**WEEKS 3 & 4**

# GEC 241- APPLIED MECHANICS II- DYNAMICS

## KINETICS OF PARTICLES

### NEWTON'S SECOND LAW OF MOTION

Kinetics is a branch of dynamics with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's law, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force. It can also be stated as: If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

The law was verified by experimentally applying known unbalanced force **F** to a particle and then measuring the acceleration **a**. A constant of proportionality m was determined from the ratio **m =F/a** since the force and acceleration are directly proportional. This positive scalar **m** is called the mass of the particle. Being constant during any acceleration, **m** provides a quantitative measure of the resistance of the particle to a change in its velocity that is its inertia.

This law can be mathematically written as

$$\boldsymbol{F} = \boldsymbol{ma} \qquad (\mathbf{1})$$

The above equation is one of the most important formulations in mechanics.

When a particle is subjected simultaneously to several forces, the formula should be replaced by

$$\sum \boldsymbol{F} = \boldsymbol{ma} \qquad (\mathbf{2})$$

where $\sum F$ represents the sum, or resultant, of all the forces acing on the particle.

### LINEAR MOMENTUM OF A PARTICLE: RATE OF CHANGE OF LINEAR MOMENTUM

When the acceleration **a** is replaced by the derivative *dv/dt* in equation (2), then

$$\sum \boldsymbol{F} = \boldsymbol{m}\frac{\boldsymbol{dv}}{\boldsymbol{dt}} \qquad (\mathbf{3})$$

Or, since the mass m of the particle is constant,

$$\sum \boldsymbol{F} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(\boldsymbol{mv}) \qquad (\mathbf{4})$$

The vector $mv$ is called the linear momentum or simply the momentum of the particle. It has the same direction as the velocity of the particle and its magnitude is equal to the product of the mass m and the speed v of the particle. Equation (4) expresses that the resultant of the forces acting on the particle is equal to the rate of change of the linear momentum of the particle.

It is in this form that the second law of motion was originally stated by Newton. Denoting by **L,** the linear momentum of the particle,

$$\mathbf{L} = \boldsymbol{mv} \qquad (\mathbf{5})$$

and by $\dot{\mathbf{L}}$ its derivative with respect to t. this can be written as

$$\sum \mathbf{F} = \dot{\mathbf{L}} \qquad (\mathbf{6})$$

It should be noted that the rate of change of the linear momentum $mv$ is zero when $\sum \mathbf{F} = 0$. Thus, if the resultant force acting on a particle is zero, the linear momentum of the particle remains constant, in both magnitude and direction. This is the principle of conservation of linear momentum for a particle which can be recognized as an alternative statement of Newton's first law.

**EQUATIONS OF MOTION**

Consider a particle of mass m acted upon by several forces then $\sum \mathbf{F} = \mathbf{ma}$. In order to solve problems involving the motion of a particle, however, it will be found more convenient to replace eqn.(2) by equivalent equations involving scalar quantities.

**Rectangular components.** Resolving each force **F** and the acceleration **a** into rectangular components, we write

$$\sum \left(F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k}\right) = m\left(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k}\right)$$

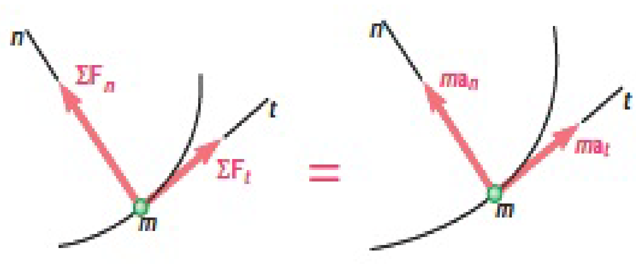
from which

$$\sum \boldsymbol{F_x} = \boldsymbol{ma_x} \qquad \sum \boldsymbol{F_y} = \boldsymbol{ma_y} \qquad \sum \boldsymbol{F_z} = \boldsymbol{ma_z} \qquad (\mathbf{7})$$

Recall that the components of the acceleration are equal to the second derivatives of the coordinates of the particle, we have

$$\sum \boldsymbol{F_x} = \boldsymbol{m\ddot{x}} \qquad \sum \boldsymbol{F_y} = \boldsymbol{m\ddot{y}} \qquad \sum \boldsymbol{F_z} = \boldsymbol{m\ddot{z}} \qquad (\mathbf{8})$$

**TANGENTIAL AND NORMAL COMPONENTS**.



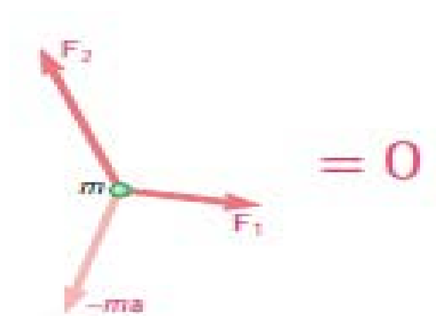

Resolving the forces and the acceleration of the particle into components along the tangent to the path (in the direction of the motion) and the normal (toward the inside of the path) and substituting into eqn.(2), we have two scalar equations

$$\sum \boldsymbol{F_t} = \boldsymbol{ma_t} \qquad \sum \boldsymbol{F_n} = \boldsymbol{ma_n} \qquad (\mathbf{9})$$

Substituting for $a_t$ and $a_n$, we have

$$\sum \boldsymbol{F_t} = \boldsymbol{m}\frac{\boldsymbol{dv}}{\boldsymbol{dt}} \qquad \sum \boldsymbol{F_n} = \boldsymbol{m}\frac{\boldsymbol{v^2}}{\boldsymbol{p}} \qquad (\mathbf{10})$$

**DYNAMIC EQUILIRIUM**



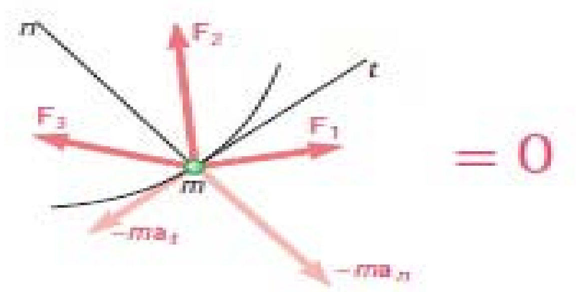

From eqn (2),$\sum \boldsymbol{F} - \boldsymbol{ma} = \boldsymbol{0}$, which expresses that if we add the vector –**ma** to the forces acting on the particle, we obtain a system of vectors equivalent to zero. The vector –**ma**, of magnitude ma and of direction opposite to that of the acceleration, is called the inertia vector. The particle may thus be considered to be in equilibrium under the given forces and the inertia vector. The particle is said to be in dynamic equilibrium and the problem under consideration can be solved by the methods developed earlier in statics.

In the case of coplanar forces, all the vectors including the inertia vector can be drawn tip-to-tail to form a closed vector polygon. Or the sums of the components of all the vectors in the figure above again including the inertia vector can be equated to zero. Using rectangular components, we therefore write

$$\sum \boldsymbol{F_x} = \mathbf{0} \qquad \sum \boldsymbol{F_y} = \mathbf{0} \quad \textbf{\textit{including inertia vector}}$$

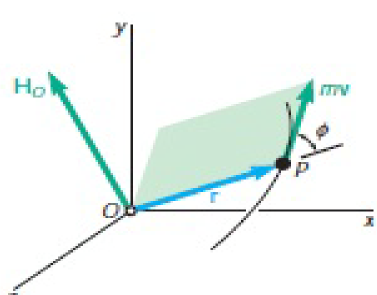
When tangential and normal components are used, it is more convenient to represent the inertia vector by its two components $-ma_t$ *and* $-ma_n$ in the sketch below. The tangential component of the inertia vector provides a measure of the resistance the particle offers to a change in speed, while its normal component (also called the centrifugal force) represents the tendency of the particle to leave its curved path. It should be noted that either of these two components may be zero under special conditions:

(1) If the particle starts from rest, its initial velocity is zero and the normal component of the inertia vector is zero at t = 0.
(2) If the particle moves at constant speed along its path, the tangential component needs to be considered.




**ANGULAR MOMENTUM OF A PARTICLE**

**RATE OF CHANGE OF ANGULAR MOMENTUM**



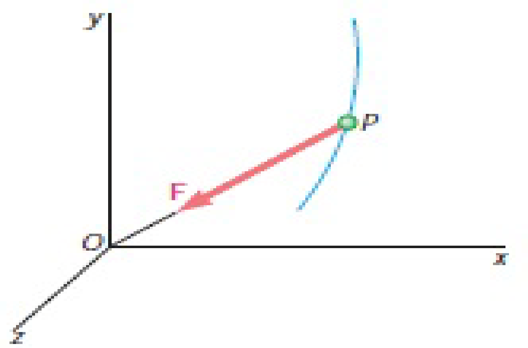
Consider a particle P of mass m moving with respect to a Newtonian frame of reference $O_{xyz}$. Recall that the linear momentum of the particle at a given instant is defined as the vector mv obtained by multiplying the velocity v of the particle by its mass m. The moment about O of the vector mv is called the moment of momentum or the angular momentum of the particle about O at that instant and is denoted by $\mathbf{H_o}$. Recalling the definition of the moment of a vector and denoting by **r** the position vector of P, we write

$$\boldsymbol{H_o} = \boldsymbol{r} \times \boldsymbol{mv} \qquad (\mathbf{11}\boldsymbol{a})$$

and note that $\mathbf{H}_O$ is a vector perpendicular to the plane containing **r** and *m***v** and of magnitude

$$\boldsymbol{H_o} = \boldsymbol{rmv}\,\mathbf{sin\,f} \qquad (\mathbf{11}\boldsymbol{b})$$

**MOTION UNDER A CENTRAL FORCE: CONSERVATION OF ANGULAR MOMENTUM**




When the only force acting on a particle *P* is a force **F** directed toward or away from a fixed point *O*, the particle is said to be moving *under a central force,* and the point *O* is referred to as the *center of force*. Since the line of action of **F** passes through *O*, we must have $\sum M_o = 0$ at any given instant. Therefore we obtain

$$\dot{\mathbf{H}}_{\mathbf{o}} = \mathbf{0} \qquad (\mathbf{12})$$

for all values of *t* and, integrating in *t*,

$$\mathbf{H}_O = \textbf{constant} \qquad \textbf{(13)}$$

We thus conclude that *the angular momentum of a particle moving under a central force is constant, in both magnitude and direction*.

Recalling the definition of the angular momentum of a particle, we write

$$\mathbf{r} \text{ X } m\mathbf{v} = \mathbf{H}_O = \textbf{constant} \qquad \textbf{(14)}$$

from which it follows that the position vector **r** of the particle *P* must be perpendicular to the constant vector $\mathbf{H}_O$. Thus, a particle under a central force moves in a fixed plane perpendicular to $\mathbf{H}_O$. The vector $\mathbf{H}_O$ and the fixed plane are defined by the initial position vector $\mathbf{r}_0$ and the initial velocity $\mathbf{v}_0$ of the particle. For convenience, let us assume that the plane of the figure coincides with the fixed plane of motion.

Since the magnitude $H_O$ of the angular momentum of the particle *P* is constant, the right-hand member in Eq. (11b) must be constant. We therefore write

$$rmv \sin f = r_0 m v_0 \sin f_0 \qquad (15)$$

This relation applies to the motion of any particle under a central force. Since the gravitational force exerted by the sun on a planet is a central force directed toward the center of the sun, Eq. (15) is fundamental to the study of planetary motion. For a similar reason, it is also fundamental to the study of the motion of space vehicles in orbit about the earth.

Alternatively, we can express the fact that the magnitude $H_O$ of the angular momentum of the particle *P* is constant by writing

$$mr^2\dot{u} = H_o = constant \qquad (16)$$

or, dividing by *m* and denoting by *h* the angular momentum per unit mass $H_O/m$,

$$r^2\dot{u} = h \qquad (17)$$

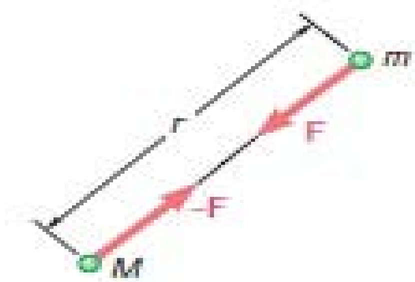
Equation (17) can be given an interesting geometric interpretation.

**NEWTON'S LAW OF GRAVITATION**

It states that two particles of masses M and m at a distance from each other attract each other with equal and opposite forces **F** and **–F** directed along the line joining the particles. The common magnitude F of the two forces is

$$F = G\frac{Mm}{r^2} \qquad (18)$$

where *G* is a universal constant, called the *constant of gravitation*.




Experiments show that the value of *G* is $(66.73 \pm 0.003) \times 10^{-12}\, m^3/kgs^2$ in SI units or approximately $34.4 \times 10^{-9}\, ft^4/lb.s^4$ in U.S. customary units. Gravitational forces exist between any pair of bodies, but their effect is appreciable only when one of the bodies has a very large mass. The effect of gravitational forces is apparent in the cases of the motion of a planet about the sun, of satellites orbiting about the earth, or of bodies falling on the surface of the earth. Since the force exerted by the earth on a body of mass *m* located on or near its surface is defined as the weight **W** of the body, we can substitute the magnitude $W = mg$ of the weight for *F*, and the radius R of the earth for r, we obtain

$$W = mg = \frac{GM}{R^2}m \quad or \quad g = \frac{GM}{R^2} \qquad (19)$$

where *M* is the mass of the earth. Since the earth is not truly spherical, the distance *R* from the center of the earth depends upon the point selected on its surface, and the values of *W* and *g* will thus vary with the altitude and latitude of the point considered. Another reason for the variation of *W* and *g* with latitude is that a system of axes attached to the earth does not constitute a newtonian frame of reference. A more accurate definition of the weight of a body should therefore include a component representing the centrifugal force due to the rotation of the earth. Values of *g* at sea level vary from 9.781 $m/s^2$, or 32.09 $ft/s^2$, at the equator to 9.833 $m/s^2$, or 32.26 $ft/s^2$, at the poles.

The force exerted by the earth on a body of mass *m* located in space at a distance *r* from its center can be found.

The computations will be somewhat simplified if we note that according to Eq. (19), the product of the constant of gravitation *G* and the mass *M* of the earth can be expressed as

$$\boldsymbol{GM = gR^2} \qquad (\mathbf{20})$$

where *g* and the radius *R* of the earth will be given their average values *g*= 9.81 $m/s^2$ and *R*= 6.37 X $10^6$ m in SI units and *g*=32.2 $ft/s^2$ and *R*= (3960 mi)(5280 ft/mi) in U.S. customary units.

The discovery of the law of universal gravitation has often been attributed to the belief that, after observing an apple falling from a tree, Newton had reflected that the earth must attract an apple and the moon in much the same way. While it is doubtful that this incident actually took place, it may be said that Newton would not have formulated his law if he had not first perceived that the acceleration of a falling body must have the same cause as the acceleration which keeps the moon in its orbit. This basic concept of the continuity of gravitational attraction is more easily understood today, when the gap between the apple and the moon is being filled with artificial earth satellites.

**KEPLER'S LAWS OF PLANETARY MOTION**

Kepler's three laws of planetary motion can be stated as follows:

1. Each planet describes an ellipse, with the sun located at one of its foci.
2. The radius vector drawn from the sun to a planet sweeps equal areas in equal times.
3. The squares of the periodic times of the planets are proportional to the cubes of the semi-major axes of their orbits.