A vehicle function which is not covered in this compendium is noise.

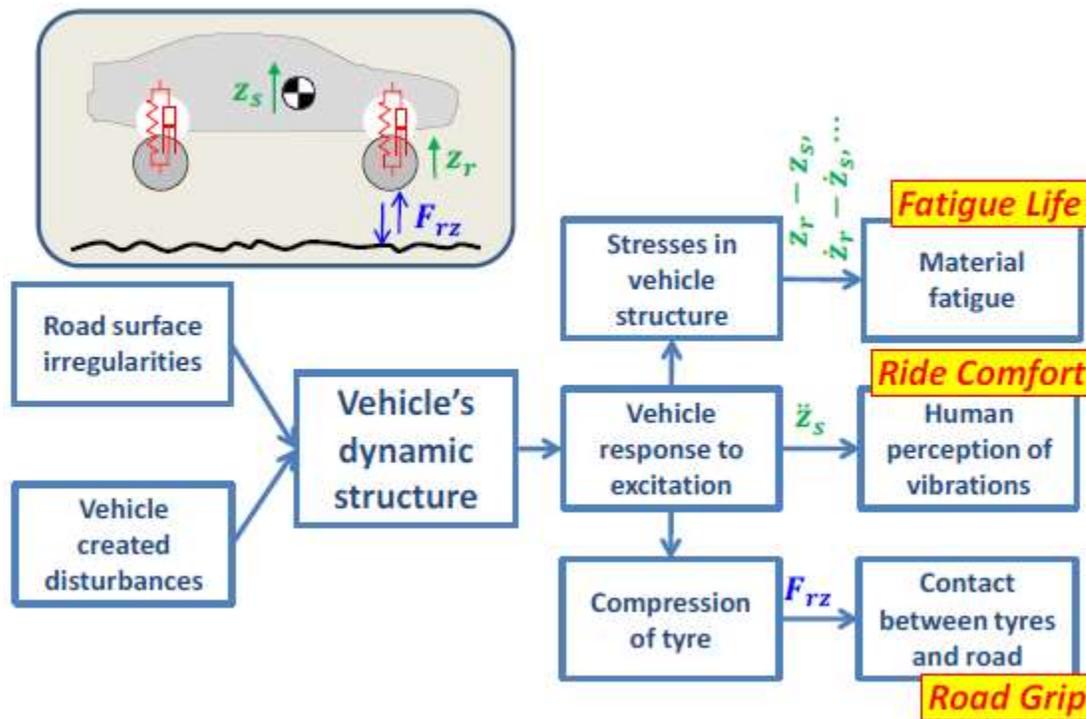


Figure 6: Different types of knowledge and functions in the area of vertical vehicle dynamics, organised around the vehicle's dynamic structure.

Suspension System

Suspension design influence ride comfort, load on suspension and road grip through how **vertical forces** and **camber and steering angles** on each wheel changes with **body motion** (heave, roll, pitch), **road unevenness** (bumps, potholes, waviness) and **wheel forces in ground plane** (from Propulsion, Braking and Steering subsystems).

Suspension in a vehicle may refer to suspension wheels (or axles), suspension of sub-frame and drivetrain and suspension of cabin (for heavy trucks). In present section, only wheel (or axle) suspension is considered.

A wheel suspension has the purpose to constrain the wheel from 6 degrees of freedom, dofs, relative the body, to 2 or 3 dofs. A steered wheel needs 3 dofs (pitch rotation, vertical translation and yaw rotation). For an un-steered wheel, also the yaw rotation should be constrained. For most steering, left and right wheels on an axle have dependent steering angles, which could be seen as 2.5 dofs/wheel.

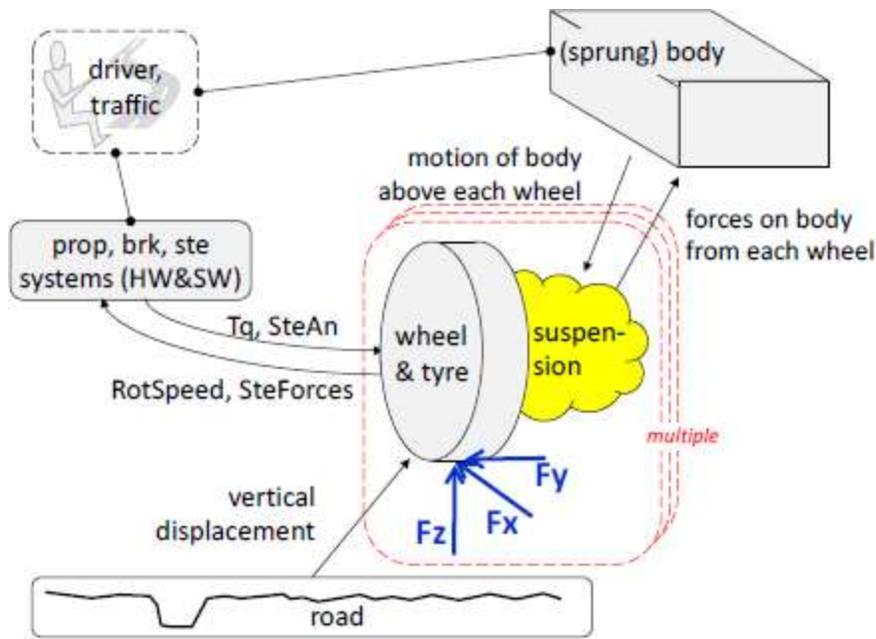


Figure 7: Individual wheel suspension described as one modular sub-model per wheel. It may be noted that a both wheel model (main geometry such as wheel radius) and tyre model (how F_x and F_y vary with tyre slip and F_z) is a part of each such sub-model.

An (axle) suspension system mainly consists of:

- **Linkage**, which has the purpose to constrain the relative motion between wheel and body. An alternative way to express this is that the linkage defines how longitudinal and lateral wheel forces are brought into the body (body = sprung mass). Effective pivot points and roll/pitch centres, mentioned earlier in this compendium, are defined through the linkage. In the real pivot points in the linkage, there are **bushings** with stiffness and damping. The bushings stiffness is much larger than the stiffness of the compliances mentioned below. For steered wheels, the linkage also has the purpose to allow steering. There are two main concepts: Individual wheel suspension and rigid axle suspension.
- **Compliances** (or springs), which has the purpose to allow temporary vertical displacement of the wheels relative to the body. There are often one spring per wheel but also a spring per axle. The second is called anti-roll bar and connects left and right wheel to each other to reduce body roll. Compliances often have a rather linear relation between the vertical displacement and force of each wheel, but there are exceptions:
 - o **Anti-roll bars** make two wheels dependent of each other (still linear). Anti-roll bars can be used on both individual wheel suspensions and rigid axle suspensions.
 - o The compliances can be intentionally designed to be non-linear in the outer end of the stroke, e.g. **bump stops**.
 - o The compliances can be non-linear during the whole stroke, e.g. **air-springs** and **leaf-springs**.
 - o The compliances can be intentionally designed to be **controllable** during operation of the vehicle. This can be to change the pre-load level to adjust for varying roads or varying weight of vehicle cargo or to be controllable in a shorter time scale for compensating in each oscillation cycle. The latter is very energy consuming and no such “active suspension” is available on market.

- **Dampers**, which has the purpose to dissipate energy from any oscillations of the vertical displacement of the wheel relative to the body. Dampers often has a rather linear relation between the vertical deformation speed and force of each wheel, but there are exceptions:

- o The dampers can be intentionally designed to be different in different deformation direction. This is actually the normal design for dampers of **hydraulic piston type**, and it means that damping coefficient is different in compression and rebound.

- o Damping in **leaf springs** is non-linear since they work with dry friction.

- o Damping in **air-springs** is non-linear due to the nature of compressing gas.

- o The dampers can be intentionally designed to be **controllable** during operation of the vehicle. This can be to change the damping characteristics to adjust for varying roads or varying weight of vehicle cargo or to be controllable in a shorter time scale for compensating in each oscillation cycle. The latter is called “semi-active suspension” and is available on some high-end vehicles on market.

The simplest view we can have of a suspension system is that there is an individual suspension between the vehicle body and each wheel. Each such suspension is a parallel arrangement of one linear spring and one linear damper. This lecture uses this simple view for analysis models, because it facilitates understanding and it is enough for a first order evaluation of the functions studied (comfort, road grip and fatigue load) during normal driving on normal roads.

Road models

In general, a road model can include ground properties such as coefficient of friction, damping/elasticity of ground and vertical position. The independent variable is either one, along an assumed path, or generally two, x and y in ground plane. In vertical dynamics in this compendium, we only assume vertical displacement as function of a path. We use x as independent variable along the path, meaning that the road model is: $z_r = z_r(x)$. The function $z_r(x)$ can be either of the types in Figure 8. We will concentrate on stationary oscillations, which by Fourier series, always can be expressed as multiple (spatial) frequency harmonic stationary oscillation. This can be specialized to either single (spatial) frequency or random (spatial) frequency. Hence, the general form of the road model is multiple (spatial) frequencies:

$$z_r = z_r(x) = \sum_{i=1}^N \hat{z}_i \cdot \cos(\Omega_i \cdot x + x_{0i}); \quad 2$$

One frequency road model

For certain roads, such as roads built with concrete blocks, a single (spatial) frequency can be a relevant approximation to study a certain single wave length. Also, the single (spatial) frequency road model is good for learning the different concepts. A single (spatial) frequency model is the same as a single wave length model ($\lambda=2\cdot\pi\Omega'$,) and it can be described as:

$$z_r = z_r(x) = \hat{z} \cdot \cos(\Omega \cdot x + x_0); \quad 3$$

Multiple frequency road models

Based on the general format in Equation [2], we will now specialise to models for different road qualities. In Figure 8, there are 4 types of road types defined. Approximately, the 3 upper of those are roads and they are also defined as PSD-plots in Figure 9. The mathematical formula is given in Equation [4] and numerical parameter values are given in Equation [5].



Smooth

The road condition is assessed as Smooth if at least 95% of the total distance is covered on properly surfaced roads of good quality. Shorter distances on poor surfaces may arise.



Rough

The road condition is assessed as Rough if the road surface is of poor quality or if the road is not properly maintained. Up to 5% of the total distance may be covered on extremely poor roads or off-road.



Very Rough

The road condition is assessed as Very Rough if more than 5% of the total distance is covered on extremely poor roads or off-road.



Cross Country

The road condition is assessed as Cross Country if a considerable amount of driving occurs in severe off-road conditions.

Figure 8: Four typical road types, whereof the upper 3 can be considered as road types. From (AB Volvo, 2011).

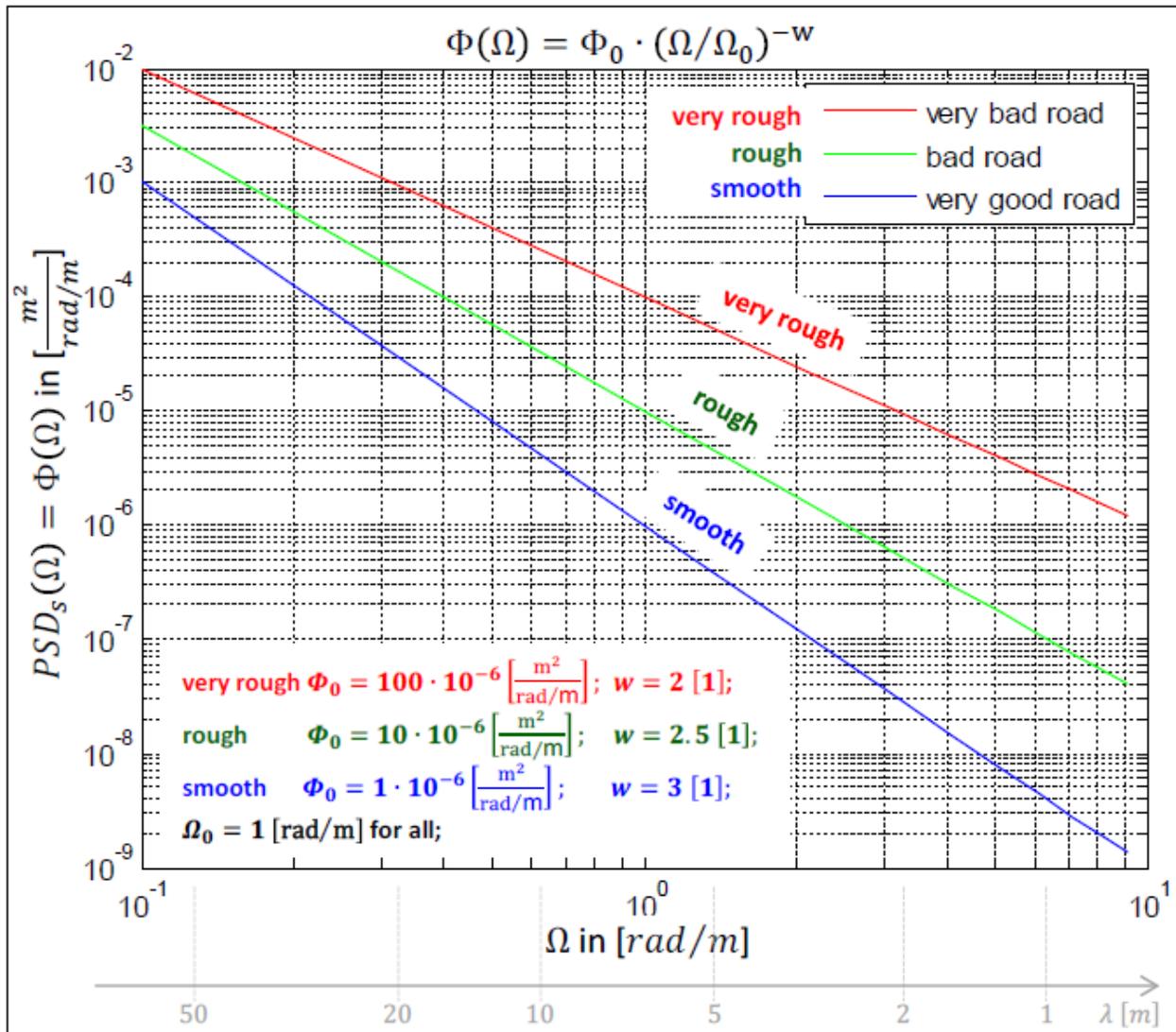


Figure 9: PSD spectra for the three typical roads in Figure 8.

$$\Phi = \Phi(\Omega) = \Phi_0 \cdot \left(\frac{\Omega}{\Omega_0}\right)^{-w} = \frac{MS_s(z_r, \Omega)}{\Delta\Omega} \dots \quad 4$$

Where $\Phi_0 = \text{road severity} \left[\frac{m^2}{rad/m} \right]$

$W = \text{road waviness} [1]$

$\Omega = \text{spatial angular frequency} [rad/m]$

$\Omega_0 = 1 [rad/m]$

Typical values are:

Very good road:

$$\Phi_0 = 1 \times 10^{-6} \left[\frac{m^2}{rad/m} \right]$$

Bad road:

$$\Phi_0 = 10 \times 10^{-6} \left[\frac{m^2}{rad/m} \right]$$

Very bad road:

$$\Phi_0 = 100 \times 10^{-6} \left[\frac{m^2}{rad/m} \right] \dots\dots\dots 5$$

The waviness is normally in the range of $w = 2.3$, where smooth roads have larger waviness than bad roads.

The decreasing amplitude for higher (spatial) frequencies (i.e. for smaller wave length) can be explained by that height variation over a short distance requires large gradients. On micro-level, in the granular level in the asphalt, there can of course be steep slopes on the each small stone in the asphalt. These are of less interest for vehicle vertical dynamics, since the wheel dimensions filter out wave length \ll tyre contact length.

- $\Omega_1, \dots, \Omega_N$
- $\hat{z}_1, \dots, \hat{z}_N$
- x_{01}, \dots, x_{0N}

Number of frequency components, N, to select is a matter of accuracy or experience. The offsets, x_{01}, \dots, x_{0N} , can often be assumed to be zero. If phase is to be studied, as in Figure 9, a random generation of offsets is suitable. See also Reference (ISO 8608).

If we generate the actual $z_r(x)$ curves for the 3 road types in Figure 9, we can plot as shown in Figure 10. To generate those plots, we have to assume different number of harmonic components (N in Equation [3]) and also randomly generate the phase for each component (each x_{0i}).

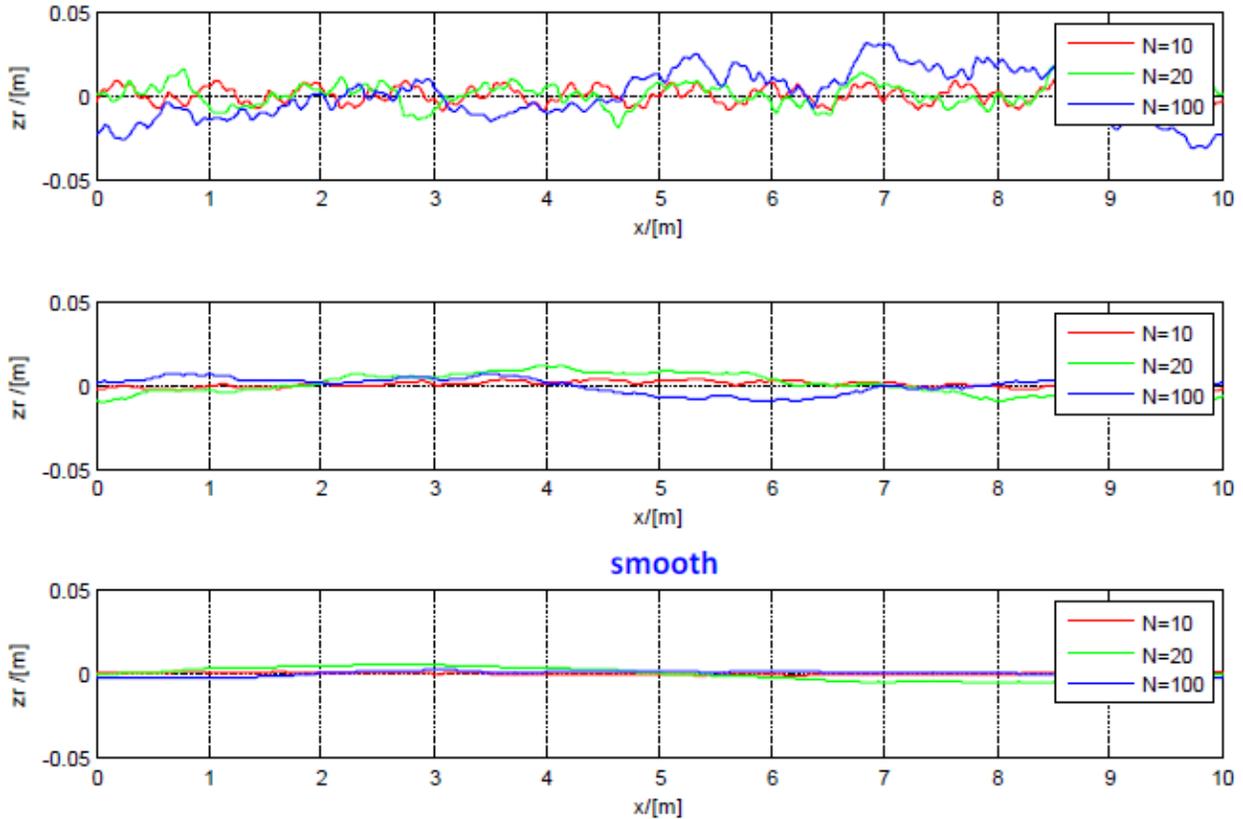


Figure 10: Road profiles, $z_r(x)$, for the three typical roads in Figure 5-5.

Ride comfort *

Function definition: (Stationary) Ride comfort is the comfort that vehicle occupants experience from stationary oscillations when the vehicle travels over a road with certain vertical irregularity in a certain speed. The measure is defined at least including driver (or driver seat) vertical acceleration amplitudes.

Ride comfort is sometimes divided into:

- **Primary Ride** – the vehicle body on it's suspension. Bounce(Heave), Pitch and Roll ≈ 0.4 Hz
- **Secondary Ride** – same but above body natural frequencies, i.e. $\approx 4..25$ Hz

Fatigue life *

Function definition: (Vehicle) Fatigue life is the life that the vehicle, mainly suspension, can reach due to stationary oscillations when vehicle travels over a road with certain vertical irregularity in a certain speed. The measure is defined at least including suspension vertical deformation amplitudes.

Beside human comfort, the fatigue of the vehicle structure itself is one issue to consider in vertical vehicle dynamics.

Single frequency

Loads on suspension spring

In particular, the suspension spring may be subject to fatigue. The variation in spring material stress is dimensioning, which is why the force variation or amplitude in the springs should be under observation. Since spring force is proportional to deformation, the suspension deformation amplitude is proposed as a good measure. Beside fatigue loads, z_u - z_{sis} is also relevant for judging whether suspension bump-stops become engaged or not. At normal driving, that limit should be far from reached, except possibly at maximum loads (many persons and much luggage).

Road grip *

Function definition: Road grip (on undulated roads) is how well the longitudinal and lateral grip between tyres and road is retained due to stationary oscillations when the vehicle travels over a road with certain vertical irregularity in a certain speed.

Functions over longer events and Functions in shorter events show the brush model explain how the tyre forces in the ground plane appears. It is a physical model where the contact length influences how stiff the tyre is for longitudinal and lateral slip. There is also a brief description of relaxation models for tyres. This together motivates that a tyre have more difficult to build up forces in ground plane if the vertical force varies. We can understand it as when contact length varies, the shear stress build up has to start all over again. As an average effect, the tyre will lose more and more grip, the more the vertical force varies.