The Carnot cycle

Carnot proposed a cycle which would give the maximum possible efficiency between temperature limits. Figure 1 shows this cycle which consists of an isothermal expansion ($pV = C$) of the gas in the cylinder from point 1 to point 2 as heat energy is supplied, followed by an adiabatic expansion (pV_ $= C$) to point 3. Between 2 and 3 the gas has cooled. The piston moves up the cylinder between 3 and 4 compressing the gas isothermally as heat energy is rejected, and between 4 and 1 the gas is compressed adiabatically. All the processes are reversible, and heat energy is supplied and rejected at constant temperature. The cycle is therefore impossible to create in practice. A lengthy proof, using the non-flow energy equation and the expressions for work done substituted into the expression for efficiency we have just derived, gives an expression for the efficiency of the cycle, i.e. the Carnot efficiency.

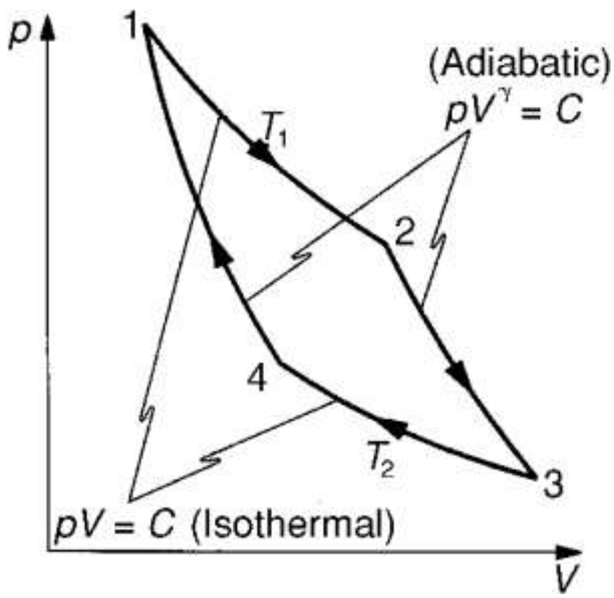

Figure 1 *Carnot cycle*

Carnot efficiency, $\eta = 1 - \frac{T_2}{T_1}$

T_2 is the lowest and T_1 the highest of only two temperatures involved in the cycle. The cycle cannot be created in practice, since we have reversible processes and must supply and reject heat at constant temperature. It does, however, supply a means of rating the effectiveness of a cycle or plant by allowing us to calculate a maximum theoretical efficiency based on maximum and minimum temperatures, even if the cycle is not operating on the Carnot cycle.

Example 1

A diesel engine works between a maximum temperature of 600°C and a minimum temperature of 65°C. What is the Carnot efficiency of the plant?

Carnot efficiency = $\eta = 1 - \frac{T_2}{T_1}$

$= 1 - \frac{(65+273)}{600+273} = 1 - \frac{338}{873} = 0.613 = 61.3\%$

The air standard cycles

In what are known as air standard cycles, or ideal cycles, the constant volume, constant pressure and adiabatic processes are put together to form theoretical engine cycles which we can show on the *p*/*V* diagram. The actual *p*/*V* diagram is different from what is possible in practice, because, for instance, we assume that the gas is air throughout the cycle when in fact it may be combustion gas. We also assume that valves can open and close simultaneously and that expansion and compressions are adiabatic. Air standard cycles are reference cycles which give an approximation to the performance of internal combustion engines.

Constant volume (Otto) cycle

This is the basis of the petrol engine cycle. Figure 2 shows the cycle, made up of an adiabatic compression, 1–2 (piston rises to compress the air in the cylinder), heat energy added at constant volume, 2–3 (the fuel burns), adiabatic expansion, 3–4 (the hot gases drive the piston down the cylinder), and heat energy rejected at constant volume, 4–1 (exhaust).

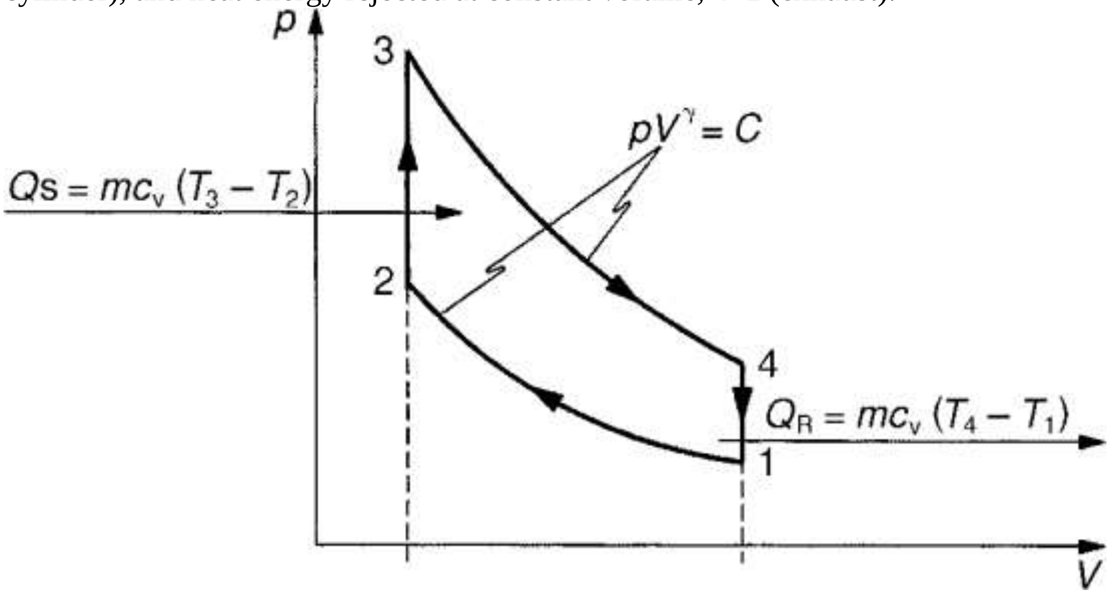

Figure 2 *Constant volume (Otto) cycle*

What can we do with this? We can calculate from our knowledge so far:

- the pressures, volumes and temperatures around the cycle;
- the work done during each of the processes and therefore the net work done;
- the heat energy transferred during each process;
- the ideal – or air standard – efficiency of the cycle using the expression we derived earlier in this chapter.

Indicated mean effective pressure, P_{mi}

The air standard efficiency of the cycle is a useful indicator of actual performance, but limited because it is not easy to decide in practice where heat energy transfers begin and end. The ideal cycle diagram – and an actual indicator diagram, which we see later – can also provide a value of *indicated mean effective pressure*, P_{mi}, which is another useful comparator. This is found by dividing the height of the diagram by its length to produce a rectangle of the same area. See Figure 3, in which the rectangle is shown hatched.

$$P_{mi} = \frac{area\ of\ diagram}{length} = \frac{A}{L}$$

The area of the diagram is work, joules = N.m. The length of the diagram is volume, i.e. m^3.

$$P_{mi} = \frac{N.m}{m^3} = \frac{N}{m^2}$$

Key notes.

- The indicated mean effective pressure can be thought of as the pressure acting on the piston over full stroke which would give the same work output. Generally speaking, the higher this is the better.
- Since the brake power of an engine is usually quoted and easier to record, instead of an indicated mean effective pressure, a value of *brake mean effective pressure*, P_{mb}, is more often used as the comparator. It is found by using the expression for indicated power with values of brake power and brake mean effective pressure substituted. This is covered later in the chapter.

Example 2.4.2

The ratio of compression of an engine working on the constant volume cycle is 8.6:1. At the beginning of compression the temperature is 32°C and at the end of heat supply the temperature is 1600°C. If the index of compression and expansion is 1.4, find:

(a) the temperature at the end of compression;

(b) the temperature at the end of expansion;

(c) the air standard efficiency of the cycle.

Figure 4 shows the cycle.

Note:

- the compression ratio is a ratio of volumes, $V1/V2$, not a ratio of pressures;
- the dimensionless ratio values of 8.6 and 1 are used directly in the equations;
- the heat energy transfer in a constant volume process is ($m.c$v.δT).

Solution.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \frac{T_2}{305} = \left(\frac{8.6}{1}\right)^{1.4-1}, T_2 = 305\ x\ 8.6^{0.4}$$

= 721.3K, temperature at end of compression.

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}, \frac{T_4}{1873} = \left(\frac{1}{8.6}\right)^{1.4-1}, T_4 = 1873\ x\ 0.42$$

= 792K, temperature attend of expansion.

Air standard efficiency,

$$= 1 - \frac{heat\ rejection}{heat\ supplied}$$

$$= 1 - \frac{m.c_v(T_4 - T_1)}{m.c_v(T_3 - T_2)},$$ m and c_v are cancel

$$= 1 - \frac{792-305}{1873-721.3} = 1 - \frac{487}{1151.7}$$

= 0.577 = 57.7% air standard efficiency.

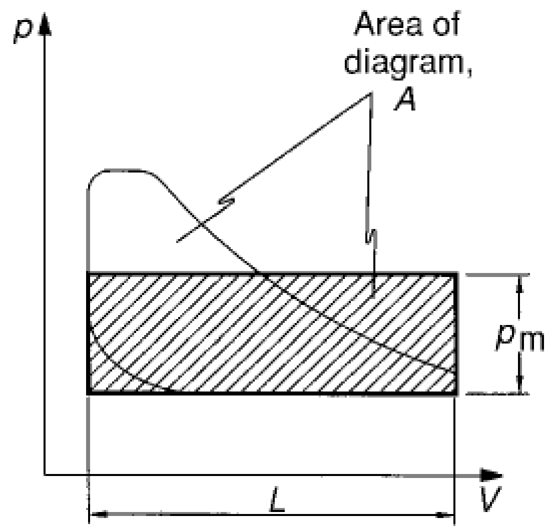

Figure 2.4.3 *Mean effective pressure*

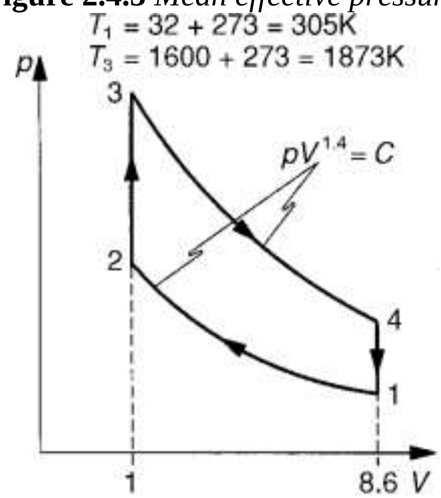

Figure 2.4.4 *Example 2.4.2*

Example 2.4.3

In an air standard (Otto) constant volume cycle, the compression ratio is 8 to 1, and the compression commences at 1 bar, 27°C. The constant volume heat addition is 800 kJ per kg of air. Calculate:

(a) the thermal efficiency;

(b) the indicated mean effective pressure, *P*mi.

c_v = 718 J/kgK

γ = 1.4

Figure 2.4.5 shows the cycle.

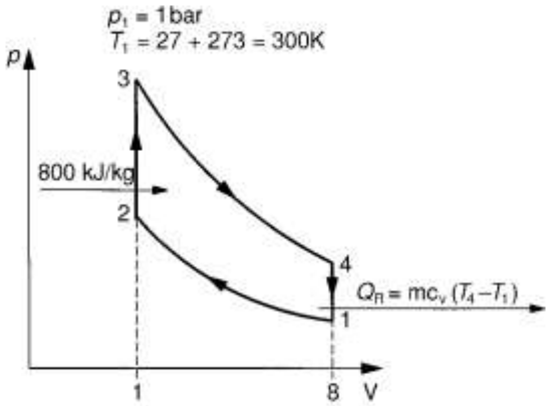

Figure 2.4.5 *Example 2.4.3*

$$\gamma = \frac{c_p}{c_v}, c_p = \gamma - c_v = 1.4\ x718$$

= 1005J/kgK, see adiabatic compression and expansion.

$$R = c_p - c_v, R = 1005 - 718 = 287\ J/kgK$$

$$P_1V_1 = m.R.T_1,\ V_1 = \frac{m.RT_1}{P_1} = \frac{1\ x\ 287\ x\ 300}{1\ x\ 10^5} = 0.861\ m^3$$

$$\frac{V_1}{V_2} = 8, V_2 = \frac{0.861}{8} = 0.1076\ m^3$$

$$Swept\ volume = V_2 - V_1 = 0.861 - 0.1076 = 0.7534m^3$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \frac{T_2}{300} = \left(\frac{8}{1}\right)^{1.4-1}, T_2 = 300\ x\ 8^{0.4} = 689.2K.$$

$$Q_{1-2} = m.c_v(T_3 - T_2)$$

$$T_3 = 689.2 + \frac{800\ x\ 10^3}{718} = 1803.4K \quad (m = 1kg)$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}, T_4 = 1803.4\ \left(\frac{1}{8}\right)^{0.4} = 785K$$

Heat energy rejected $= Q_{4-1} = m.c_v(T_4 - T_1)$

= 718(785 -300) = 348.2kJ

Work = heat supplied – heat rejected

= 800 – 348.2 = 451.8 kJ

Air standard efficiency = $\eta = 1 - \frac{heat\ rejection}{heat\ supplied}$

$$1 - \frac{451}{800} = 1 - 0.5647 = 0.435 = 43.5\%$$

Mean effective pressure, $P_{mi} = \frac{area\ of\ diagram}{length}$

$$= \frac{work}{Swep\ volume}$$

$$= \frac{451.8}{0.7534}\ 599.7\ kN/m^2$$

The indicator diagram
A real-life *p*/*V* diagram is called an *indicator diagram*, which shows exactly what is happening inside the cylinder of the engine. This plot is useful because it allows us to find the work which the engine is doing and therefore its power, and it also enables us to see the effect of the timing of inlet, exhaust and fuel burning, so that adjustments can be made to improve cycle efficiency. In the case of a large slow-speed engine, like a marine diesel engine which typically rotates at about 100 rpm, an indicator diagram can be produced by screwing a device called an engine indicator onto a special cock on the cylinder head of the engine. The indicator records the pressure change in the cylinder and the volume change (which is proportional to crank angle), and plots these on *p*/*V* axes using a needle acting on pressure sensitive paper wrapped around a drum. This produces what is known as an 'indicator card'.
Figure 2.4.10 shows the indicator. The spring in the indicator can be changed to suit the maximum cylinder pressure, so that a reasonable plot can be obtained. Such a mechanical device is not satisfactory for higher-speed engines, but the same result can be plotted electronically.

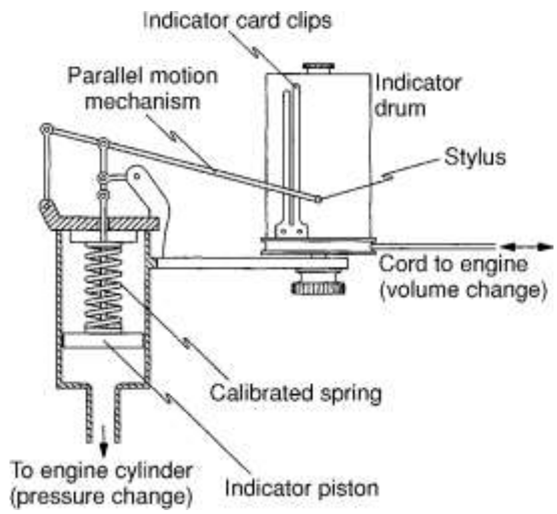

Figure 2.4.10 *Engine indicator*

In both cases we get an actual *p*/*V* diagram from within the cylinder, and just as we were able to find work done from our air standard cycles by finding the area within the diagram, so we can find the actual work done, and therefore the power of the engine, by finding the area of the indicator diagram. Of course in this case, the curves are not 'ideal', and the equations cannot be used, but the area within the diagram can be found by some other means, such as by using a planimeter.
Indicated power
As you might expect, the power calculated from the indicator diagram is called the *indicated power* of the engine. It is the power developed inside the cylinder of the engine. As we saw earlier, a value of indicated mean effective pressure can be found by dividing the area of the diagram by its length, but in this case, we must multiply the result by the spring rate of the indicator spring. This gives an 'average' cylinder pressure, used in the expression for indicated power, and it is also used as an important value for comparison between engines.
Indicated power is given by the formula,
$$Indicated\ power, ip = P_{mi}\ A.L.n$$
Where:

$P_{mi} = mean\ effective\ pressure\ \left(\frac{N}{m^2}\right)$
A = area of piston (m^2)
L = length of strike (m)
N = number of power strokes per second.

The verification of this expression can be seen in two ways.
First, we know that the area under the *p*/*V* diagram is work done. The product (*P*mi.*A*.*L*) gives this area since *P*mi is the height of the rectangle and the volume change is given by length multiplied by the area of the bore. The *n* term then imposes a time element which 'converts' the work done to power in kW.
Second, we can use the well-known work done expression from mechanics, work = force × distance. The force on the piston is (pressure × area, i.e. *P*mi × *A*), and this force operates over a distance equal to the length of stroke, *L*. The *n* term then gives power.
Putting the units into our expression for indicated power,

$$ip = \frac{N}{m^2}\ x\ m^2\ x\ m\ x\ \frac{1}{s} = \frac{N.m}{s} = \frac{J}{s} = watts$$

Key points

- The number of power strokes per second is the same as the rev/s for a 2-stroke engine, because there is a power stroke every revolution of the crank.
- For a 4-stroke engine, *n* is the rev/s divided by 2 because there is a power stroke once every two revolutions of the crank.

Brake power

Brake power is the power actually available at the output shaft of the engine. It would be a wonderful world if all the power developed in the cylinders was available at the output shaft, but unfortunately this is not the case because of the presence of friction. This absorbs a certain amount of power, called the friction power. The brake power is, therefore, always less than the indicated power, and this is expressed by the mechanical efficiency of the engine, η_m.

$$\eta_m = \frac{bp}{ip}$$

To find the brake power, it is necessary to apply a braking torque at the shaft by means of a *dynamometer*. The simplest form of this is a rope brake dynamometer which consists of a rope wrapped around the flywheel carrying a load. See Figure 2.4.11. More sophisticated types used on high-speed engines are hydraulic or electrical. They all do the same job in allowing the value of braking torque applied to the engine to be measured. This value is put into the formula for rotary power, i.e. $P = T\omega$, where *T* is the torque in N.m and ω is the speed of rotation in rad/s. ω can be inserted as $2\pi n$, since there are 2π radians in one revolution and *n* is the rev/s. We then have the usual form of the equation for brake power,

$$bp = 2\pi n.T$$

Putting in the units, we have,

$$bp = \frac{1}{s}\ x\ N.m = \frac{N.m}{s}\ \frac{J}{s} = watts$$

Note again that 'rev' is dimensionless, as is 2π.
For the rope-brake dynamometer in Figure 2.4.10, the friction load on the flywheel is,

$$(W - S)\ \text{newtons}$$

Where *W* is the applied weight and *S* is the spring balance reading.
The friction torque is,

$$(W - S) \times r$$

Where r is the radius of the flywheel.
The brake power is then given by,

$$\text{bp} = (W - S) \times r \times \omega \text{ watts}$$

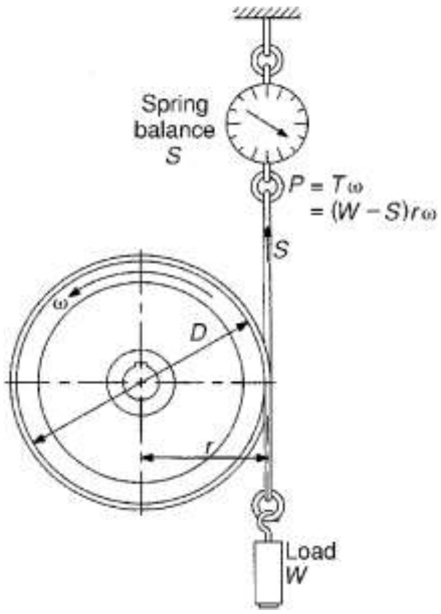

Figure 2.4.11 *Rope brake dynamometer*

Key point
When dealing with brake power, remember that we are dealing with the power output from the engine, i.e. from *all* the cylinders combined in a multi-cylinder engine. We usually assume that each cylinder is delivering the same power.

Brake mean effective pressure, P_{mb}

It was explained (see page 36) that a value of *brake mean effective pressure*, P_{mb}, is used as a comparator between engines, because it is easier to find than indicated mean effective pressure, P_{mi}. Brake mean effective pressure is calculated from the indicated power formula with brake power and P_{mb} substituted,

$$\text{bp} = P_{mb} \times A \times L \times n$$