2.0 Simple Mechanism

As earlier mentioned a machine is a device which receives energy and transforms it into some useful work. A machine consists of a number of parts or bodies. We shall study here the Mechanisms of various parts or bodies from which the machine is assembled.

Kinematic Link or Element

Each part of a machine, which moves relative to some other parts, is known as a Kinematic link (or simply link) or element. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. A link should have the following characteristics.

1. It should have relative motion and
2. It must be a resistant body. (A body is said to be a resistance if it is capable of transmitting the required forces with negligible deformation).

Types of Links

1. **Rigid link:** A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links don not exists. However as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.
2. **Flexible Link:** A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.
3. **Fluid Link:** The link which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

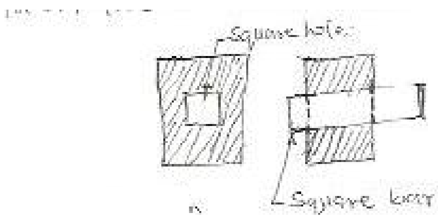
Kinematic Pair

Two links which are connected together is such as way that their relative motion is completely or successfully constrained forms a kinematic pair.

Types of Constrained Motions

The following are the three types of constrained motions:

1. **Completely constrained motion:** When the motion between a pair is limited to a definite direction irrespective of the direction of force applied then the motion is said to be a completely constrained motion.



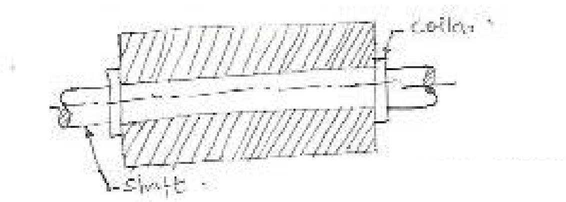


Fig.2.1 **Square bar in a square hole** Fig.2.2. **Shaft with collar in a circular hole**

For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

The motion of a square bar in a square hole as in fig. 2.1 and motion of a shaft with collars at each end in a circular hole, as in fig. 2.2 are examples of completely constrained motion.

2. **Incompletely constrained motion:** when the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole as shown in fig. 2.3, is an example of an incompletely constrained motion as it may either rotate or slide in a hole.

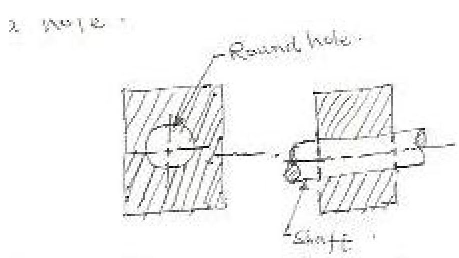


Fig. 2.3 Shaft in a Circular hole

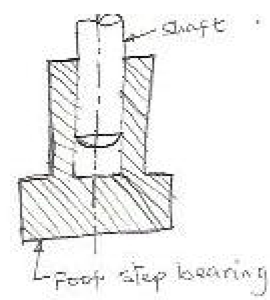


Fig. 2.4. Shaft in a foot Step b

3. **Successfully constrained motion:** When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other meant, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing in fig. 2.4, the shaft may rotate in a bearing or may move upwards (in completely constrained motion), but if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion.

Kinematic Chain

When the Kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e completely or successfully constrained motion), it is called a Kinematic chain. This is to say that a Kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two parts and the relative motion between the links or elements is completely or successful constrained. For example, the crankshaft of an engine forms a Kinematic pair with the bearings which a fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with connecting rod forms a third and the piston with the cylinder forms a fourth pair. The combination of these links is a Kinematic chain.

Types of Kinematic chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following are the three types of Kinematic chains with four lower pairs.

1. Four bar chain or quadric cycle chain
2. Single slider crank chain, and
3. Double slider crank chain.

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as Mechanism, while the mechanism with more than four links is known as compound mechanism. When a mechanism is required to transmit power or to do some particular type of work, it becomes a machine.

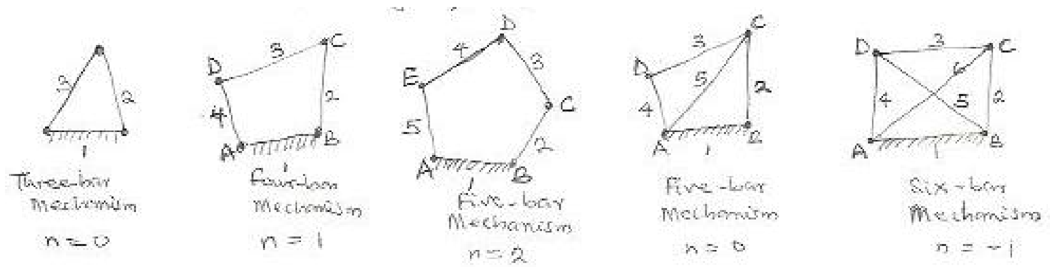
Number of Degree of Freedom for Plane Mechanism

In the Design or analysis of a mechanism, one of the most important concern is the number of degree of freedom (also called movability) of the mechanism. It is defined as the number of input parameters (usually pair variables) which must be independently controlled is order to bring the mechanism into a useful engineering purpose.

Application of Kutzbach Criterion to Plane Mechanisms

Kutzbach criterion for determining the number of degrees of freedom or movability (n) of a plane mechanism is

$$n = 3(l - 1) - 2j - h.$$



Note:

a. When n = 0, then the mechanism forms a structure and not relative motion between the links is possible.
b. When n = 1, then the mechanism can be driven by a single input motion.
c. When n = 2, then two separate input motions are necessary to produce constrained motion for the mechanism.
d. When n = -1 or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure.

Velocity in Mechanisms

There are two important methods of determining the velocity of any point on a link in a mechanism out of many.

1. Instantaneous centre method
2. Relative velocity method.

- **Velocity of a point on a link by Instantaneous centre method:** The instantaneous centre method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

- **Number of Instantaneous centre in a mechanism**

  The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links.

  The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time.

  Mathematically, number of instantaneous centres

N= $\frac{n(n-1)}{2}, where\ n = Number\ of\ links$

- **Types of Instantaneous centres**

The instantaneous centres for a mechanism are of the following types

1. Fixed instantaneous centres,
2. Permanent instantaneous centres
3. Neither fixed nor permanent instantaneous centres.

The first two types i.e. fixed and permanent instantaneous centres, are together known as primary instantaneous centres and the third type is known as secondary instantaneous centres.

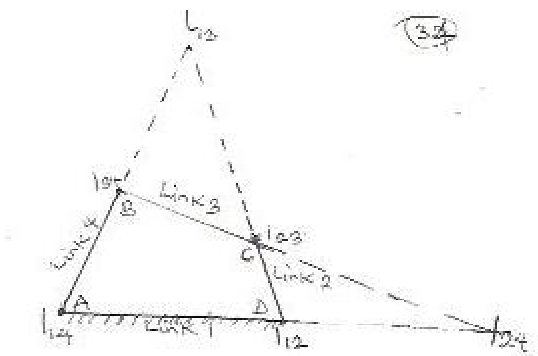


Fig. 2.5: Types of instantaneous centre

Consider a four bar mechanism ABCD as shown is fig. 2.5. the instantaneous centres (N) in a four bar mechanism is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

The instantaneous centres I_{12} and I_{14} are called the fixed instantaneous centres as they remain in the same place for all configuration of the mechanism. The instantaneous centres I_{23} and I_{34} are the permanent instantaneous centres as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are neither fixed nor permanent instantaneous centres as they vary with the configuration of the mechanism.

Method of locating Instantaneous Centres in a Mechanism

Consider a pin jointed four bar mechanism as shown which in fig. 2.6 (a). The following procedure is adopted for locating instantaneous centres.

1. First, Determine the number of instantaneous (N) by using the relation.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

The instantaneous centres I_{12} and I_{14} are called the fixed instantaneous centres as they remain in the same place for all configuration of the mechanism. The instantaneous centres I_{23} and I_{34} are the permanent Instantaneous centres as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{14} are neither fixed nor permanent instantaneous centres as they vary with the configuration of the mechanism.

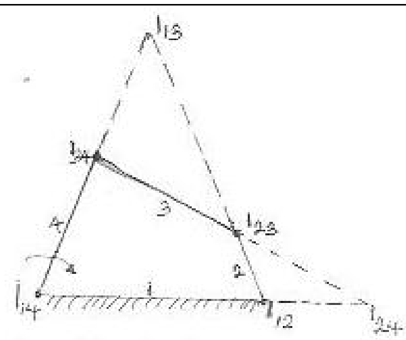
Method of Locating Instantaneous Centres in a Mechanism

Consider a pin jointed four bar mechanism as shown in fig.2.6 (a). The following procedure is adopted for locating instantaneous centres.

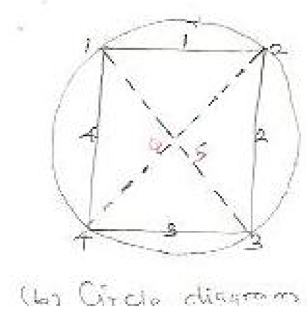
1. First, Determine the number of instantaneous centre (N) by using the relation N = $\frac{n(n-1)}{2}$, *where* $n = $ *Number of links*

   In this case, N = 6
2. Make a list of all the Instantaneous centres in a mechanisms, since for a four bar mechanism, there are six instantaneous centres, therefore these centres are listed as shown in the table below (known as book – keeping table).
3. Locate the fixed and permanent instantaneous centres by inspection I_{12} and I_{14} are fixed instantaneous centres and I_{23} and I_{34} are permanent instantaneous centres.

Links	1	2	3	4
Instantaneous centres (6 in number)	12 13 14	23 24	34	-



(a) Four bar Mechanism

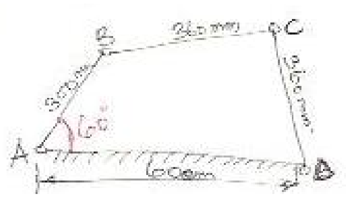


(b) Circle diagram

4. Locate the remaining neither fixed nor permanent instantaneous centres (or secondary centres) by Kennedy's theorem. This is done by circle Diagram as shown in fig. 2.6(b), mark points on a circle equal to the number of links in a mechanism. In this case, mark 1, 2, 3 and 4 on the circle.
5. Join the points by solid lines to show that these centres are already found. In the circle Diagram these lines are 12, 23, 34 and 14 to indicate the centres I_{12}, I_{23}, I_{34} and I_{14}.
6. In order to find the other two instantaneous, join two such points that the line joining them form two adjacent triangles in the circle diagram. i.e. In the case of fig.2.6(b), join 1 and 3 to form the triangle 123 and 341 and the Instantaneous centre I_{13} which lie on the intersection of I_{12} I_{23} and I_{14} and I_{34}, produced if necessary on the mechanisms. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{24}, produced if necessary, on the mechanism. This I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it.

Example 1

In a pin jointed four bar mechanism, as shown in the fig. below AB 300mm, Bc = CD = 360mm, and AD =600mm. The angle BAD = 60. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC.



Solution

N_{AB} = 100 r.p.m.

$$\omega_{AB} = \frac{2\pi x 100}{60} = 10.47\, rad/s$$

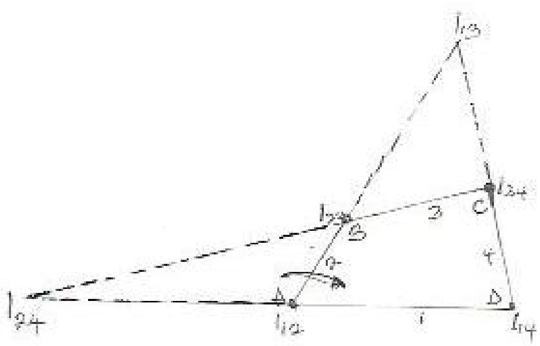
Since crank length AB= 300mm = 0.3m, therefore velocity of point B on link AB.

$$V_B = \omega_{AB} \times AB = 10.47 \times 0.3$$
$$= 3.14\, m/s$$

Location of Instantaneous centre

Follow the method of locating instantaneous centre as previously discussed..

i.e N = 6. ………………………….. (n = 4)



Angular velocity of the link BC

$$\omega_{BC} = \frac{VB}{I_{13}B} = \frac{3.141}{0.5}$$

= 6. 282 rad/s.

Example 2

Locate all the instantaneous centres of the slider crank mechanism as shown below. The lengths of crank OB and connecting rod AB are 100mm and 400mm respectively, if the crank rotates clockwise with an angular velocity of 10 rad/s, find.

(a) Velocity of the slider A and

(b) Angular velocity of the connecting rod AB.

Solution

Given: ω_{oB} = 10rad/s, OB = 100mm = 0.1m

Linear velocity of the crank OB

$V_{oB} = V_B = \omega_{oB} \times OB$

= 10 x 0.1 = 1m/s.

Location of instantaneous centres.

Since there are four links (n = 4), therefore the number of instantaneous centres

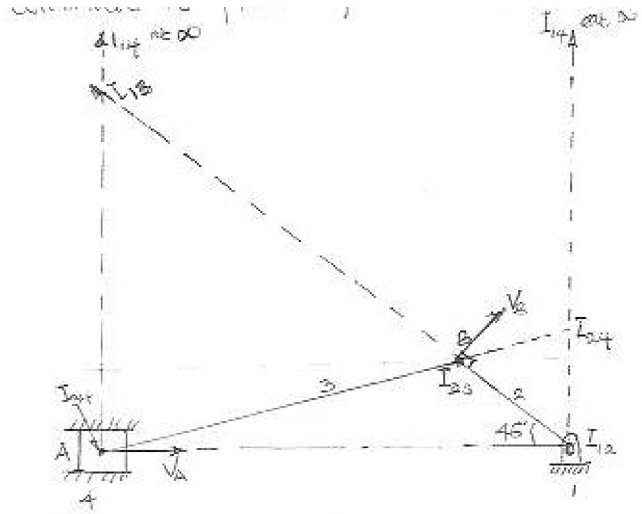
$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

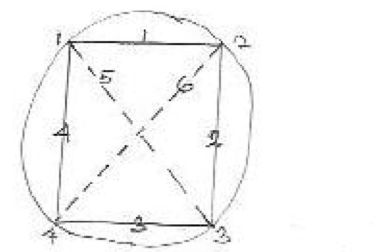
2. For a four link mechanism, the book keeping table may be drawn as previously discussed.

3. Locate the fixed and permanent instantaneous centre by inspection. These centres are I_{12}, I_{23} and I_{34} as shown below, since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will lie at infinity.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Kennedy's theorem. This is done by circle diagram.

5. Continue as previously discussed.





By measurement, we find that

$I_{13}A$ = 460mm = 0.46m, and $I_{13}B$ = 560mm = 0.56m.

1. Velocity of the slider A

   Let V_A = velocity of the slider A

We know that

$$\frac{V_a}{I_{13A}} = \frac{V_B}{I_{13}B} \; or \quad V_A = V_B \, x \frac{I_{13}A}{I_{13}B}$$

$$\frac{1\times 0.46}{0.56}$$

$$= 0.82\, m/s.$$

2. Angular velocity of the connecting rod AB

let ω_{AB} = Angular velocity of the connecting and AB.

From,

$$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B} = \omega AB$$

$$\omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56}$$

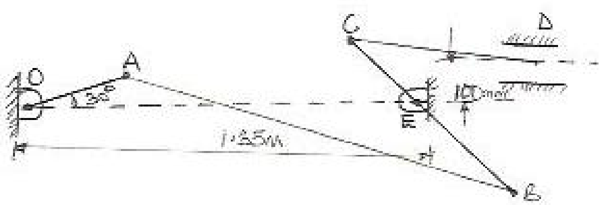
$$= 1.78\, rad/s$$

Example 3

A mechanism as shown below has the following dimensions: OA = 200mm, AB = 1.5, BC =600mm; CD = 500mm and BE = 400mm.

Locate the instantaneous centres. If the crank OA rotates Uniformly at 120 r.p.m clockwise, find

a. The velocity of B, C and D
b. The angular velocity of the link, AB, BC, and CD.



Solution

$$N_{oA} = 120 \text{ r.p.m or } \omega_{0A} = \frac{2\pi \times 120}{60} = 12.57\, rad/s.$$

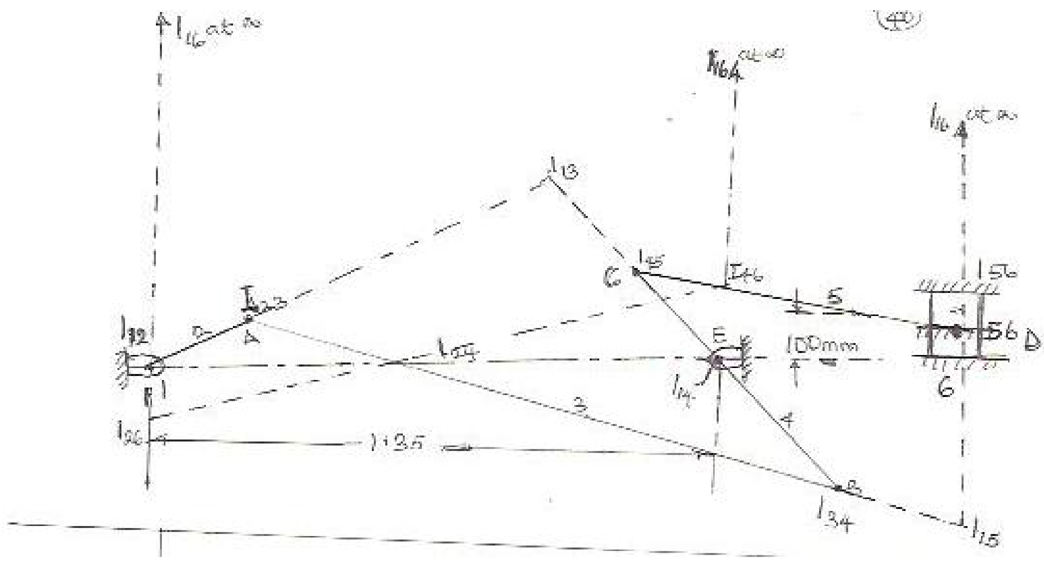
Since the length of crank OA = 200mm = 0.2m, therefore linear velocity of crank OA

$V_{oA} = V_A = \omega_{oA} \times OA =$ 12.57x 0.2

2.514 m/s.

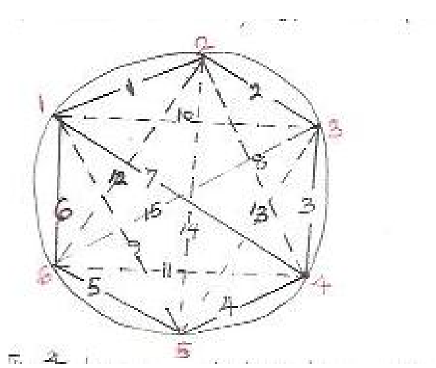
Location of instantaneous centres

1. Since n = 6, N = 15
2. Make a list of the instantaneous centres in a mechanism. Since the mechanism has 15 instantaneous centres, therefore these centres are listed in the book keeping table.



Links	1	2	3	4	5	6
Instantaneous Centre 15 in numbers	12 13 14 15 16	23 24 25 26	34 35 36	45 46	56	

3. Locate the fixed and permanent instantaneous centre by inspection. These centres are I_{12}, I_{23}, I_{34}, I_{45}, I_{16} and I_{14} as shown above
4. Locate the remaining neither fixed nor permanent instantaneous centres by Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1,2,3,4,5 and 6 as 14 to indicate the centre I_{12}, I_{23}, I_{34}, I_{45}, I_{56} and I_{14} respectively (see fig. below)



5. Joint point 2 to 4 by a dotted line to form the triangles 124 and 234. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the

instantaneous centre I_{24} lies on the intersection of I_{12} I_{14} and I_{23} I_{34} produced if necessary. Thus centre I_{24} is located. Mark number 8 on the dotted line 24 (because seven centres have already been located)

6. Now join point 1 to 5 by a dotted line to form the triangle 1 4 5 and 1 5 6. The side 1 5, common to both triangle, is responsible for completing the two triangles. Therefore the instantaneous centres I_{15} lies on the intersection of I_{15} is located. Mark number 9 on the dotted line 15.
7. Join point 1 to 3, …… for No 10
8. Join point 4 to 6 …… for No 11
9. Join point 2 to 6 ……for No 12
10. Similarly, the 13th, 14th and 15th instantaneous centres (i.e. I_{35}, I_{25} and I_{36}) may be located by joining the point 3 to 5, 2 to 5 and 3 to 6 respectively.

By measurement, we find that

I_{13} A = 840mm = 0.84m, I_{13}B = 107mm = 1.07m; I_{14}B = 400mm = 0.4m

I_{14}C = 200mm = 0.2m; I_{15} C = 740mm = 0.74m; I_{15}D = 500mm = 0.5.

a) Velocity of Points B, C, and D

Let V_B, V_C and V_D = velocity of the point B, C and D respectively

$$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$$

$$V_B = \frac{V_A}{I_{13}A} \ x \ I_{13}B = \frac{2.514}{0.84}$$

$$= 3.2 mls$$

Again

$$\frac{V_B}{I_{14}B} = \frac{V_c}{I_{14}C}$$

$$V_C = \frac{V_B}{I_{14}B} \ x \ I_{14}C = \frac{3.2}{0.4} \ x \ 0.2$$

$$= 1.6 mls$$

Similarly,

$$\frac{V_C}{I_{15}C} = \frac{V_D}{I_{13}D}$$

$$V_C = \frac{V_D}{I_{15}C}\ x\ I_5 D = \frac{1.6}{0.74} \times 0.5$$

$$= 1.08 mls$$

2. Angular velocity of the link AB, BC and CD

Let $\omega_{AB}, \omega_{AC}\ and\, \omega_{CD}$ = Angular velocity of the links AB, BC, and CD reporting

$$\omega_{AB} = \frac{V_A}{I_{13}A} = \frac{2.514}{0.84} = 2.99 rad/s$$

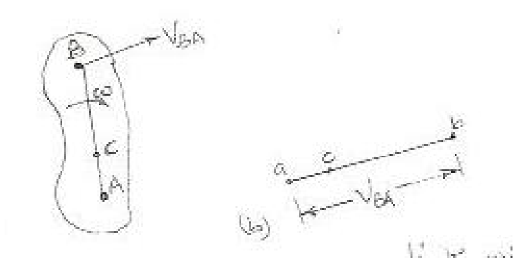
$$\omega_{BC} = \frac{V_B}{I_{14}B} = \frac{3.2}{0.4} = 8 rad/s$$

$$\omega_{CD} = \frac{V_C}{I_{15}C} = \frac{1.6}{0.74} = 2.16 rad/s$$

Velocity in mechanism (relative velocity method)

Motion of a Link

Consider two point A and B or a rigid link AB as shown in fig. 2.7(a), let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is this obvious, that the relative motion of B with respect to A must be perpendicular to AB.



(a) (b)

Hence, velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) Diagram.

The relative velocity of B with respect to A (i.e. V_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in fig. 2.7(b)

Let ω = Angular velocity of the link AB about A.

We know that the velocity of any point B with respect to A.

$V_{BA} = \overline{ab} = \omega.AB$ .... (i)

Similarly, the velocity of any point C on AB with respect to A

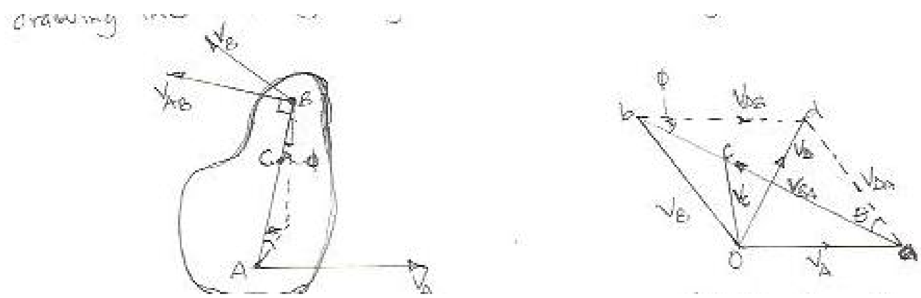
$V_{CA} = \overline{ac} = \omega.AC$ .... (ii)

From equation (i) and (ii).

$$\frac{V_{CA}}{V_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega.AC}{\omega.AB} = \frac{AC}{AB}$$

Velocity of a point on a link by relative velocity method

Consider two points A and B on a link as shown in fig 2.8(a). Let the absolute velocity of the point A ie. V_A is known in magnitude and direction and the absolute velocity of the point B ie. V_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in fig. 2.8(b).



(a) Motion of points on a link (b) Velocity Diagram

The velocity diagram is drawn as follows:

1. Take some convenient point O, known as pole
2. Through O, draw Oa parallel and equal to V_A, to some suitable scale
3. Through a, draw a line perpendicular to AB of fig. 2.8(a). this line will represent the velocity B with respect to A, i.e, V_{BA}.
4. Through O, draw a line parallel to V_B intersecting the line of V_{BA} at b.
5. Measure ob, which gives the required velocity of point B (V_B), to the scale.

Note:

1. The vector ab which represents the velocity of B with respect to A (V_{BA}) is known as velocity of image of the link AB.
2. The absolute velocity of any point C on AB may be determined by dividing vector ab at c in the same ratio as C divides AB. In other words

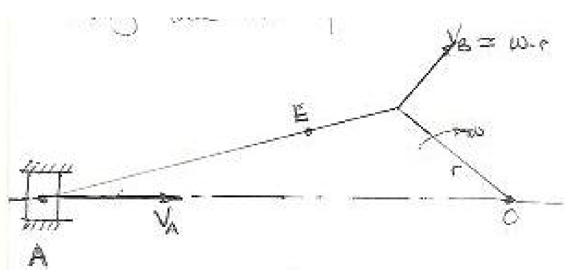
$$\frac{ac}{ab} = \frac{AC}{AB}$$

3. The absolute velocity of any other point D outside AB as shown in fig. 2.8(a), may also be obtained by completing the velocity triangle abd and similar to triangle ABD, as shown in fig. 2.8(b).
4. The angular velocity of the link AB may be found by dividing relative velocity of B with respect to A (i.e. BA) by the length of link AB. Mathematically, angular velocity of the link AB.

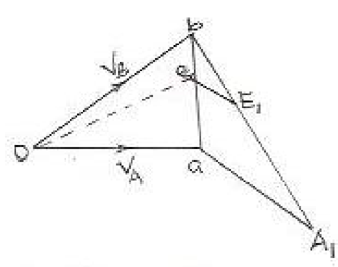
$$\omega_{AB} = \frac{V_{BA}}{AB} = \frac{ab}{AB}$$

Velocities in slider crank mechanism

A slider crank mechanism is shown in fig. 2.9(a). The slider is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity W rad/s. therefore the velocity of B i.e V_B is known in magnitude and direction. The slider reciprocates along the line of stroke A0.



(a) Slider Crank Mechanism

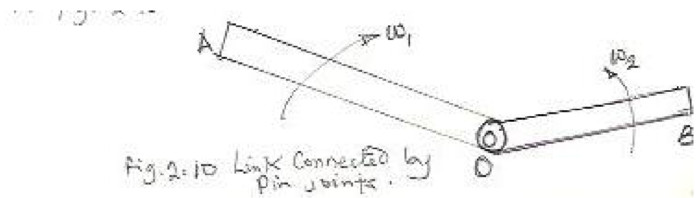


(b) Velocity Diagram

Rubbing velocity at a pin joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.

Consider two links OA and 0B connected by means of pin joints O as shown in fig. 2.10



According to the definition,

Rubbing velocity at the pin joint o

= $(\omega_1 - \omega_2)$r, if the links move in the same direction

= $(\omega_1 + \omega_2)$r, if the links move in the opposite direction.

Note:

When the pin connects one sliding member and other turning member; the angular velocity of the sliding member is zero. In such cases

Rubbing velocity at the pin joint = ω.r

Example 1

In a four bar chain ABCD, AD is fixed and is 150mm long. The crank AB is 40mm long and rotates at 120 r.p.m clockwise, while the link CD = 80mm Oscillates about D. Bc and AD are of equal length. Find the angular velocity of CD when angle BAD=$60^{0.}$

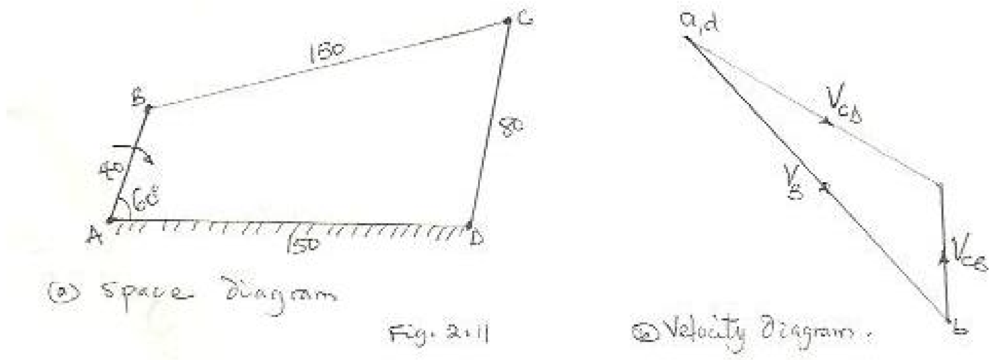
Solution

$N_{BA} = 120$ r.p.m, $\omega_{BA} = \frac{2\pi X 120}{60} = 12.568 rad/s$

The length of crank AB = 40mm = 0.04m, therefore velocity B with respect to A or velocity of B (since A is a fixed point).

$V_{BA} = V_B = \omega_{BA}$ x AB = 12.568 x 0.04

= 0.503mls



First of all, draw the space diagram to some suitable scale, as shown in fig. 2.11(a) above and the velocity diagram, as shown in fig 2.11(b) as discussed below.

1. Since the link AD is fixed, therefore point a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity B with respect to A or simply velocity of B (ie. V_{BA} or V_B) such that

   Vector ab = $V_{BA} = V_B = 0.503$ mls

2. Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (ie. V_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. V_{CD} or V_C). The vector bc and dc intersect at C.

   By measurement, we find that

   $V_{CD} = V_C =$ vector dc = 0.385mls

We know that, CD = 80mm = 0.08m

-: Angular velocity of link CD,

$$W_{CD} = \frac{V_{CD}}{DC} = \frac{0.385}{0.08}$$

= 4.8 rad/s. (clockwise about D)

Example 2: The crank and connecting rod of a theoretical steam engine are 0.5m and 2m long respectively, the crank makes 180r.p.m in the clockwise direction. When it has turned 45^0 from the inner dead centre position, determine:

a. Velocity of piston
b. Angular velocity of connecting rod.

c. Velocity of point E on the connecting rod 1.5m from the gudgeon pin

d. Velocity of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50mm, 60mm and 30mm respectively.

e. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.

Solution

Given: $N_{B0} = 180$ r.p.m or $W_{B0} = \frac{2\pi 180}{60} = 18.852\, rad/s$

Since the crank length 0B = 0.5m therefore linear velocity of B with respect to 0 or velocity of B (because o is a fixed point).

$V_{B0} = V_B =$ B0 x 0B

18.852 x 0.5

9.426mls

a. Velocity of piston

First of all, draw the space diagram, to some suitable scale. Also the velocity diagram as discussed below.

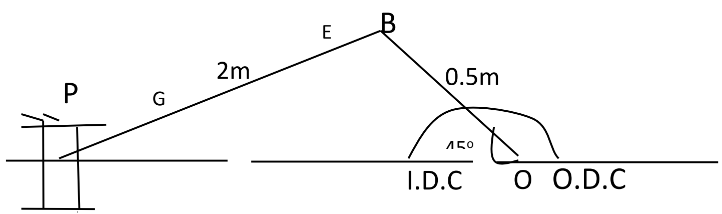
i. Draw vector ob perpendicular to B0, to some suitable scale, to represent the velocity of B with respect to 0 or velocity of B such that

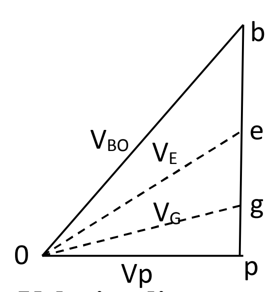
Vector 0b = $V_{B0} = V_B = 9.426$ mls.

ii. From point b, draw vector bp perpendicular to Bp to represent velocity of p with respect to B (ie. V_{PB}) and from point 0, draw vector op parallel to op parallel to po to represent velocity of p with respect to O (ie. V_{po} or simply V_p). The vector bp and op intersect at point p.

By measurement, the position of piston P,

V_p = vector op = 8.15mls





Velocity diagram

Space diagram

b. Angular Velocity of connecting rod

From the velocity diagram, the velocity of P with respect to B,

V_{PB} = Vector bp = 6.8mls

Since the length of connecting rod PB is 2m, therefore angular velocity of the connecting rod

$$\omega_{PB} = \frac{V_B}{PB} = \frac{6.8}{2} = 3.4\, rad/s$$

c. Velocity of point E on the connecting rod

By measurement, velocity of point E.

V_E = vector oe = 8.5 mls

Or

$$\frac{BE}{Bp} = \frac{be}{bp}\, or\, be = \frac{BE\, x\, bp}{Bp}$$

d. Velocity of rubbing

Diameter of crank shaft pin 0,

d_o = 50mm = 0.05m

Diameter of crank – pin at B,

dp = 60mm = 0.06m

And diameter of cross – head pin,

d_c = 30mm = 0.03m

Velocity of rubbing at the pin of crank shaft

$$= \frac{d_o}{2} \times \omega_{BO} = \frac{0.05}{2} x 18.85$$

= 0.47mls

Velocity of rubbing at the pin of crank

$$\frac{d_B}{2}(\omega_{B0} + \omega_{PB}) = \frac{0.06}{2}(18.85 + 3.4) = 0.47 m/s$$

……………… (ω_{BO} is clockwise and ω_{PB} is anticlockwise)

Velocity of rubbing at the cross-head

$$= \frac{d_c{}^c}{2} x\, \omega_{PB} = \frac{0.03}{2} x 3.4 = 0.051 mls$$

e. The position of point G on the connecting rod which has the least velocity relative to crank shaft is determined by drawing perpendicular from o to vector bp. Since the length of og will be the least, therefore the point g represents the required position of G on the connecting rod.

By measurement,

Vector bg = 5 m/s

The position of point G on the connecting rod is obtained as follows

$$\frac{bg}{bp} = \frac{BG}{Bp}$$

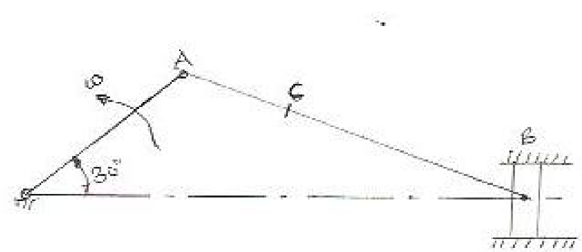
$$BG = \frac{bg}{bp} x\, Bp$$

$$= \frac{5}{6.8} x\, 2$$

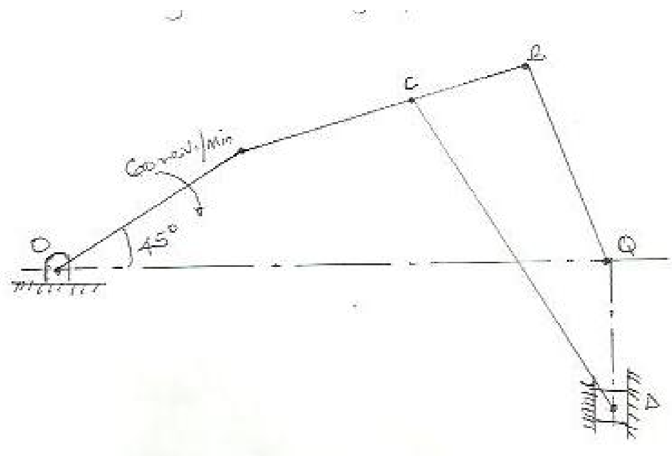
$$= 1.47m$$

<u>**Assignment**</u>

1. The Crank OA of the engine Mechanism as shown below rotates at 3600rev/min anticlockwise. OA =100mm, and the connecting rod AB is 200mm long. Find (a) the piston velocity (b) the angular velocity of AB (c) the velocity of point C on the rod 50mm from A.



2. The figure below shows a four-bar mechanism OABQ with a link CD attached to C the mid-point of AB. The end D of link CD is constrained to move vertically. OA=1m, AB=1.6m, QB=1.2m, OQ=2.4m and CD=2m. For he position shown, the angular velocity of Crank OA is 60 rev/min clockwise; find (a) the velocity of D (b) the angular velocity of CD; (c) the



Acceleration in Mechanism

Acceleration Diagram for a link

Consider two points A and B on a rigid link in the fig 2.13 below. Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link AB.

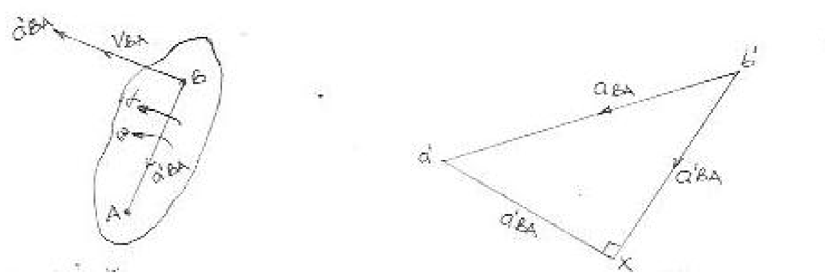


Fig. 2.13 (a) Link (b) Acceleration diagram

1. The centripetal or radial component of the acceleration of B with respect to A

   $a^1BA = \omega^2 \ x \ length \ of \ link \ AB = \omega^2 x \ AB = V^2{}_{BA} / BA$

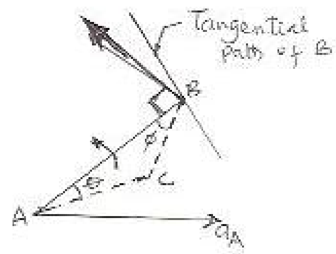
   This radial component of acceleration acts perpendicular to the velocity V_{BA}, In other words, it acts parallel to the link AB

2. The tangential component of the acceleration of B with respect to A,

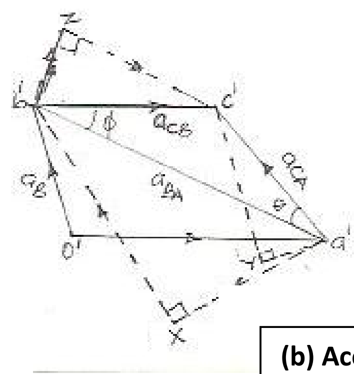
   $a^1BA = \alpha \times Length \ of \ the \ Link \ AB$
   $= \alpha \times AB$

This tangential component of acceleration acts parallel to the velocity V_{BA}. In other words it acts perpendicular to the link AB.

Acceleration of a point on a Link



(a) Points on a link



(b) Acceleration Diagram

Consider two points A and B on the rigid link, as shown in Fig. 2.14(a)

Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

1. From any point o^1, draw vector o^1a^1 parallel to the direction of absolute acceleration at point A i.e., a_A to some suitable scale, as shown in fig. 2.14(b)
2. We know that the acceleration of B with respect to A i.e. a_{BA} has the following two components;
   (i) Radial component of the acceleration of B with respect to A i.e. a^r_{BA} and
   (ii) Tangential component of the acceleration B with respect to A i.e. a^t_{BA}. These two component are mutually perpendicular.
3. Draw vector a^1x parallel to the link AB (because radial component of the acceleration of B with respect to A will pass through AB), such that

   Vector $a^1x = a^1BA = \frac{V^2_{BA}}{AB}$

   Where, V_{BA} = Velocity of B with respect to A

Note: The value of V_{BA} may be obtained by drawing the velocity diagram as previously discussed.

4. From point x, draw vector xb^1 perpendicular to AB or vector a^1x because tangential component of B with respect to A i.e. a^tBA, is perpendicular to radial component a^r_{BA}) and through o^1 draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B. The vector xb^1 and o^1b^1 intersect at b^1. Now the values of a_B may be measured, to the scale
5. By joining the points a^1 and b^1 we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a^1b^1 is known as acceleration image of the link AB.
6. For any other point C on the link draw triangle $a^1b^1c^1$ similar to triangle ABC. Now vector b^1c^1 represent the acceleration of C with respect to A i.e. a_{CA}. As mentioned earlier, a_{CB} and a_{CA} will each have two components as follows:

(i) a_{CB} has two components: a^r_{CB} and a^t_{CB} as shown by triangle b^1zc^1, in which b^1z is parallel to BC and ZC^1 is perpendicular to b^1Z or BC.

(ii) a_{CA} has two components: a^r_{CA} and a^t_{CA} as shown by triangle a^1yc^1 in which a^1y is parallel to AC and yc^1 is perpendicular to a^1y or AC.

7. The angular acceleration of the link AB is obtained by dividing the tangential component of the acceleration of B with respect to $A\left(a^t{}_{BA}\right)$ To the length of the link. Mathematically, angular acceleration of the link AB,

$$\alpha_{AB} = \frac{a^t{}_{BA}}{AB}$$

Acceleration in the slider Crank mechanism

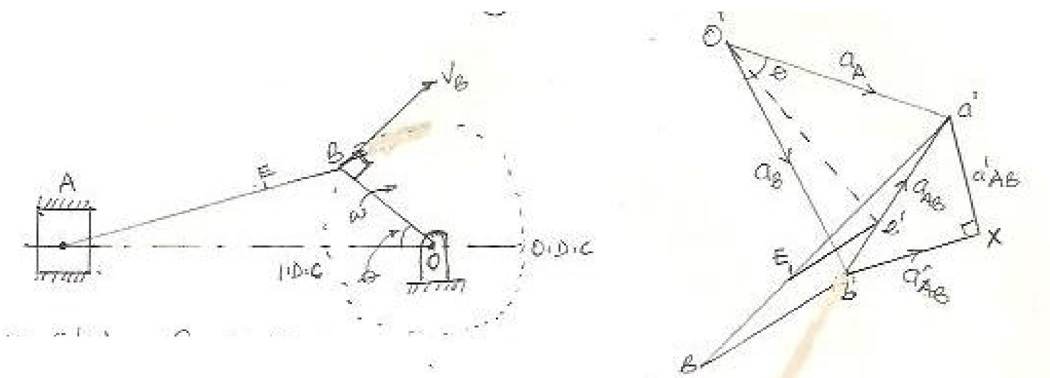
A slider crank mechanism is shown in fig. 2.13 (a). Let the crank OB makes an angle θ with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity ω_{BO}, rad/s

.: Velocity of B with respect to O or velocity of B (because O is a fixed point).

$V_{BO} = V_B = W_{BO} \times OB$, acting tangentially at B

We know that centripetal or radial acceleration of B with respect to O or acceleration of B (because O is a fixed point)

$$a_{Bo} = a_B = \omega^2_{BO} \times OB = \frac{V^2_{BO}}{OB}$$



(a) Slider crank mechanism **(b) Acceleration Diagram**

Fig. 2.13 Acceleration in the slider crank mechanism.

The acceleration diagram as shown above in fig 2.13(b) may be drawn as follows.

1. Draw vector ob parallel to BO an set off equal in magnitude of $a_{BO} = a_B$ to some suitable scale.
2. From point b^1, draw vector b^1x parallel to BA. The vector b^1x represents the radial component of the acceleration of A with respect to B whose magnitude is given by:

   $$a_{AB} = \frac{V^2_{AB}}{BA}$$

   since the point B moves with constant angular velocity, therefore there will be no tangential component of the acceleration.

3. From point x, draw vector xa^1 perpendicular to b^1x (or AB). The vector xa^1 represents the tangential component of the acceleration of A with respect to B ie. A^t_{AB}
4. Since the point A reciprocates along AO, therefore the acceleration must be parallel to AO, intersecting the vector xa^1 at a^1. Now the acceleration of the piston or the slider A(a_A) and a^t_{AB} may be measured to the scale.

5. The vector $b^1 a^1$ which is the sum of the vector $b^1 x^1$ and x^1a^1, represents the total acceleration of A with respect to B i.e a_{AB}. The vector b^1a^1 represents the acceleration of the connecting rod AB.
6. The acceleration of any other point on AB such as E may be obtained by dividing the vector b^1a^1 at e^1 in the same ratio as E divides AB. In other words

$$\frac{a^1e^1}{a^1b^1} = \frac{AE}{AB}$$

7. The angular acceleration of the connecting rod AB may be obtained by dividing the tangential component of the acceleration of A with respect to B a^t_{AB} to the length of AB. In other words, angular acceleration of AB,

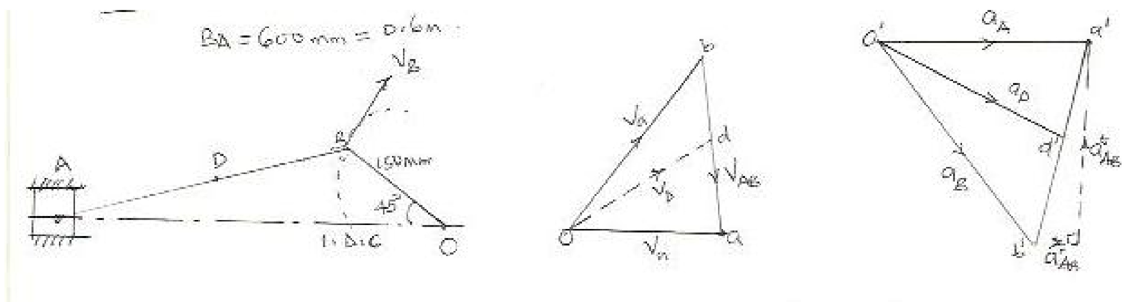
$$\text{AB} = \frac{a^t_{AB}}{AB} clockwise\ of\ B.$$

Example 1. The crank of a slider mechanism rotates clockwise at a constant speed of 300 r.p.m. the crank is 150mm and the connecting rod is 600mm long. Determine:

(a) linear velocity and angular acceleration of the mid point of the connecting rod,

(b) angular velocity and angular acceleration of the connecting rod, at a crank angle of 45^0 from inner dead centre position.

Solution: Given: N_{BO} = 300 r.p.m or $\omega_{BO} = 2\pi\ X \frac{300}{60}$ = 31.42 rad/s; OB = 150mm = 0.15m, BA = 600mm = 0.6m.



Linear velocity of B with respect to O or velocity of B.

$V_{BO} = V_B = \omega_{BO}$ x OB

= 31.42 x 0.15

= 4.713mls

Linear velocity of the mid point of the connecting rod.

- Draw space diagram, to some suitable scale
- For velocity diagram

1. Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of represent the velocity of B i.e V_{BO} or V_B, such that

   Vector ob = $V_{BO} = V_B$ = 4.713 mls.
2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e V_{AB}, and from point 0 draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. V_A. The vectors ba and o_a intersect at a.

   By measurement, the velocity of A with respect to B

   V_{AB} = vector ba = 3.4mls

   And velocity of A, V_A = Vector oa = 4m/s
3. To find the velocity of the midpoint D of the connecting rod AB, divide the vector b_a at d in the same ratio as D divides AB in the space diagram.

   $$bd/ba = BD/BA$$

4. Join od. Now the vector od represents the velocity of the mid point of the connecting rod i.e V_D.

By measurement,

V_D = vector od = 4.1m/s.

Acceleration of the midpoint of the connecting rod.

We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a_{BC}^{r} = a_B = \frac{V_{Bo}^2}{OB}$$

$$= \frac{(4.713)^2}{0.15}$$

$$= 148.1 mls^2$$

and the radial component of the acceleration of A with respect to B

5. $a_{AB}^{r} = \frac{V^2 AB}{BA}$

$$= \frac{(3.4)^2}{0.6} 19.3 m/s^2$$

Now, the acceleration diagram.

1. Draw vector o[1] b[1] parallel to Bo, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B ie. $a_{Bo}^{r}\ or\ a_B = 148.1 mls^2$
2. The acceleration of A with respect to B has the following two component

a. The radial component of the acceleration of A with respect to B ie. a_{AB}^{r} and

b. The tangential component of the acceleration of A with respect to B ie. a_{AB}^{t}. These two component are mutually perpendicular.

Therefore form point b^1, draw vector b^1x parallel to AB to represent a^r_{AB} = 19.3m/s and from point x draw vector xa^1 perpendicular to vector b^1 x whose magnitude is yet unknown.

3. Now from O^1, draw vector O^1 a^1 parallel to the path of motion of A (which is along AO) to represent the acceleration of A ie. a_A. The vectors x_a ^{1} and o^1a^1 intersect at a^1. Join a^1b^1.
4. In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector a^1b^1 at d^1 in the some ratio as D divides AB.

$$\text{i.e. } b^1d^1 / b^1a^1 = BD/BA$$

5. Join O^1d^1. The vector O^1d^1 represents the acceleration of mid point D of the connecting rod.

By measurement

$$A_D = \text{vector } o^1d^1 = 117\text{m/s}^2$$

b. Angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{V_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 rads^2 \text{ (Anticlockwise about B)}$$

c. Angular acceleration

By measurement, from the acceleration diagram

$$a^t_{AB} = 103 mls^2$$

Angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 rad./s^2 \ (clockwise\ about B).$$