Gear Box

A gear box for machine tool should possess the following requirements:

- Providing adequate spindle speeds
- Transmit desired power
- Provide smooth silent operation of the transmission and accurate rotation of the spindle without vibrations.
- Simple construction in respect of total number of shafts, gears, clutches, bearing and control system components.
- Ease of carrying out preventive maintenance and to make the adjustments in bearing and clutches etc. by taking care of easy access.

Gear box may be built either into the spindle head (head stock), or be designed as a separate unit. Gear boxes designed as a separate unit integrated with spindle head provides a more compact spindle drive and also easy to assemble. However , in these, vibrations from the gear box may be transmitted to the spindle gear boxes and also heat produced in the gear box may heat the spindle head.

In the case of gear boxes with a divided drive, the gear box and spindle head (head stock) are designed as separate units and the gear box is linked to the spindle head through some types of transmission. In these gear boxes, the heat produced in the gear box by friction losses and vibrations are not transmitted to the spindle head.

Design of gear trains

In sliding cluster-type gear boxes, each shaft carries a number of gears, which mesh with the gears on other shafts. To ensure that all the gears on the two shafts mesh properly, the sum of the meshing gears on the two carrying shafts must be equal, if the gear modules are equal. At the same time, the gear trains should provide the designed transmission ratios. The number of teeth for the gear trains for various transmission ratios, can be computed as follows:

1. First, calculate all the required transmission ratio between the two shafts; say i_h, i_m, i_l, …, etc
2. Convert the transmission ratios to fractions, with both numerator and denominator as integers

   $$say,\ i_h = \frac{a_1}{b_1};\ i_m = \frac{a_2}{b_2};\ i_l = \frac{a_3}{b_3}$$

   where, $a_1, a_2, a_3 \ldots b_1, b_2, b_3 \ldots$ are integers

3. Add the numerator and denominator,
   Say $(a_1 + b_1);\ (a_2 + b_2);\ (a_3 + b_3)\ldots,\ etc$
4. Find the lowest common multiplier (L.C.M) of the sum of the numerators and denominators, for all the transmission ratio, i.e. find LCM of $(a_1 + b_1);\ (a_2 + b_2);\ (a_3 + b_3)$
5. The sum of the meshing gears on the two shafts, is an integer multiple of the L.C.M.

$\therefore$ Sum of the teeth of the meshing gears = N x L.C.M, where N is an integer. The value of N depends upon the minimum number of teeth, allowed on the smallest gear. This generally ranges from 18 to 22 teeth.

6. Find the minimum sum of the meshing gears (t_{sm}), from the minimum number of the teeth on the pinion, using the following formula:

$$t_{s\min} = \frac{(a+b)\times t_{\min}}{\textit{lesser between a and b}}$$

a, b: Values of a, b for maximum transmission ratio i_1

t_{min}: Minimum no. of teeth in a pinion (18-22)

t_{smin}: Sum of the minimum number of teeth in the gear trains.

This (t_{min}) should be increased to the nearest higher integer multiple of the L.C.M., to get the final sum of teeth of the meshing gears [t_s]

7. The sum can be divided in 2 parts according to the transmission ratio.

$$t_1 = t_s \frac{a_1}{a_1 + b_1}$$

$$t_2 = t_s - t_1$$

Example :

Design a 2-stage, 6-speed gear box, for 125 to 710 R.P.M spindle speeds. Draw a ray diagram for a 1,500 R.P.M. input motor, and calculate the no. of teeth for all the gears. Minimum no. of teeth =20.

Solution:

$$\phi = z-1\sqrt{\frac{N_{\max}}{N_{\min}}} = {}^{(6-1)}\sqrt{\frac{710}{125}}$$

$$= 1.415$$

The rotary speeds will be: 125, 125 x 1.415, 125 x 1.415^2, 125 x 1.415^3, 125 x 1.415^4, and 125 x 1.415^5, i.e. 125, 180, 250, 355, 500 and 710 R.P.M.

The motor speed 1,500 R.P.M must be reduced to 2 speeds, convenient for the intermediate shaft II. The maximum possible reduction is determined by the maximum transmission ratio 4.0, for the straight spur gear.

For the intermediate shaft II

$$N_{II\,\min} = 1{,}500 \times \frac{1}{4} = 375 \; R.PM$$

This must be reduced to minimum spindle R.P.M. 125, in the next stage

Transmission ratio = $i_{II,\, III,\, I} = \frac{375}{125} = 3$

The fourth speed, i.e. 355 R.P.M, will be obtained with the same transmission ratio $\frac{1}{3}$, from the other higher speed on the intermediate shaft II. It can be arrived at as below:

$$i_{II,,\, III,\, I} = 3 = \frac{N_{in}}{N_{out}} = \frac{N_{in}}{355}$$
$$\therefore \; N_{in} = 355 \times 3 = 1{,}065 \; R.P.M$$

The middle transmission ratio = $i_{II,\ III,\ m}$

$$i_{II,,\, III,\, m} \; \frac{N_{in}}{N_{out}} = \frac{375}{180} = 2.08$$
$$Also,\; i_{II,,\, III,\, m} = \frac{1{,}065}{500} = 2.13$$
$$\therefore i_{II,,\, III,\, m} = 2$$
$$i_{II,,\, III,\, h} = \frac{N_{in}}{N_{out}} = \frac{1{,}065}{710} = 1.5 = \frac{3}{2}$$
$$Also \; \frac{375}{250} = \frac{3}{2}$$

We can draw the ray diagram (Fig. 9). The next stage is to find the number of teeth for the individual gears and clusters.

The transmission ratio between shafts I (motor) and II (intermediate) are:

$$i_{II,,\, III,\, m} \; \frac{N_{in}}{N_{out}} = \frac{1{,}500}{375} = 4 = \frac{4}{1}$$
$$i_{II,,\, III,\, h} = \frac{N_{in}}{N_{out}} = \frac{1{,}500}{1{,}065} = 1.408 = \frac{10}{7}$$

Adding numerators and denominators of the fractional numbers

$$a_1 + b_1 = 1 + 4 = 5;\ a_2 + b_2 = 7 + 10 = 17$$
$$L.C.M.of\ 5\ and\ 17 = 5 \times 17 = 85$$

The sum of actual number of teeth must be an integer multiple of the L.C.M. = 85.

$$i_{\max} = 4$$
$$t_{\min} = 20\ (given)$$

Sum of minimum no. of teeth = $s_{\min} = \dfrac{(a+b)t_{\min}}{lesser\ between\ a\ and\ b}$

$$t_{s\min} = \frac{(1+4)\times 20}{1} = 100$$

The nearest, higher integer multiple of L.C.M = 85 is 85 x 2 = 170

From :

$$t_1 = \frac{1}{1+4} \times 170 = 34;\ and\ from$$
$$t_2 = 170 - 34 = 136$$
$$t_3 = \frac{7}{17} \times 170 = 70$$
$$t_4 = 170 - 70 = 100$$

For transmission between shafts II and III,

$$i_{II,,\ III,\ I} = 3; i_{II,,\ III,m} = 2; i_{II,,\ III,h} = \frac{3}{2}$$
$$a_3 + b_3 = 1 + 3 = 4;\ a_4 + b_4 = 1 + 2 = 3;\ a_5 + b_5 = 2 + 3 = 5$$
$$L.C.M\ of\ 4,3,\ and\ 5 = 4 \times 3 \times 5 = 60.$$

The sum of the teeth for meshing gears on shafts II and III, must be an integer multiple of the L.C.M. = 60.

Maximum transmission ratio = $i_{II,,\ III,I} = \dfrac{3}{1}$

$$\therefore t_{s\min} = \frac{1+3}{1} \times 20 = 80$$

The next integer multiple of L.C.M = 60 x 2 = 120. It is necessary to check that this sum (120) gives adequate center distance between shafts II and III, and that shaft III will not obstruct the biggest gear on shaft II. The biggest gear on shaft II is no. 2 with 136 teeth

Outside radius of gear no. 2 = $\frac{m(t_2+2)}{2}$

$$= \frac{m(136+2)}{2} = 69\,m$$

$$m = \text{mod}\,ule\ of\ \ gear$$

For t_s=120, the center distance between shafts II and III = $m\frac{t_s}{2} = m\frac{120}{2} = 60m.$

As 60 m is less than the outside radius of 69 m of gear no. 2, the center distance between shafts II and III must be increased, i.e. t_s must be increased. The next higher multiple of L.C.M 60, is 60 x 3 = 180 m, and center distance between shafts II and III = $m = \frac{180}{2} = 90m.$

As 90 m is much bigger than the outside radius of 69 m of gear no.2, there is no risk of gear no.2 obstructing shaft III.

For t_s= 180

$$t_5 = \frac{1}{1+3} \times 180 = 45;\ \ and\ \ t_6 = 180 - 45 = 135$$

$$t_7 = \frac{1}{1+2}\ \times 180 - 60;\ and\, t_8 - 180 - 60 - 120$$

$$t_9 = \frac{2}{2+3} \times 180\ =\ 72;\ and\ \ t_{10} = 180 - 72 = 108$$

Figure 10. shows the layout of the gear in the gear box. The circled numbers show the gear, and the corresponding suffix used, in specifying the number of teeth i.e. the number of teeth in gear no. 7= t_7. Also note the following points: for gear box layout.

1. Clustered gears are mounted on splined shafts (I and III)
2. Axial travel of clusters is limited by stop collars, to facilitate blind-positioning and prevent overshoot
3. In the 3-gear cluster, the biggest gear is placed in the center
4. There should be a difference of at least 4 teeth between adjacent gears in a cluster
5. Fixed, single gears on shaft II are provided with grub screw clamping, to prevent axial motion.
6. Center distance between adjacent shafts should be large enough to provide is positive clearance between the shaft and the biggest gear on the adjacent shaft and the biggest gear on the adjacent shaft. (Refer to example 1.)
7. For the same module, the sum of teeth (t_s) between the meshing gears on the two shafts should be equal.

EXAMPLE 2.

Design a 9-speed, two-stage gear box for spindle speeds 30-500 R.P.M. Use a 1,500 R.P.M. motor. Minimum no of teeth in the pinion = 20. Use belt transmission for reducing motor speed to a level suitable for the gear box input shaft.

Solution: $\phi = \sqrt[(9-1)]{\frac{500}{30}} = 1.421 = 1.414\ (s \tan dard)$

Nearest standard step ratio = 1.414. The speeds will therefore be:

30, 30 x 1.414, 30 x 1.414^2 … 30 x 1.414^8

= 30, 42.45, 60.06, 85, 120.27, 170.18, 240.8, 340.73, and 482.14.

Range Ratio = $\frac{482.14}{30} = \frac{48}{3} \cong 16$

For drawing a ray diagram between spindle shaft III and the preceding shaft II, take t_1=4 max. Therefore, for the preceding intermediate shaft II, the minimum speed will be:

$$N_{II} \min = N_{III} \min \times i \max = 30 \times 4 = 120\, R.P.M$$

For maximum speed 480 of shaft III minimum transmission ratio $i = \frac{1}{2}$ can be used $\frac{N_i}{N_6} = \frac{N_i}{180} = \frac{1}{2};\ N_i = \frac{480}{2} = 240 : \max\ RPM\ of\ shaft\ II$

This speed will have transmission ratio (4), with the corresponding lower speed on the spindle. Connect the 240 R.P.M. point on intermediate shaft II with the $60\left(\frac{240}{4}\right)$ R.P.M. point on the spindle.

For finding the middle speed on intermediate shaft II, find the speeds corresponding with spindle speeds 42 and 340 R.P.M.

For maximum transmission ration (4), the middle speed on the intermediate shaft – 42 x = 168 R.P.M.

For minimum transmission ratio $\frac{1}{2}$, the middle speed on the intermediate shaft $= 340 \times \frac{1}{2} = 170$

Select 170 R.P.M and mark the point on the intermediate shaft. Connect this point (170) with the point for 42 and 340 R.P.M on the spindle shaft line III.

That still leaves 3 central spindle speeds: 85, 120, and 170 R.P.M, unconnected. Connect them to the points for 120, 170 and 240 R.P.M. respectively, on the intermediate shaft, to finish the ray diagram between the spindle and the intermediate shaft. [Fig. 2]

For the ray diagram between the gear box input shaft I, and the intermediate shaft II, extend the left-most line connecting speeds 30 and 120, to meet the input shaft line I at the 480 R.P.M (120 x 4) point. Connect the 480 R.P.M. point of the input speed to the speed points 170, and 240 R.P.M on the intermediate shaft, to complete the ray diagram for the gear box.

The motor speed of 1,500 R.P.M should be reduced to 480 R.P.M by belt transmission. The motor will have a small (say 125) diameter pulley. The diameter of the pulley, on the input shaft of the gear box will be = $D = 125 \times \frac{1,500}{480} = 390.6$

Figure 11. also shows the single ray for transmission between the motor and the gear box input shaft I.

Let us find the number of teeth for various gears.

First, we have to convert the transmission ratios into fractions, where both, the numerators and denominators are integers. Next, find the L.C.M of the sums of the numerators and denominators of all the transmission ratios.

$$i_{II,,\,III,\,I} = 4 = \frac{a_1}{b_1} \therefore a_1 + b_1 = 1 + 4 = 5$$

$$i_{II,,\,III,\,m} = \frac{485}{170} = \frac{48}{17} \therefore a_2 + b_2 = 17 + 48 = 65$$

$$i_{II,,\,III,\,h} = \frac{1}{2} = a_3 + b_3 = 2 + 1 = 3$$

$$As\ 65 = 13 \times 5,$$

$$L.C.M = 13 \times 5 \times 3 = 195$$

$$t_{\min} = 20\ (given)$$

$$t_{\max} = 4$$

$$t_s \min = \frac{1+4}{1} \times 20 = 100$$

The sum of the minimum no. of meshing teeth [ts] will be the nearest, higher integer multiple of the L.C.M found earlier, i.e. 195. Nearest higher integer multiple = 195 x 1 = 195

Referring to (Fog. 2.23b)

$$t_1 = \frac{a_1}{a_1 + b_1} \times 195 = \frac{1}{1+4} \times 195 = 39$$
$$t_2 = 195 - 39 = 156$$
$$t_3 = \frac{a_2}{a_2 + b_2} \times ts = \frac{17}{17+48} \times 195 = 51$$
$$t_4 = 195 - 51 = 144$$
$$t_5 = \frac{a_3}{a_3 + b_3} \times 195 = \frac{2}{2+1} \times 195 = 130$$
$$t_6 = 195 - 130 = 65$$

For transmissions between shafts II and III,

$$i_{II,, III, I} = \frac{4}{1} = \frac{a_4}{b_4} \therefore a_4 + b_4 = 5$$
$$i_{II,, III, m} = \frac{125}{85} = \frac{25}{17} \therefore a_5 + b_5 = 42 = 14 \times 3$$
$$i_{II,, III, h} = \frac{1}{2} = a_6 + b_6 = 3$$

$$t_{\min} = 20; \quad t_{\max} = 4$$
$$\therefore t_s \min = \frac{1+4}{1} \times 20 = 100$$

Nearest higher integer multiple of L.C.M 210 is: 210 x 1=210

$t_s = 210$

Let us check for interference between the biggest gear on shaft II (t_2) and shaft III, for the center distance corresponding with the sum of teeth (t_s) 210. Center distance between shafts II and III = $m = \frac{t_s}{2} = 105m$

Outside radius of the biggest gear 'No. 2' $= \frac{(t_2 + 2)}{2} m$

$$= \frac{156 + 2}{2} m$$
$$= 79m$$

As 79 m is less than the center distance of 105 m, shaft III would not interfere (obstruct) rotation (or even assembling) of gear t_2. We can now find the number of teeth for individual gears, nos. 7 to 12.

$$t_7 = \frac{a_4}{a_4 + b_4} \times t_5 = \frac{1}{5} \times 210 = 42;\ t_8 = 210 - 42 = 168;$$

$$t_9 = \frac{17}{42} \times 210 = 85;\ t_{10} = 210 - 85 = 125;\ t_{11} = \frac{2}{3} \times 210 = 140;\ t_{12} = 210 - 140 = 70$$

For finding actual sizes of the gears, we will have to find the tooth module (size) and width of the gear, as explained earlier in Gear design. (eqns and Tables 13- 15).

Internal gears

These are compact and strong, and run smoothly. In internal gears, the driver and the driven gears run in the same direction. The bigger internal gear meshes with a smaller external gear (Fig. 12). The correction is usually applied radially outward for the internal gear, and considered negative. Also, $(x_1+x_2)=0$. The cutter used for cutting the internal teeth should have at least a dozen lesser teeth, than the internal gear.

$$t_2 - t_{cutter} \geq 12;\ Also,\ t_{cutter} \geq 16$$
$$t_1 \geq 14;\ and\ \ t_2 - t_1 \geq 12$$

The formulae for pitch, outside (addendum), and base, and root diameters of gears stated earlier, can also be used for internal gears. The center distance (C_i) can be found from the following equation:

$$C_i = \frac{m(t_2 - t_1)}{20} cms$$

For the shaping cutter for an internal gear,

$$C = 0.05\ (t_2 - t_c)m$$

For changed center distance (C'),

$$\text{Working pressure angle} = \alpha' n = \cos^{-1}\left[\frac{C}{C'}.\cos\alpha_n\right]$$

$$x_1 + x_2 = \left\{\frac{t_2 - t_1}{2\tan\alpha_n}\right\}(Inv\ \alpha' n = Inv\ \alpha n)$$

It will be useful to enlarge the inner (addendum) diameter (D_i) of the internal gear, and increase the outside diameter of the pinion, to reduce interference during operation. When $t_1>16$.

$$D_i = D_p - 1.2m = (t_2 - 1.2)m$$
$$d_o = d_p + 2.5m = (t_1 - 2.5)m$$

The tooth depth (h) is not changed, and the outside (dedendum) diameter of the internal gear (D_r) and, the root diameter of the pinion (d_r), are increased accordingly.

$$D_r = D_i + 4.5m$$
$$d_r = d_o - 4.5m$$

Example 2.17: Determine the important diameters of 1.5 module gears, having an internal gear and pinion with a transmission ratio of 1.75, pressure angle = 20°. Enlarge the gear and pinion to reduce interference. The internal gear is shaped with a 16-teeth cutter.

Solution: Considering the constraints for the number of teeth.,

$$t_1 \geq 14;\ t_2 - t_1 \geq 12$$
$$t_2 = 1.75 t_1$$
$$If\ \ we\ take\ t_1 = 20$$
$$t_2 = 1.75 \times 20 = 35$$
$$t_2 - t_1 = 35 - 20 = 15,\ i.e > 12$$
$$D_p = 35 \times 1.5 = 52.5$$
$$D_i = 52.5 - 1.2 \times 1.5 = 50.7$$
$$d_p = 20 \times 1.5 = 30$$

$$d_o = 30 + 2.5 \times 1.5 = 33.75$$
$$D_r = 50.7 + 4.5 \times 1.5 = 57.45$$
$$d_r = 33.75 - 4.5 \times 1.5 = 27$$
$$C = 0.5(35 - 20) \times 1.5 = 11.25mm$$