LECTURE NOTE

ON

GEC 224 (STRENGTH OF MATERIALS)

INTRODUCTION

External forces applied to a body have the tendency to deform the body which develops an internal résistance against the deforming forces. This resistance increases with the increase in deforming forces but only up to a certain limit, beyond which the deforming forces will cause the failure of that body. The ultimate internal resistance, to the eternal forces, offered by a body depends upon the type of deformation taking place and the nature of material of which the body is made.

In strength of materials the internal effects produced and the deformations of bodies caused by externally applied forces are studied.

STRESS AND STRAINS

STRESS

External forces acting on a rigid body are termed as load. All externally applied loads deform an elastic material. As the material undergoes deformation it sets up internal résistance to the deformating forces. The quantum of internal resisting forces correspondingly increases with increase with the increase in externally applied loads only up to a certain limit beyond which any increase in applied loads will continue the process of deformation to the stage of failure. The deformation is known as <u>strain</u> and the resisting forces are called <u>stresses</u>. Stress per unit area is called unit stress and the total internal force within a single member is generally called total stress.

Stress which are normal to the plane on which they act are called <u>direct stresses</u>. There are only two basic stresses which are:

(i) The normal stress: Which act normal to the stress surfaces under consideration. These are either tensile or compressive stresses.

(ii) The shearing stresses: Which act parallel to the stressed surfaces under consideration. A member could be experiencing any of the basic stress or combination thereof.

The force transmitted across any section, divided by the area of that section, is that the intensity of stress or, for brevity, the stress (σ) = P/A, $measure\ in\ N/m^2$

- Tensile stress: Consider a straight bar of uniform x-section
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The fig. above is being subjected to a pair of collinear forces acting in opposite directions and coinciding with the axis of the bar. If the forces are directed away from the bar, then the bar tends to increase in length under the action of applied forces and the stresses developed in the bar are tensile. Tensile stress may be denoted by ∂t.

- Compressive stress. In the above case, if the forces are directed towards the bar (fig. below) then the bar tends to shorten in length under the action of the applied forces. The stresses developed in the bar are compressive and may be denoted by ∂c.

STRAIN:

Strain is a measure of the deformation produced in a member by the load. Direct stresses, tensile or compressive, produce change in length in the direction of the stress. If a rod of length l is in tension and the stretch or elongation produced is x , then the directs strain (E) is define as the ratio.

$$\frac{Elongation}{original\ length} = change\ in\ length\ for\ unit\ length$$

$$or\quad E = x/l$$

STRESS-STRAIN CURVES FOR TENSION

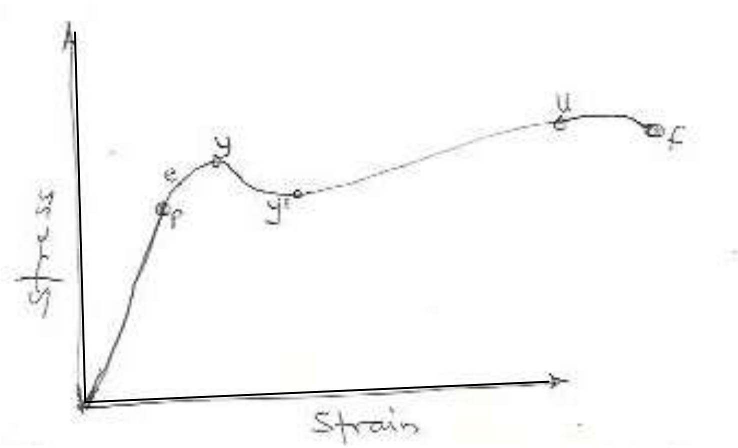
Behavior of materials subjected to tension is studied by plotting curves of stresses and corresponding strains observed by gradually increasing axially applied load to the point of failure of the specimen. Such curves, for different materials, differ in shape.

Stress-Strain Curves for ductile materials

Mild steel is the most commonly used ductile material. A specimen mild steel in tension loaded with gradually increasing loads shows initially the strains that are proportional to the stresses. Beyond a certain point P fig. below, known as limit on proportionality, the stress-strain curves does not remain linear. The specimen, if stressed beyond e, known as elastic limit, does not return back to its original position when the load is removed.

Property of materials to recover their original position on removal of loads is termed as elasticity.

If the specimen is laded beyond e then on unloading the specimen, a certain amount of strain called permanent set is retained by the specimen. With further increase in load, strain goes on increasing along ey to y^1. Immediately beyond the point y there is an

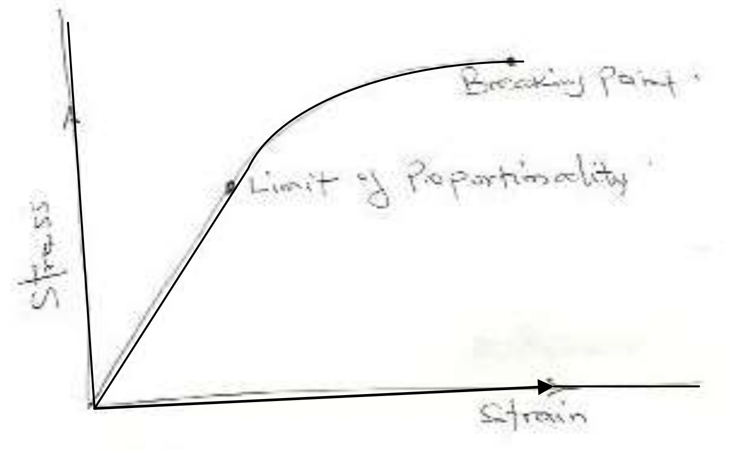


increase in strain even though there is no increase in stress. The stress corresponding to the point y is called the yield stress.

At the yield stress the material begins to flow. At u the stress is the maximum and is known as ultimate stress. Beyond u the bar elongates even with decrease in stress and finally fails at a stage corresponding to point f. The ratio of maximum load, that the specimen is capable of sustaining, to its original area of cross-section is termed as ultimate stress of the material.

After U the specimen is greatly reduced in cross-section area. At f, the point of failure, the reduced area is the least and this phenomenon is known as necking.

Stress-strain curve for brittle materials



Brittle materials have a very low proportionality point and do not show the yield point.

HOOKE'S LAW. Principle of superposition: It states that for materials subjected to simple tension or compression within elastic limit, the stress is proportional to the strain. Mathematically, it can be expressed as

$$\frac{Stress}{Strain} = constant \ (called\ the\ young's\ Modules\ or\ the\ modules\ of\ elasticity, usually,\ denoted\ by\ E$$

Or $\quad \frac{\sigma}{\in} = \text{E}$

For a bar of uniform cross-section A and length l this can be written

$$E = \frac{pl}{Ax}$$

Example 1: The following results were obtained in a tensile test on a mild-steal specimen of original diameter 2cm, aro guage length 4cm. At the limit of proportionality the load was 80,000N and the extension 0.048mm. The specimen yielded at a load of 85,000N, and the maximum load with stood was 150,000N.

When the two parts were fitted together after being broken, the length between guage points was found to be 5.56cm, and the diameter at the neck was 1.58cm.

Calculate the Young's Modulus and the stress at the limit of proportionality, the yield stress and ultimate tensile stress; also the percentage elongation and contraction.

Solution:

P = 80,000N, A = π^2_{mm}, L=40mm, x = 0.048

From E = $\frac{pl}{Ax}$

$$\therefore E = \frac{80000 \ x \ 40}{\Pi \, x \, (10)^2 \ x \ 0.048}$$

$$= 213{,}000 N / mm^2$$

Stress at limit of proportionality = $P/A = \frac{80{,}000}{\Pi \ x \, 100}$

= 254.7 N/mm^2

= 255N/mm^2

Yield stress = $\frac{85000}{\Pi \, x 100}$

= 478N/mm^2

Percentage Elongation = $\frac{5.56 - 4}{4} \ x \, 100 = 39\%$

Percentage Contraction = $\frac{2^2 - 1.58^2}{2^2} \ x \, 100 = 38\%$

Ex. 2: A square steel rod 20mm x 20mm in section is to carry an axial load (compressive) of 100KN. Calculate the shortening in length of 50mm. E – 2.14 x 10^8 KN/m^2

Solution

A = 0.02 x 0.02 = 0.0004m^2

l = 50mm or 0.05m

p = 100KN

E = 2.14 x 10^8 KN/m^2

Shortening of the rod ∂l

Stress, $\delta = \frac{P}{A}$

$$\therefore \delta = \frac{100}{0.0004} = 250000\,KN/m^2$$

Also, E = $\frac{Stress}{Strain}$

Strain = $\frac{Stress}{E} = \frac{250000}{2.14\ x\ 10^3}$

Or $\frac{\partial l}{l} = \frac{250000}{2.14 x 10^8}$

$$\therefore \partial l = \frac{25000}{2.14\ x\ 10^8}\ x\ l$$

$$= \frac{25000}{2.14\ x\ 10^8}\ x\ 0.05$$

$$= 0.0000584\,m\ or\ 0.0584\,mm$$

POISON'S RATIO

If a baby is subjected to a load, its length changes; ratio of this change in length to the original length is known as linear or primary strain. Due to this load, the dimensions of the body change; in all directions at right angles to its line of application the strain thus produced are called lateral or secondary or transverse strains and are of nature opposite to that of primary strains. For example, if the load is tensile, there will be an increase in length and a corresponding decrease in cross-sectional area of the body. In this case, linear or primary strain will be tensile and lateral or secondary or transverse strain will be compressive.

The ratio of lateral strain to linear strain is known as Poisson's ratio i.e. Poisson's ratio

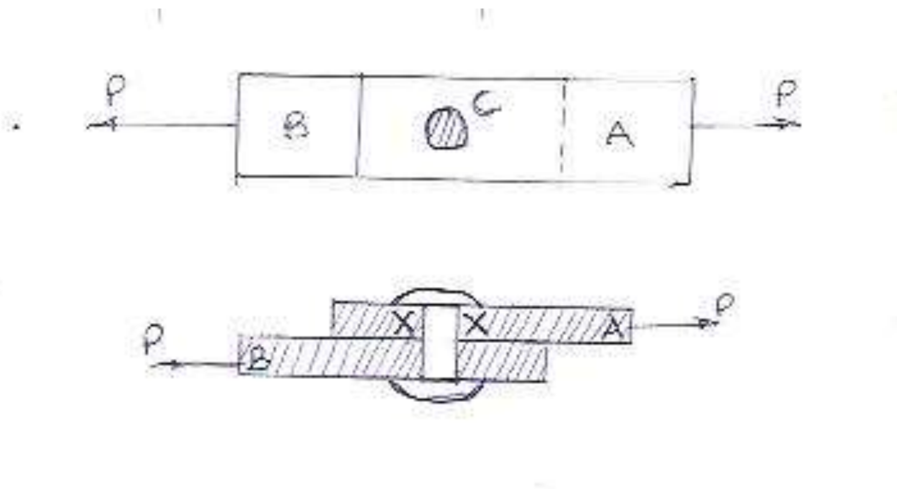
N = $\frac{lateral\ strain\ or\ transverse\ strain}{Linear\ or\ primary\ strain} = \frac{1}{m}$

Where,

M = is a constant and its value varies between 3 and 4 of different materials.

Shear Stress

Consider two plates A and B (fig. 1) joined together by a rivet C. if the plates carried a tensile load P then the rivet may shear along the plane XX (fig. b). If d is the diameter of the rivet, then the area of X – selection of the rivet subjected to shear is



$$A \frac{\Pi d^2}{4}$$

and the shear stress

$$\sigma = \frac{P}{A}$$

Or $\sigma = \frac{4P}{\Pi d^2}$

It should be noted that the applied load is tangential to the resisting area and therefore shear stress is also termed as tangential stress. The tensile and compressive stresses on the other hand are caused by forces acting perpendicular to the areas of resisting those forces and as such these stresses are termed as <u>direct stresses</u> or direct stresses.

Example 1. A hollow cast-iron cylinder 4m long, 300mm out diameter, and thickness of metal 50mm is subjected to a central load on the top when standing straight. The stress produced is 7500Kn/m^2. Assume Young's Modulus for cast-iron as 1.5 x 10^8 KN/m^2 and find (i) magnitude of the load, (ii) longitudinal strain produced, and (iii) total decrease in length

SOLUTION

1. Given: D = 300mm = 0.3m (outer diameter)

   t = 50mm = 0.05m (thickness)

   d = D-2t = 0.3 – 2 x 0.05 = 0.2 (inner diameter)

   l = 4m

   ∂ = 75000KN/m^2

   E = 1.5 x 10^8 KN/m^2

(i) Magnitude of the load P:

From $\delta = \frac{P}{A}$

P = $\sigma\ x\ A = 75999\ x\ \frac{\Pi}{4}\left(D^2 - d^2\right)$

= 7500 x $\frac{\Pi}{4}\ (0.3\ -\ 0.2\ )^2$

= 2945.2KN

(ii) Longitudinal Strain produced, e:

Strain, $(\ell) = \frac{Stress}{E} = \frac{75000}{1.5\ x\ 10^8}$

= 0.0005

(iii)Total decrease in length dl;

Using the relation

Strain = $\frac{Change\ in\ length}{Original\ length} = \frac{dl}{l}$

0.0005 = $\frac{dl}{l}$

$\therefore dl = 0.0005\ x\ 4$

= 0.002m = 2mm

Example 2. A steel bar is 900mm long; its two ends are 40mm and 30mm in diameter and the length of each rod is 200mm. The middle portion of the bar is 15mm in diameter and 500mm long. If the bar is subjected to an axial tensile load of 15KN, find the total extension.

Take E = 200GN/m^2

Given : Load, P = 15KN

Area, $A_1 = \frac{\Pi}{4}\, x\, 40^2 = 1256.6 mm^2 = 0.001256 m^2$

Area, $A_2 = \frac{\Pi}{4}\, x\, 15^2 = 176.7 mm^2 = 0.0001767\, m^2$

Area, $A_3 = = \frac{\Pi}{4}\, x 30^2 = 706.8 mm^2 = 0.0007068 m^2$

Length: l_1 = 200mm = 0.2m, l_2 = 500mm = 0.5m and l_3 = 200mm = 0.2m

Total extension of the bar:

Let dl_1, dl_2 and dl_3 be the extensions in the parts 1, 2 and 3 of the steel bar respectively.

Then, $dl_1 = \frac{Pl_1}{A_1E_1}, dl_2 = \frac{Pl_2}{A_2E_1}, dl_3 = \frac{Pl_3}{A_3E} \left[\therefore E = \frac{\partial}{\ell} = \frac{P/A}{\frac{\partial l}{l}} \right]$

Total extension of the bar,

$$dl = dl + dl_2 + dl_3 = \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E} = \frac{P}{E} \left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right]$$

$$= \frac{15 x 10^3}{200\ x\ 10^9} \left[\frac{0.20}{0.001256} + \frac{0.50}{0.0001767} + \frac{0.20}{0.0007068} \right]$$

$$= 0.0002454 m = 0.2454 mm$$

Example.3 The bar in the fig below is subjected to a tensile load of 50KN. Find then diameter of the middle portion if the stress is limited to 130MN/m^2. Find also the length of the middle portion if the total elongation of the bar is 0.015mm. Take E=200GN/m^2

Magnitude of Tensile load, P = 50KN

Stress - $\partial = 130 Mw/m^2$

Total Elongation of the bar ((∂l) = 0.15mm = 0.15 x 10^{-3}m

E = 200 GN/m^2

From $\delta = \frac{P}{A} = \frac{50\ x\ 1000}{\left(\pi/4\right)d^2} = 130\ x\ 10^6$

$$d = \left[\frac{50\ x\ 1000}{\left(\pi/4\right) x\ 130\ x\ 10^6}\right]^{\frac{1}{2}}$$

= 0.0221m

= 22.1mm

Length of the middle portion

Let the length of the middle portion = x metre

Stress in the end portions, $\sigma^1 = \frac{50\ x\ 1000}{\pi/4\ x \left(\frac{40}{1000}\right)^2}$

= 39.79 x 10^6 N/m^2

From, $dl = \frac{P}{A}\ x\ \frac{l}{E}$

$\therefore$ Elongation of the end portion $(dl) = \delta\ x\ \frac{0.25 - x}{E}$

Also, elongation of the end portions of extension of the middle portion = 0.15 x 10^{-3}

$$\frac{39.79\ x 10^6\ x\ (0.25 - x)}{200\ x\ 10^9} + \frac{130\ x 10^6\ x X}{200\ x\ 10^9} = 0.15\ x\ 10^3$$

$$39.79\ x\ 10^6\ x\ (0.25 - x) + 130 x 10^6,\ we get$$

$$0.25 - x + 3.267x = 0.754$$

$$x = 0.222m = 222m$$

Example 4. A steel rod of 20mm diameter and 500cm long is subjected to an axial pull of 3000kg. Determine (i) the intensity of stress, (ii) the strain (iii) the elongation of rod. Take E = 2.1 x 10^8 KN/m^2

Diameter of rod = 20mm = 0.02m

Length of the rod = 500cm = 0.5m

Load= 3000kg = 30000 N = 30KN

Cross sectional area of the rod is A = $\frac{\Pi d^2}{4} = \frac{\Pi \, x \, 0.02}{4} = 3.14 \; x \, 10^4 m^2$

(i) Intensity of Stress $\sigma = \frac{P}{A} = \frac{30000}{3.14 x 10^4} = 0.955 \; x \, 10^8 \, N/m^2$

$=0.955 \text{ x } 10^5 \text{KN/m}^2$

$= 955 \text{ x } 10^4 \text{KN/m}^2$

(ii) Strain $\in = \frac{\sigma}{E} = \frac{955 \; x \, 10^4}{2.1 \; x \; 10^8} = 0.955 \; x \, 10^8 \, N/m^2$

$=454.76 \text{ x } 10^{-4}$

$= 0.000455$

$$\text{dl} = \frac{Pl}{AE} = \frac{30 \, x \, 0.5}{3.14 \, x \, 10^{-4} \; x \; 2.1 x \, 10^8} = \frac{150}{3.14 \; x \; 2.1 \; x 10^4} = \frac{150}{65940}$$

iii. Elongation: $0.002275 m = 2.275 \; x 10^{-3}$

$= 0.0227 mm = 0.0002275 m$

FORCE AND MOMENTS

Forces.

Definition (i.) A force a push or pull.

(ii) Force is that which changes or tend to change the existing state of rest or of uniform motion of a body in a straight line it is measured in Newton (N).

A force is completely defined when known:

1. It magnitude (how much, size).

2. It direction.

3. Point of application – i.e. the point on the body at which the force acts.

General Condition for Equilibrium of Co-planar forces.

For a number of co-planar force to be in equilibrium, the following condition must be satisfied.

1. The sum of the force in one direction must be equal to the sum of the force in the opposite direction.

2. Sum of component in a perpendicular direction must be equal to sum of component in opposite direction.
3. Sum of the clockwise moment of the force(s) about any point must be equal to the sum of anticlockwise moment about the same Point.

Effects of a force.

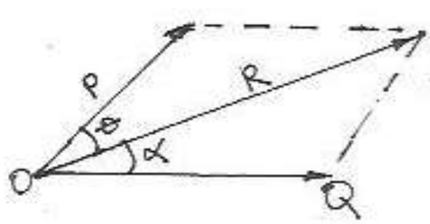
A force may produce the following effects a body on which it acts.

1. It may change the motion of a body on which it act i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of body.

Principle of Parallelogram of Forces.

It two force acting at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, the resultant of the forces will be represented both in magnitude and direction by the diagonal of the parallelogram drawn form that point.

Two or more vectors acting on a given object in various directions can be combined to give a single vector which will produce the same effect as all the vectors put together. The single vector is called the resultant.



Resultant of vectors
(Vectors sum)

If two vectors P and Q act on an object, in the same direction the resultant is P+Q (algebraic sum). If the two vectors act indirectly opposite directions, the resultant is P-Q (algebraic sum). If the vectors P and Q are inclined as shown above , the resultant R is the diagonal drawn through O.

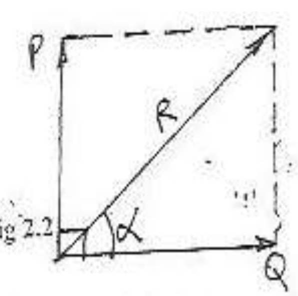


Fig 2.2

In vector notation R = P + Q

(Vectors sum) if the angle

Between P and Q is ɵ = 90, the

magnitude of R = $\sqrt{P^2 + Q^2}$

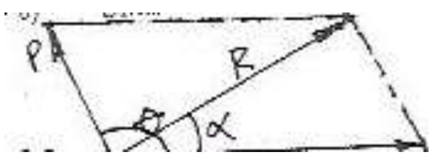
and the direction of R is $\alpha = \tan^{-1}(P/Q)$

Direction of the resultant

R is the magnitude of the diagonal of the rectangle if the angle between P and Q is obtuse as in fig: 2.3 below. The resultant R is given by the magnitude of the diagonal which has value.

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$$

The direction of R, given by angle α can be found by scale drawing or Sine rule



$$\frac{R}{\sin(180 - \theta)} = \frac{P}{\sin\alpha}$$

R = magnitude of the diagonal

.

Where P and Q = Force whose resultant is required to be found, θ = Angle between the force P and Q, and α = Angle which the resultant force makes with one of the force.

Note: if the angle (α) which the resultant force makes with the other force Q.

$$\text{then } \tan\alpha = \frac{p\sin\theta}{Q + p\cos\theta}$$

Ex.1 Two forces of 100N and 150N are acting simultaneously at a point. What is the resultant of these two forces the angle between is 45°?

Solution.

Given: P = 100N, Q = 150N and Q = 45°

We known that resultant of the two forces,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{100^2 + 150^2 + 2 \times 100 \times 150\cos 45°}$$

$$= \sqrt{10000 + 22500 + 30000 \times 0.707}$$

$$= 250N.$$

Ex: 2 Two forces act at an angle of 120°. The bigger force is 40N and the resultant is perpendicular to the smaller one. Find the Smaller force.

Solution:

Given: AOC = 120°, P = 40N and ‹BOC = 90°

From the geometry of the figure, we find the ‹AOB,

$$\alpha = 120° - 90°$$

$$= 30°$$

Let Q = Smaller force in N.

We know that;

$$\tan \alpha = \frac{p \sin\theta}{Q + p\cos\theta}$$

$$\text{Tan } 30° = \frac{Q \sin 120°}{40 + Q\cos 120°} = \frac{Q \sin 60°}{40 + Q(-\cos 60°)}$$

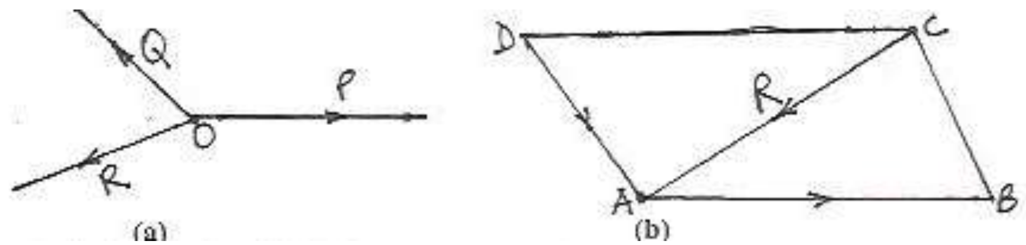
$$0.577 = \frac{Q \times 0.8660}{40 - Q \times 0.5} = \frac{0.866Q}{40 - 0.5Q}$$

$$40 - 0.5Q = \frac{0.866Q}{0.577} = 1.5Q$$

$$2Q = 40, \text{ or } Q = 20N.$$

TRIANGLE OF FORCES

If three forces acting at a point is represented in magnitude and direction by three sides of a triangle taken in other (taken the same way round), they will be in equilibrium.



Let the forces P, Q, R act at the point O.

Let the P be represented in magnitude and direction by AB

Let the Q be represented in magnitude and direction by BC

Let the R be represented in magnitude and direction AC

R = P + Q (the method of vectors addition).

Complete the parallelogram ABCD.

BC = AD (opposite side of parallelogram).

:. AD represent the forces Q.

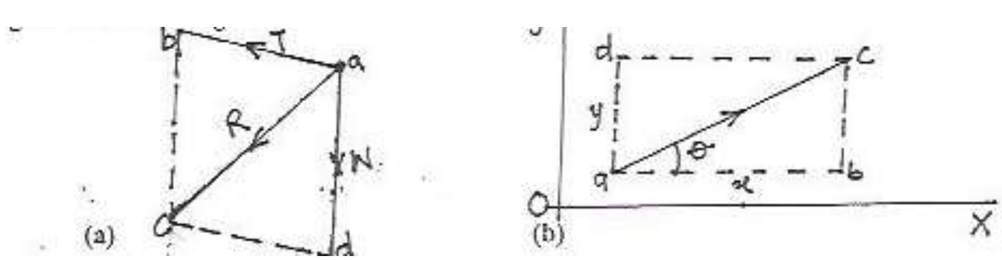
By the parallelogram of the forces, the resultant of the force AB and AD is represented by AC. i.e. the resultant of AB, BC, CA, = the resultant of the AB, AD,CA.

The resultant of AC, CA. But the resultant of AC and CA = O

:. The 3 forces are in equilibrium

<u>EQUILIBRANT</u>

The force W and T may also be represented by the two sides' ab and ad of the parallelogram abcd below



The resultant ac may be balanced by an equal and opposite force ca called the equilibrant

RESOLUTION OF A FORCE.

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is generally resolved along two mutually perpendicular directions. Infact, the resolution of a force is the reverse action of the addition of the component.

The fig. above shows a force R = resolve into force x = ab and Y = ad along the two perpendicular direction Ox and Oy respectively.

Ab = ac $\cos\theta$ i.e $x = R\cos\theta$

And ad = ac$\sin\theta$ or $y = R\sin\theta$.

Principle of Resolution

It states "The algebraic sum of the resolved parts of a number of forces, in a given direction, is equal to resolved part of their resultant in the same direction "

Method of Resolution for the Resultant force.

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e. ΣH).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e. ΣV).
3. The resultant R of the given force will be given by the equation:
   $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

4. The resultant force will be included at an angle Q, with the horizontal, such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Note: The value of the angle θ will vary depending upon the values of ΣV and ΣH as discussion below.

1. When ΣV is tve, the resultant makes an angle between 0°. and 180°. But when ΣV is –ve, the resultant makes angle between 180° and 360°.
2. When ΣH is + ve, the resultant makes an angle from 0° to 90° and 270° to 360°. But when ΣH is -ve the resultant makes an angle between 90°and 270°

Example 1

A triangle ABC has its side AB = 40mm along positive x-axis and side BC = 30mm along positive y-axis. Three forces of 40N, 50N and 30N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces

Solution.

The system of given forces is shown below.

From the geometry of the fig. we find that the triangle ABC is a right angled triangle in the side AC = 50mm moreover,

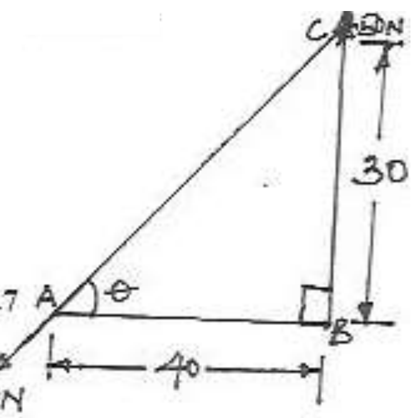


Fig 2.7

$$\sin\theta = \frac{30}{50} = 0.6$$

And

$$\cos\theta = \frac{40}{50} = 0.8$$

Resolving all the forces horizontally (i.e. along AB)

$\Sigma H = 40 - 30\cos\theta = 40 - (30 \times 0.8) = 16N.$

And now resolving all the forces vertical (i.e. along BC)

$\Sigma V = 50 - 30\sin\theta\ 50 - (30 \times 0.6) = 32N$

We known that magnitude the resultant force,

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(16)^2 + (32)^2}$

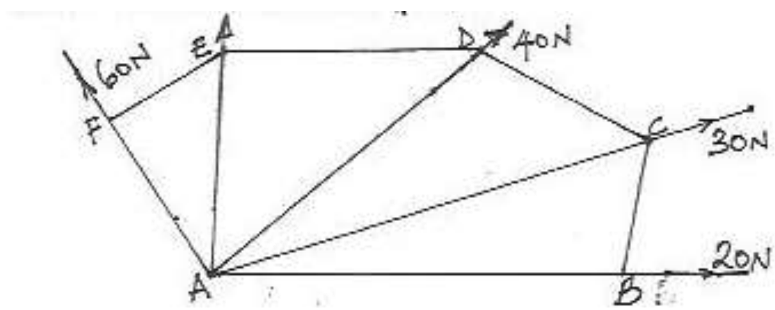
$= 35.8N$

Note: Since both the values of ΣH and ΣV are positive, therefore the resultant force lies between 0° and 90°.

Example .2 The forces 20N, 30N, 40N, 50N, 60N, are acting at one of the angular point of a regular hexagon, toward the other five angular points taken in order. Find the magnitude and direction of the resultant force.

Solution.

The system of given forces is shown in the fig. below.



$\Sigma H = 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°$

$= 20 \times 1 + 30 \times 0.8660 \quad + 40 \times 0.5 + 50 \times 0 + 60 \times (-0.5)$ N.

$= 36.0$N ------------- (i)

and now resolving the force vertically (i.e. at right ‹ to AB).

$\Sigma V = 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120°$

$= 20 \times 0 + 30 \times 0.5 + 40 \times 0.8660 + 50 \times 1 + 60 \times 0.8660$

$= 151.6$N

We known that magnitude of the resultant force

$R = (\Sigma H)^2 + (\Sigma V)^2$

$= (36.0)^2 + (151.6)^2$

$= 155.8$N

Direction of the resultant forces.

Let θ = Angle, which the resultant force makes with the horizontal (i.e. AB)

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = 76°39$Ans.

Or Note: Since both the values of ΣH and ΣV are positive therefore the resultant force lies between 0° and 90°.

Ex 3 . The following force act at a point:

1.20N inclined at 30° towards North of East.
2.25N toward North
3.30N toward North West, and
4.35N inclined at 40° toward south of west.

Find the magnitude and direction of the resultant force

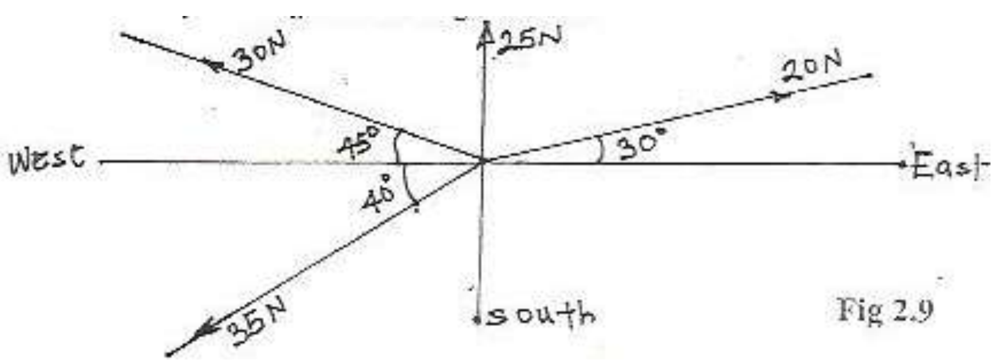


Fig 2.9

Magnitude of the resultant force.

Resolving all the force horizontally (i.e. along East – West line)

ΣH = 20cos30°+ 25cos90° + 30cos135° +35cos220°N

= 20 x 0.866 + 25 x 0 + 30(-0.707) + 35 (-0.766)

= -30.7N

And now resolving all the force vertically (i.e. along N-S line

ΣV = 20sin30° + 25sin90° + 30sin135° + 35sin220°N

= (20 x 0.5) + (25 x 1.0) + (30 x 0.707) + 35(-0.64

= 33.7N.

We known that magnitude of the resultant of the force

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \quad = \sqrt{(-30.7)^2 + (33.7)^2}$

$= 45.6N$

Direction of the resultant force.

Let θ = Angle, which the resultant force makes with that East.

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{33.7}{-30.7} = -1.098$$

Or θ = 47°42

Since ΣH is – ve and ΣV is + ve, therefore θ lies between 90° and 180°.

: Actual θ = 180°- 47°42

= 132°18

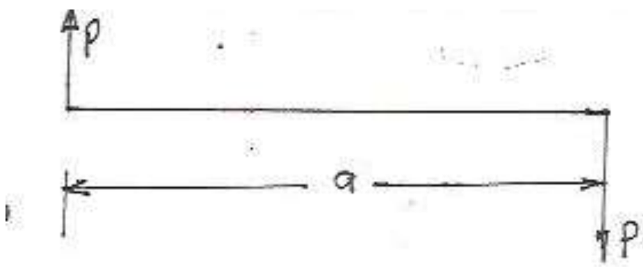
COUPLE

A pair of two equal and unlike parallel force (i.e. force equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a **couple.**

As a matter of fact, a couple is unable to produce any translatory motion (i.e. motion in a straight line). But it produce a motion of rotation in the body, on which it acts. The simple of a couple one of the forces applied the key of a lock, while locking or unlocking it.

Arm of a Couple.

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple as shown in the fig. below.



Moment of a Couple

The moment of a couple is the product of the force (i.e. one of the forces of two equal and opposite parallel forces) and the arm of the couple. Mathematically.

Moment of a couple = P x

P = Magnitude of the force, and

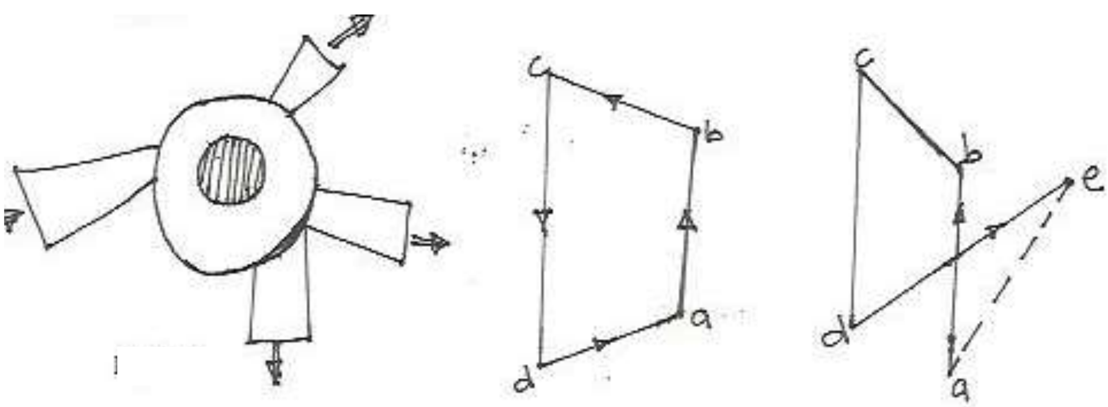
a = Arm of the couple.

<u>**Polygon of Forces**</u>.

Suppose the four forces 1, 2,3and 4 acting at the joint shown below to be in balance, they may then be represented by the four sides of the polygon abcd,

This is a close polygon since the force is in equilibrium. If the forces are not in balance the polygon will not close and require closing line gives the equilibrant or the equal and opposite force depending on the sense in which it is taken.

It is an extension of Triangle law of force for more than two forces, which states “if a number of forces acting simultaneously on a particle, be represented magnitude and



direction, by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude and direction, by the closing side of the polygon, taken in opposite order.”

Graphical (vector) Method for the Resultant Force.

It is another name for finding out the magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below.

1. Construction of space diagram showing the various forces (or loads) along with their magnitude and line of action.
2. Use of Bow’s notations. All the forces in the space diagram are named by using the Bow’s notation. It is a convenient method in which every forces (or load) is named by two capital letters, placed on its either side in the space diagram.
3. Construction of vector diagram (force diagram). It mean the construction of a diagram from a convenient point and then go on adding all the forces vectorically one by one (keeping in view the directions of the forces) to some suitable scale.

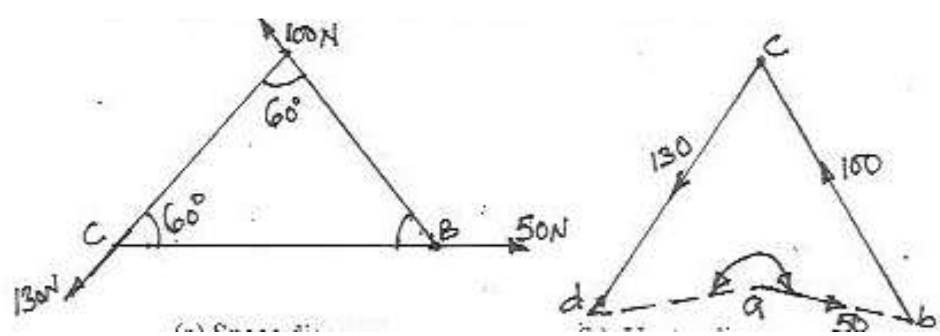
Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

Example: 1. A particle is acted upon by three forces equal to 50N, 100N, and 130N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant forces.

Solution.

The system of given forces is shown below

First of all, name the forces according to Bow’s notations as shown below. The 50N force is named as AB, 100N forces as BC and 130N force as CD, such that DA gives the resultant of these three forces.



(a) Space diagram (b) Vector diagram

Now draw the vector diagram for the given system of forces as shown in fig.2.12 (b) above and as discussed below,

Select some suitable point a and draw ab equal to 50N to some suitable scale and parallel to the 50N force of the space diagram.

2. Through b, draw bc equal to 100N to the scale and parallel to the 100N force of the space diagram.

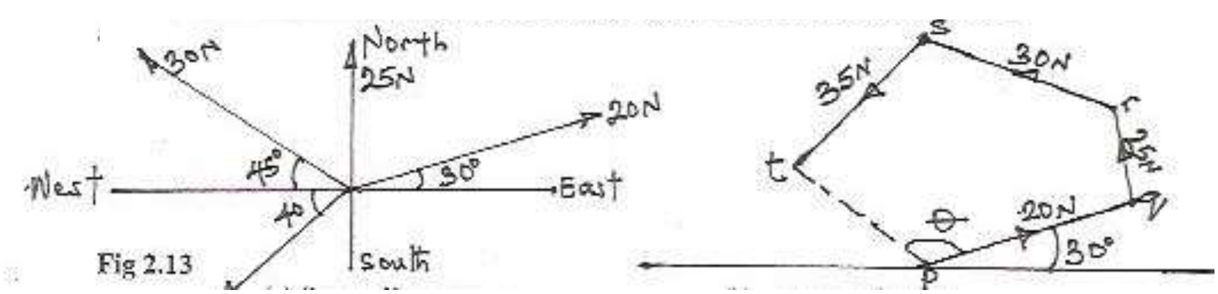
3. Similarly through c draw cd equal to 130N to the scale and parallel to the 130N force of the space diagram.

4. Join ad, which gives the magnitude as well as direction of the resultant force.

5. By measurement, we find the magnitude of the resultant force is equal to 70N and acting at an angle of 200° with ab.

Example.2 The following force act at a point

I 20N inclined at 30° toward North of East.
II 25N towards North West and
III 30N inclined at 40° toward South of West.
Find the magnitude and direction of the resultant force.
<u>Solution.</u>
The system of given is shown below. First of all, name the forces according to Bow's notations below. The 20N force is named as PQ 25N force as QR, 30N force as RS and 35N force as ST



(a) Space diagram (b) Vector diagram

Now draw the vector diagram for the given system of forces as shown in (b) and as discussed below.

1.Select some suitable point P and draw Pq equal to 20N to some suitable scale and parallel to the force PQ.

2.Through q, draw qr equal to 25N to the scale and parallel to the force QR of the space diagram.
3.Now through r draw rs equal to 30N to the scale and parallel to the forces RS of the space diagram.
4.Similarly, through S, draw st equal to 35N to the scale and parallel to the force ST of the space diagram.
5.Joint pt, which gives the magnitude as well as direction of the resultant force.
6.By measurement, we find that the magnitude of the resultant force is equal to 45.6 and acting at an angle of 132° with the horizontal i.e. East – West line.

MOMENT OF A FORCE

It is the turning effect produced by a force, on the body an the which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force. Mathematically, moment M = P x l

where,

P = Force acting on the body, and

l = perpendicular distance between the point, about which the moment is required and the line of action of the force.

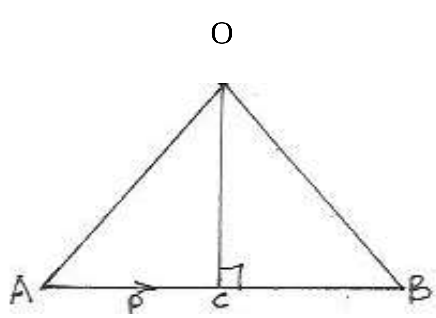
Graphical Representation of Moment

Consider a force represented in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out as shown in fig. below. From O, draw OC perpendicular to AB. Join OA and OB.

Now moment of the force P about O

= P x OC = AB x OC

Put AB x OC is equal to twice the area of the triangle AOB



Units of moment

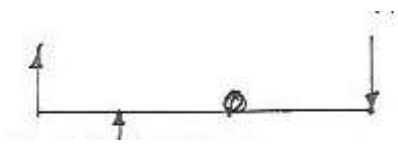
Since the moment of a force is the product of force and distance, therefore the unit of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in metres, then the units of moment will

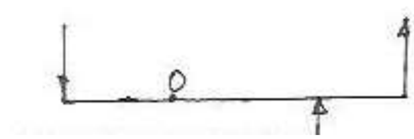
be and Newton-metre (briefly written as N.M). Similarly the unit of moment may be KN-M (i.e. KN XM), N –mm (i.e. NXmm) etc.

Types of Moments

Broadly speaking the moment are of the following two types.

1. Clockwise moments
2. Anticlockwise moments.





(a) Clockwise moments (b) Anticlockwise moments

Clockwise Moment.

It is the moment of a force, whose effect is to turn or rotate the body, in the opposite direction in which the hands of a clock move as shown above.

Anticlockwise Moments

It is the moment of a force, whose effect is to turn or rotate the body in the opposite direction in which the hands of a clock move as shown in (b) above.

Note: The general convention is to take clockwise moment as positive and anticlockwise moment as-ve

Centre of Gravity (C.G)

This is the point where the whole weight of a body appears to be acting irrespective of its position.

Centroid

The plane figures (like triangle, quadrilateral, circle, etc) have only area, but no mass. The centre of area of such figures is known as centroid. The method of finding out the centroid is the same as that of finding out the centre of gravity of a body. In many books, the authors also write C.G. for centroid and vice versa.

Centre of Gravity of Plane Figures

The plane geometrical figure (such as T-section, I-section, L-section, etc) have only areas but no mass. The C.G of such figures is found out in the same way as that of solid bodies. The centre of area of such figure is known as centroid, and coincides with the C.G of the figure. It is a common practice to use C.G for centroid and vice versa.

Let x and y be the co-ordinates of the C.G with respect to same axis of reference,

then $$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots\ldots\ldots}{a_1 + a_2 + a_3 + \cdots\ldots\ldots}$$

Similarly $$\bar{y} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots\ldots\ldots}{a_1 + a_2 + a_3 + \cdots\ldots\ldots}$$

Where a_1, a_2, a_3 …………etc is the area into which the whole figure is divided.

x_1, x_2, x_3 …………etc are the respective co-ordinates of the area

On x-x axis with respect to same axis of reference.

y_1, y_2, y_3 ……………etc are the respective coordinates of the area

On Y-Y axis with respect to same axis of the reference.

Centre of Gravity of Symmetrical Sections.

Sometime, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the C.G. of the body is very much simplified, as we have only to calculate either X or Y. This is due to the reason that the C.G of the body will lie on the axis of symmetry.

Example 1.

Find the center of gravity of a 100mm x 150mm x 30mm T-section.

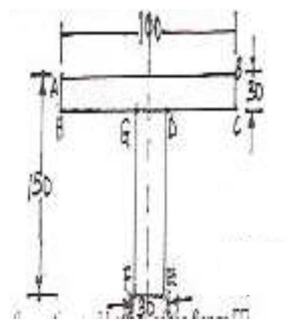
Solution

As the section is symmetrical about Y-Y axis, bisecting the web, therefore its C.G will lie on this axis. Split up the section into two rectangles ABCH and DEFG as shown below. Let bottom of the flange FE be the axis of reference.

I. Area ABCH

$a = 100 \times 30 = 3000mm^2$

$y = (150 - \frac{30}{2}) = 135mm$



2. Area DEFG

$a = 120 \times 30 = 3600mm^2$

$y = \frac{120}{2} = 60mm$

We known the distance between C.G of the section and bottom of the flange FE,

$$y = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 + 135) + (3600 \times 600)}{3000 + 3}mm$$

$= 94.1mm.$

Example 2.

Find the center of gravity of a channel section 100 x 50 x 15mm

Solution

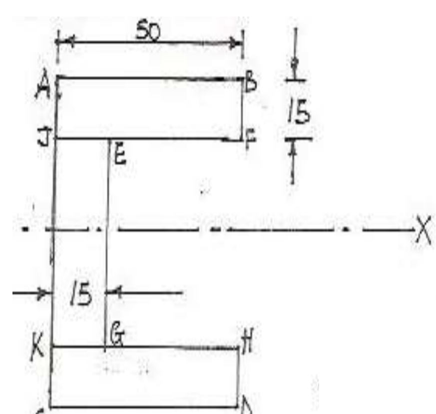
As the section is symmetrical about X-X axis, therefore its C.G will lie on this axis. Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown. Fig 2.19 let the face AC be the axis of reference.

I. Area ABFJ

$a = 50 \times 15 = 750mm^2$

$x = \underline{50} = 25mm$

II. Area EGKJ



$a = (100 - 30) \times 15mm^2$

$= 1050m$ I

III. Area CDHK

$a = 50 \times 15 = 750mm^2$

$x = \frac{50}{2} = 25mm$

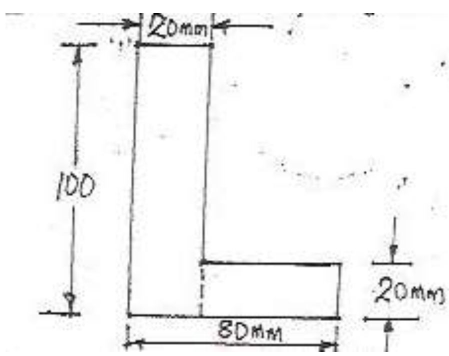
$$X = \frac{a\,x + a\,x + a\,x}{a + a + a}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750}$$

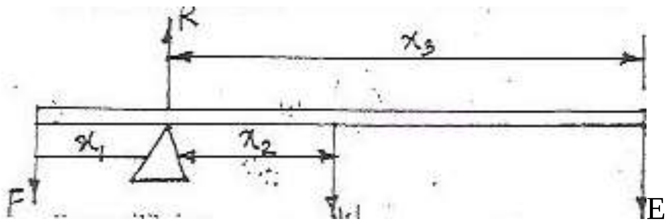
$= 17.8mm$

Practice Questions

Find the centroid of an unequal angle section 100mm x 80mm x 20mm



Parallel Force acting on a Single Pivot beam



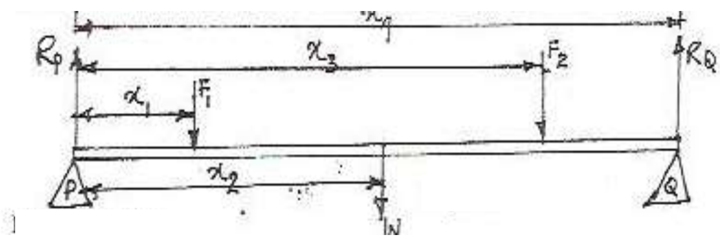
For equilibrium

a. $E + W + F = R$

b. Taking moment about the point

$FX_1, = Wx_2 + Ex_3$

Parallel forces acting on a two – pivot beam



For equilibrium

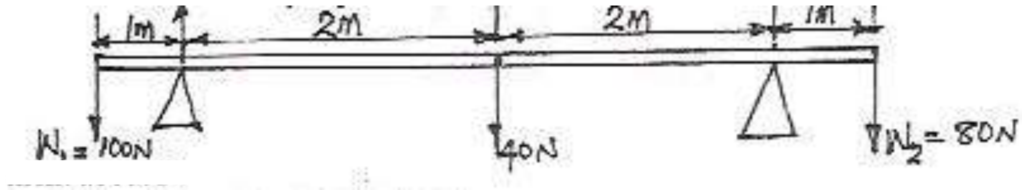
Taking moment about pivot P,

$W + F_1 + F_2 = Rp + R_Q$

Taking moment about pivot Q

$F_1 x_1 + Wx_2 + F_2 x_3 = R_Q x_4$

Ex.3 A uniform beam 6cm long and weighing 40N rest on two supports P and Q placed 1m from each end of the beam. Weight of 100N and 80N are hung from the end of the beam near P and Q respectively. Calculate the reactions at the supports.



Taking moment about P

(100 x 1) Nm + (R x 4m) = (80 x 5) + (40 x 2)

100Nm + 4R = (400 + 80)Nm

(4R)m = (480 – 100)Nm

$R = \frac{380}{4} = 95N$

Similarly, take moment about Q

Total anticlockwise moment = Total clockwise moment is

(100 x 5) + (40 x 2) = (R x 4) + (80 x 1)

80 + 4R = (500 + 80)Nm

4R = 580 – 80

$R_1 = \frac{500}{4}$

OR

$R_1 + R_2 = W1 + W2 + W3$

$R_1 + R_2 = 220N$

$R_1 = (220 - 95)$

= 125N.

SIMPLE FRAMES AND STRUCTURES.

Introduction

A frame may be defined as a structure, made up of several bars, riveted or welded together. These are made up of angle irons or channel sections, and are called members of the frame or framed structure. Though member are yet for calculation purposes, the joints welded or riveted together, at their joints are yet assumed to be hinged or pin-jointed. The determination of forces in a frame is an important problem in Engineering-science, which can be solved by the application of the principles of either stat ice or graphics. In this chapter, we shall be using the principles of either static or graphics.

Types of frames

Though there are many types of frames, yet from the analysis point of view, the frames may be classified into the following to group:

1. Perfect frame. 2. Imperfect frame.

Perfect Frame

A perfect frame is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape.

The simplest perfect frame is a triangle, which contains three members and three joints as shown below. It will be noted that if such a structure is loaded shape will not be should be three members to prevent distortion. Increase a joint, to a triangular fame, we require two members as shown by dotted lines . Thus we see that for every additional joint, to a triangular frame, two members are required,

The number of members, in a perfect frame, may also be expressed by the relation:

$$n = (2j - 3)$$

where n = No of members and

j = No of joints

Imperfect Frame

An imperfect frame is that which does not satisfy the equation:

$$n = (2j - 3)$$

Or in other words, it is a frame in which the numbers of members are more or less than (2j-3). The imperfect frames may be further classified into the following two types:

1. Deficient frame. 2. Redundant frame.

Deficient Frame

A deficient frame is an imperfect frame, in which the numbers of members are less than (2j-3).

Redundant Frame: A redundant is an imperfect frame, in which the number of member are more than (2j – 3).

Assumptions for Forces in the Members of a Perfect Frame

Following assumptions are made, while finding out the forces in the members of a perfect frame:

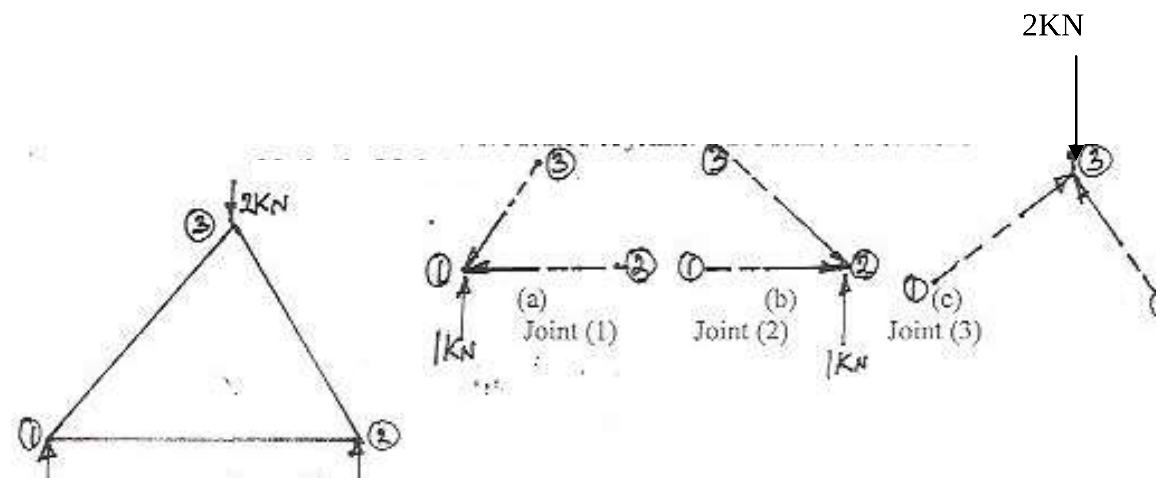
1.All the members are pin jointed,
2.The frame is loaded only at the joints.
3.The frame is a perfect one
4.The weight of the members, unless stated otherwise, is regarded as negligible in comparison with the other external forces or loads acting on the truss,

The forces in the various members of a perfect frame may be found out either by analytical method or graphical method, but in this chapter, we shall discuss the analytical method only.

Analytical Methods for the Forces

The following two analytical methods for finding out the forces, in the members of a perfect frame, are important from the subject point view:

1. Method of joint. 2. Method of section.



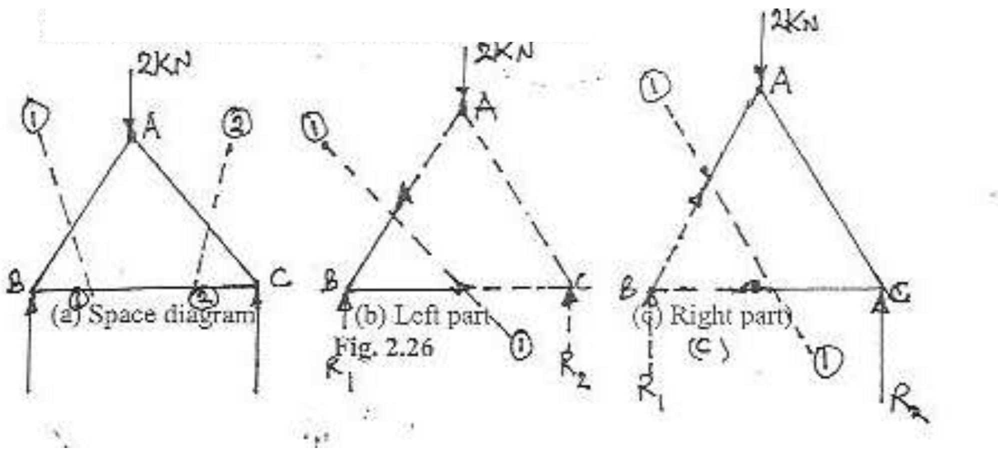
Space Diagram

In this method, each and every joint is treated separately as a free body in equilibrium as shown in fig. 2.25 (a), (b), (c) and (d). The unknown forces are then determine by equilibrium equation viz, $\sum V = 0$ and $\sum H = 0$. i.e. sum of all the vertical force as well as the horizontal forces is equated to zero.

Note. 1. The member of the frame may be named either by Bow's method or by the joint at their ends.

2. While selecting the joint, for calculation work, care should be taken that at any instant, the joint should not contain more than two members, in the forces are known.

Method of selections (or method of Moment)



(a) Space diagram (b) left part (c) Right part

This method is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method, a section line is passed through the member or members, in which the forces are required to be found out as shown in (a) above.

A part of the structure, on any one side of the section line, is then treated as a free body in equilibrium under the action of external. Force as shown in (b) and (c). The unknown forces are then found out by the application of equilibrium or the principle of static's i.e. $\sum M = 0$

Notes. 1. To start with, we have shown section line 1-1 cutting the member AB and BC. Now in order to find out the force in the member AC, section line 2-2 may be drawn

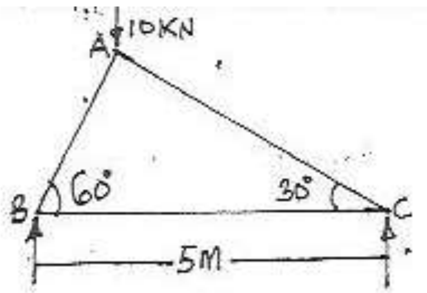
2. While drawing a section line, care should always be taken not to cut more than three members, in which the forces are unknown.

FORCE TABLE

Finally, the results are tabulated showing the members, the magnitudes of force and their nature. Sometimes, tensile force is represented with a +ve sign and compressive force with a –ve sign.

Note. The force table is generally prepared when force in all the members of a truss are required to be found out.

Example 1. The truss ABC shown in the fig. below has a span of 5 meters. It is carrying a load of 10kN at its apex.



Find the force in the member AB, AC and BC.

Solution:

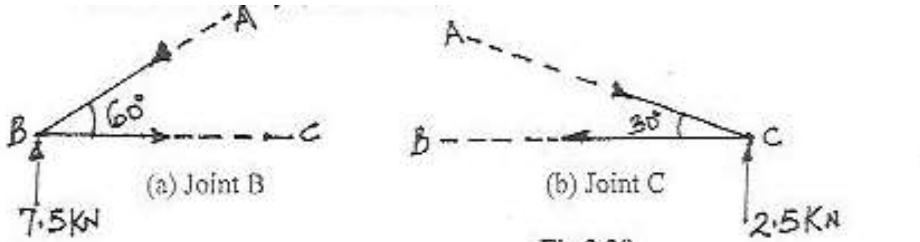
From the geometry of the truss, we find that the load of 10kN is acting at a distance 1.5m from the left hand support i.e., B and 3.75m from C. Taking moment about B and equating the same.

$$R_2 \times 5 = 10 \times 1.25 = 12.5$$

$$R_2 = 12.5/5 = 2.5kN$$

and $$R_1 = 10 - 2.5 = 7.5KN$$

The example may be solved by the method of joint or the method of section. But we shall solve it by the methods.



(a) Joint B

(b) Joint C

Fig 2.28

First of all consider joint B. Let the direction of the force PAB and PBC (or PBA and PCB) be assumed as shown in fig. 2.28 (a). Resolving the force vertically and equating the same,

$$PAB \sin 60° = 7.5$$

$$PAB = \frac{7.5}{\sin 60°} = \frac{7.5}{0.866} \text{ KN (compression)}$$

And now resolving the force horizontally and equating the same,

$$PBC = PAB\cos 60° = 8.660 \times 0.5$$

$$= 4.33 \text{KN}$$

$$PAC \sin 30° = 2.5$$

$$PAC = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5}$$

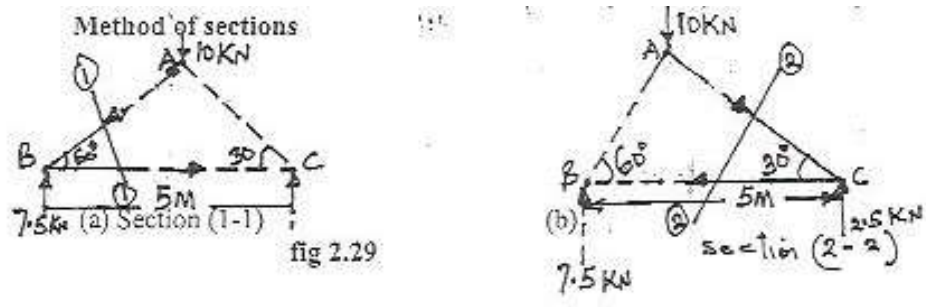
$$= 5.0 \text{KN (compression)}$$

and now resolving the force horizontal and equating the same,

$$PBC = PAC \sin 30° = 5.0 \times 0.866 \text{kN}$$

= 4.33KN (Tension)…. as already obtained.

Method of sections



First of all, pass section (1-1) cutting the truss into two parts (one part shown by firm lines and the other by dotted lines) through the member AB and BC of the truss as shown in fig. 2.29 (a). Now consider equilibrium of the left part

of the truss (because it is smaller than the right part). Let the direction of the forces PAB and PBC be assumed as shown in fig. (a)

Taking moment of the force acting in the part of the truss only about the joint C and equating the same,

$$P_{AB} \times 5\sin60° = 7.5 \times 5$$

$$P_{AB} = \frac{7.5 \times 5}{5\sin60°} = \frac{7.5}{0.866}$$

$$= 8.66KN \text{ (compression).}$$

and now taking moment of the forces acting in the left part of the truss only about the joint A and equating the same,

$$P_{BC} \times 1.25\tan60° = 7.5 \times 1.25$$

$$P_{BC} = \frac{7.5 \times 1.25}{1.25\tan60°} = \frac{7.5}{1.732}$$

$$= 4.33KN \text{ (Tension)}$$

Now pass section (2-2) cutting the truss into two part through the member AC and BC. Now consider the equilibrium of the right part of the truss (because it smaller left part). Left the directly of the forces PAB and PBC assumed as show in fig. (b).

Taking moment of the forces acting in the part of the truss only about the joint B and equating the same,

$$PAC \times 5\sin30^\circ = 2.5 \times 5$$

$$PAC = \frac{2.5 \times 5}{5\sin30^\circ} = \frac{25}{0.5} = 5KN \text{ (compression)}$$

and now taking moment of the forces acting in the right part of the truss only about the joint A and equating the same,

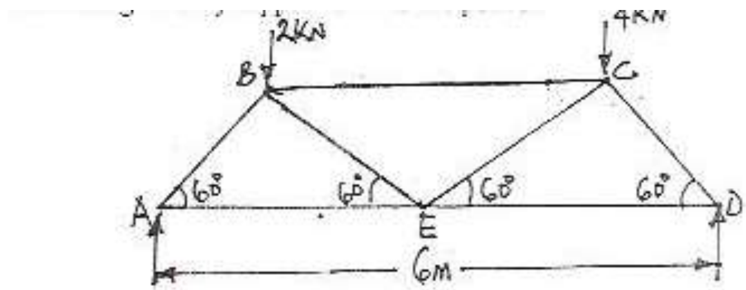
$$PBC \times 3.75\tan30^\circ = 2.5 \times 3.75$$

$$PBC = \frac{2.5\ 3.75}{3.75\tan30^\circ} = 4.33KN \text{ (Tension) as already mentioned)}$$

Now, tabulate the results as given below:

S/N	Member	Magnitude of force in kN	Nature of force
1	AB	8.66	Compression
2	BC	4.33	Tension
3	AC	5.0	Compression

Example 2. The fig. below shows a warren girder consisting of seven members each of 3m length freely supported at its end points.



The girder is loaded at B and C as shown. Find the force in all the member of the girder, indicating whether the force is compressive or tensile.

Solution. Taking moment about A and equating the same,

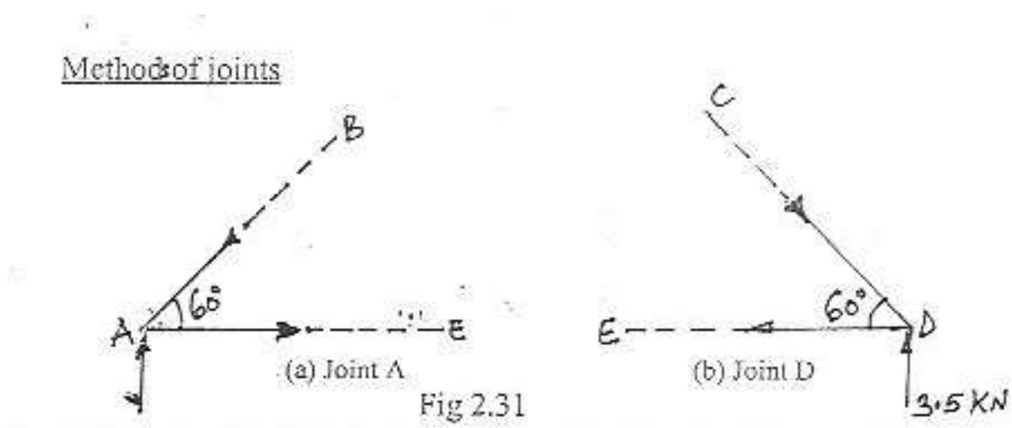
$$R_D \times 6 = (2 \times 1.5) + (4 \times 4.5) = 21$$

$$R_D = 21/6 = 3.5KN$$

and $$R_A = (2 + 4) - 3.5 = 2.5KN$$

The example may be solved by the method of joint or method of section but we shall it by both the methods.

Method of joints



(a) Joint A (b) Joint D

First of all, consider the joint A. Let the direction of PAB and PAE be assumed as shown in fig 2.31. Resolving the force vertically and equating the same,

$$PAB \times \sin 60^\circ = 2.5$$

$$PAB = \frac{2.5}{\sin 60^\circ} = \frac{2.5}{0.866}$$

$$= 2.887 KN$$

and now resolving the force horizontally and equating the same,

$$PAE = PAB \cos 60^\circ \; 2.887 \times 0.5 = 1.444 KN \text{ (Tension)}$$

Now consider the joint D. Let the direction of the forces PCD and PED be assumed as shown in fig 2.31 (b).

Resolving the forces vertically and equating the same,

$$PCD \times \sin 60^\circ = 3.5$$

$$PCD = \frac{3.5}{\sin 60^\circ} = \frac{3.5}{0.866}$$

$$= 4.042 KN \text{ (compression)}$$

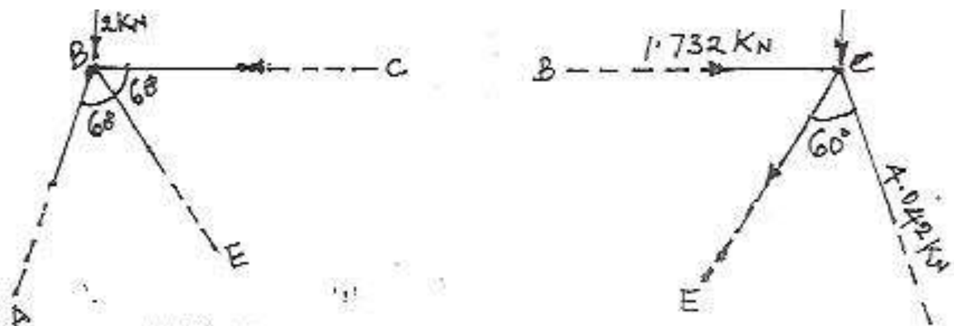
and now resolving the force vertically and equating the same,

$$PDE = PCD \cos 60^\circ = 4.042 \times 0.5$$

$$= 2.021 KN \text{ (Tension)}$$

Now consider the joint B. We have already found that force in member AB i.e. PAB is 2.887kN (Compression). Let the direction of the force PBC and PBE assumed as shown in fig. 2.32 (a)

4kN



(a) Joint B (b) Joint C

Resolving the force vertically and equating the same,

$$P_{BE}\sin 60° = P_{AB}\sin 60° - 2.0$$

$$= 2.887 \times 0.866 - 2.0 = 0.5KN$$

$$\therefore P_{BE} = \frac{0.5}{\sin 60°} = \frac{0.5}{0.866} = 0.577KN \text{ (Tension).}$$

and now resolving the force horizontally and equating the same,

$$P_{BC} = 2.887\cos 60° + 0.5777\cos 60° \text{ KN}$$

$$= (2.887 \times 0.5) + (0.5777 \times 0.5) \text{ KN}$$

$$= 1.732KN \text{ (compression}$$

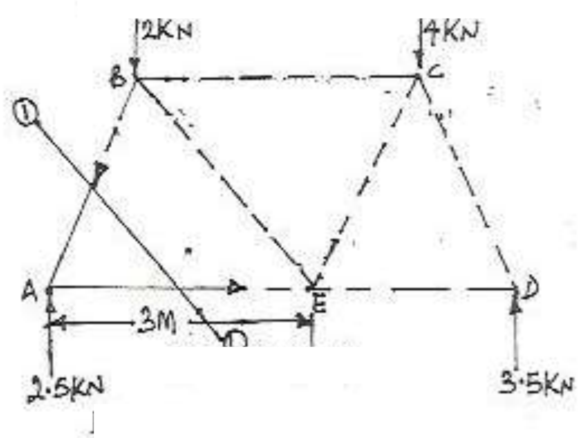
Now consider joint C. We have already found out that the forces in the member BC and CD (i.e. PBC and PCD) are 1.732kN (Compression) and 4.042kN (Compression) respectively. Let the direction of PCE be assumed as shown in Fig.2.32.(b). Resolving the force vertically and equating the same,

$$P_{CE}\sin 60° = 4 - P_{CD}\sin 60° = 4 - (4.042 \times 0.866)$$

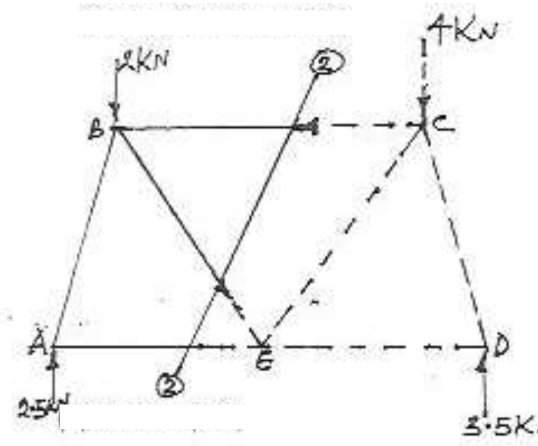
$$P_{CE} = \frac{0.5}{\sin 60°} = \frac{0.5}{0.866} = 0.577KN \text{ (compression)}$$

Method of sections

First of all, pass section (1-1) cutting the truss through the member AB and AE. Now consider equilibrium of the left part of the truss. Let the directions of the forces PAB and PAE be assumed as shown below.



(a) Section (1-1)



(b) Section (2-2)

Taking moment of the forces acting in the left the truss only, about the joint E and equating the same,

PAE x 3 sin60° = 2.5 x 3

PAB = 2.5/sin60° = 2.5/0.866 = 2.887KN (Tension)

Now pass section (2-2) cutting the truss through the member BC,BE and AE. Now consider equilibrium of the left of the truss. Let the directions of the forces and PBE be assumed as shown in Fig 2.33 (b). Taking moment of the forces acting in left part of the truss only, about the joint E and equating the same,

PBC x 3 Sin60° = (2.5 x 3) – (2 x 1.5) = 4.5

PBC = 4.5/3 Sin60° = 4.5/3 x0.866 = 1.732KN (compression)

and now taking moment of the forces acting in the forces acting in the part of the truss only, about the joint A equating the same,

PBE x 3 Sin60° = PBC x 3Sin60° - (2 x 1.5)

= (1.732 x 3 x 0.8660) – 3.0 = 1.5

PBE = 1.5/3Sin60° = 1.5/30.866   = 0.577KN (Tension)

Now pass section (3-3) cutting the truss through the member BC, CE and ED. Now consider the equilibrium of the right part of the truss. Let the direction of the forces and be assumed as shown in Fig. 17.12(a). Taking moment of the forces in the right part of the truss only, about the joint D equating the same,

PCE x 3 Sin60° = (4 x 1.5) – (PBC x 3Sin60°)

= 6.0 – (1.732 x 3 x 0.866) = 1.5

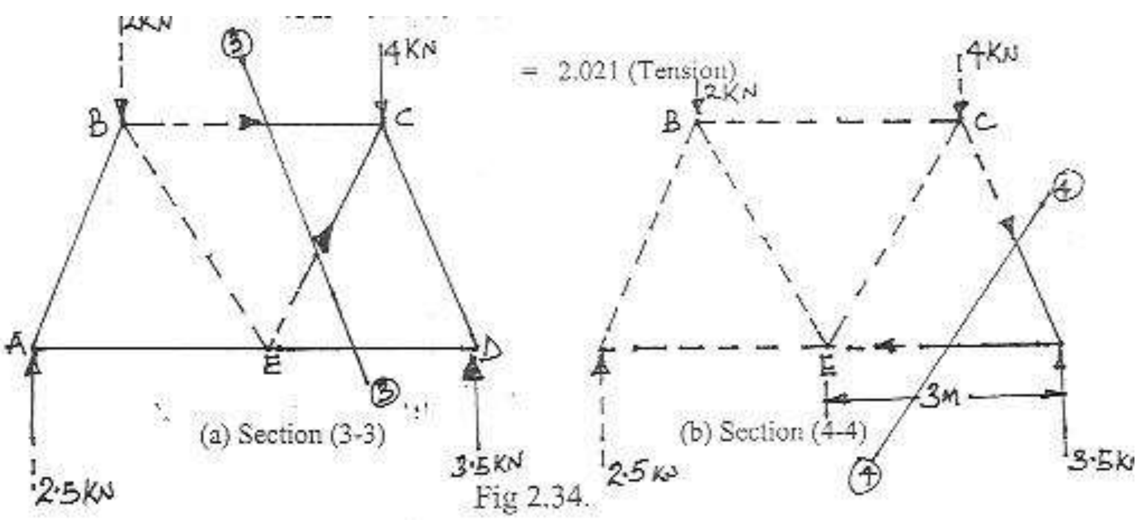
PCE = 1.5/3Sin60° = 1.5/3 x 0.866 = 0.577KN (compression)

and now taking of the forces in the right part the truss only about the joint C and equating the same,

PDE x 3Sin60° = 3.5 x 1.5 = 5.25

PED = 5.25/3Sin60° = 5.25/3 x 0.866

= 2.021 (Tension)



(a) Section (3-3)   (b) Section (4-)

Now pass section (4-4) cutting the truss through the member CD and DE. Let the directions of the forces PCD be assumed as shown in Fig. 2.34 (b). Taking moments of the forces acting in the right part of the truss only about the joint E and equating the same,

$$PCD \times 3Sin60° = 3.5 \times 3$$

$$\therefore PCD = 3.5/sin60° = 3.5/0.866$$

$$= 4.402KN \text{ (compression)}$$

Now tabulate the result as given below:

S.No.	Member	Magnitude of force	Nature of force
1	AB	2.887	Compression
2	AE	1.444	Tension
3	CD	4.042	Compression
4	DE	2.021	Tension
5	BE	0.577	Tension
6	BC	1.732	Compression
7	CE	0.577	Compression

Kinematics

Velocity: This is defined as the rate of change of displacement linear and angular of a body with respect to the time. Velocity is a vector quantity, to specify it completely the magnitude, direction and sense must be known.

Acceleration: The acceleration of a body is the rate of change of its velocity linear or angular with respect to time. A body accelerates if there is a change in either the magnitude, direction or sense of its velocity and can thus accelerates without change in speed, as in the case of a body moving in a circular path with uniform speed.

Displacement: Displacement is defined as the distance moved by a body with respect to a certain fixed point. The displacement may be along a straight or a curved path.

Equations of Uniformly accelerated Motion.

Let a body having linear motion accelerates uniformly from an initial velocity **u** to a final velocity **v** in time t; let the acceleration be **a** and the distance from the initial position be **s.**

Then

$V = u + at$

$S = ut + \frac{1}{2}at^2$, $s = \frac{u+u}{2} \times t = v_{av} \times t$

$V^2 = u^2 + 2as$

The corresponding equations for angular motion are:

$\omega_2 = \omega_1 + \alpha t$

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $s = \frac{\omega_0 + \omega}{2} \times t$

$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$

Where ω_2 *and* ω_1 are the initial and final angular velocities respectively, θ is the angle turned through in time t and $\alpha$is the angular acceleration.

Note: If a body is rotating at the rate of N rpm (revolution per minute), then its angular velocity,

$\omega = \frac{2\pi N}{60}$ rad/ s

Non-Uniform acceleration

If the non acceleration is a function of time , distance of velocity, it must be expressed in the form

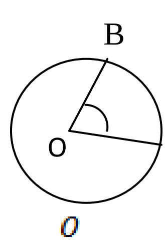
$a = \frac{dv}{dt} = \frac{d}{dt}\frac{ds}{dt} = \frac{d^2s}{dt^2}$ ………….. *Note:* $v = \frac{ds}{dt}$

OR

$a = \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds} = v \times \frac{dv}{ds}$

Relationship between Linear and Angular Quantities of Motion.

Consider a body moving along a circular path from A to B as shown below,



Let r = Radius of the circular path

θ = Angular displacement in radians

S= Linear displacement

V = Linear velocity

ω = Angular velocity

a = Linear acceleration, and

α = Angular acceleration

Find the geometry of the figure, we know that

$$s = r.\theta \;\; or \;\; \theta = s/r$$

` Also, from linear velocity

$$V = \frac{ds}{dt} = \frac{d(r.\theta)}{dt} = r.\frac{d\theta}{dt} = r.\omega$$

and linear acceleration

$$a = \frac{dv}{dt} = \frac{d(r.\omega)}{dt} = r.\frac{d\omega}{dt} = r.\alpha$$

Note:

In the case of a wheel or cylinder which rolls without slip on a flat surface

Example 1

There motion of a particle is given by a= t^3-3t^2+5. Where a is the acceleration in m/s^2 and t is the time in seconds. The velocity of the particle at t=1 second is 6.25m/s and the displacement is 8.30metres. calculate the displacement and the velocity at t=2 seconds.

Given: a = t^3 - 3t^2 +5

From a = $\frac{dv}{dt}$,

$$\frac{dv}{dt} = t^3 - 3t^2 + 5$$

or $dv = \left(t^3 - 3t^2 + 5\right)dt$

Integrating both sides

$$V = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + c_1$$

$$= \frac{t^4}{4} - t^3 + 5t + c_1 \text{ ....}(i)$$

where c_1 is the first constant of integration, from the question, when t=1s, v=6.25m/s, therefore substituting these values of t and v in equation (i).

$$6.25 = 0.25 - 1.5 + c_1$$

$$= 4.25 + c_1$$

$$or\ c_1 = 6.25 - 4.25$$

$$= 2$$

Now, substituting the value of c_1 in equation (i)

$$V = \frac{t^4}{4} - t^3 + 5t + 2 \text{ ....}(ii)$$

When velocity at t = 2seconds

$$V = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2$$

$$= 8m/s$$

When displacement at t=seconds

From, V=$\frac{ds}{dt}$, therefore equation (ii) may be written as

$$\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

OR $$ds = \left(\frac{t^4}{4} - t^3 + 5t + 2\right)dt$$

Integrating both sides

$$S = \frac{t^4}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + c_2 \text{ ....}(iii)$$

Where ,

c_2 is the second constant of integration

When t=1s, s=8.30m, substituting these values in equation (iii),

$$8.30 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + c_2$$

$$= 4.3 + c_2$$

$$c_2 = 8.3 - 4.3$$

$$= 4$$

Substituting the values of c_2 in equation (iii)

$$s = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 4.$$

Substituting the value of t = 2 seconds in this equation, we have

$$S = \frac{2^5}{20} - \frac{2^4}{4} + \frac{5 \times 2^2}{2} + 2 \times 2 + 4$$

$= 15.6$m.

Example 2.

A wheel accelerates uniformly from rest to 200rpm in 20 seconds. What is its angular acceleration? How many revolutions does the wheel make in attaining the speed of 2000 rpm.

Solution

Given: $N_o = 0$, or $\omega = 0$; N = 2000 r.p.m or $\omega = 2\prod \times \frac{2000}{60} = 209.5 rad/s$,

t = 20 seconds

Let $\alpha =$ Angular acceleration in rad/s^2

From $\omega = \omega_o + \alpha t$ or $209.5 = 0 + \alpha \times 20$

$$\alpha = \frac{209.5}{20}$$

$$= 10.475 rad/s^2$$

$$\theta = \frac{(\omega_o + \omega)t}{2} = \frac{(0 + 209.5)}{2}$$

Since the angular distance moved by the wheel during one revolution is 2π radians, therefore the number of revolutions made by the wheel,

$$n = \theta/2\pi = \frac{2095}{2\pi} = 333.4$$

Practice Questions

(1) A horizontal bar 1.5metres long and of small cross-section rotates about vertical axis through one end. It accelerates uniformly from 1200r.p.m to 1500 r.p.m in an interval of 5 seconds. What is the linear velocity at the beginning and end of the interval? What are the normal and tangential components of the acceleration of the mid-point of the bar 5 seconds after acceleration begins?

(2) The displacement of a point is given by $S = 2t^3 + t^2 + 6$, where S is in metres and t in seconds. Determine the replacement of the point when the velocity changes from 8.4m/s to 18m/s. Find also the acceleration at the instant when the velocity of the particle is 30m/s.

Mass, Force, Weight and Momentum

- **Mass**: It is the amount of matter contained in a given body, and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.
- **Weight:** It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with distance of the body from the centre of the earth, therefore the weight of the body will vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a force.
- **Force:** It is an important factor in the field of Engineering science, which may be defined as an agent, which produces or tend to produce, destroy or tend to destroy motion.
- **Momentum:** The momentum of a body is the product of its mass and velocity. Mathematically.

Momentum = mass x velocity

Let M = mass of the body

u = Initial velocity of the body

v = final velocity of the body

a = constant acceleration, and

t = Time required (in seconds) to change the velocity from u to v.

now, initial momentum = M.u

and final momentum = m.v

$\therefore$ change of momentum= m.v – m.u

and rate of change of Momentum = $\frac{m.v - m.u}{t} = \frac{m(v-u)}{t} = m.a$

Newton's Law of Motion

1. Everybody continues in its state of rest or of uniform motion in a straight line, unless acted upon by some external force.
2. The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the forces acts.
3. To every action, there is always an equal and opposite reaction.

From the second law,

Force α rate of change of momentum

α mass x rate of change of velocity

i.e F = kma

where k is a constant

The unit of the quantities are chose so as to make the value of k unity.

i.e. F = ma.

Which is the force required to give a mass of 1kg an acceleration of $1m/s^2$

- **Impulse:** The impulse of a constant force (f) acting for a time t is the product ft. if during this time, the velocity change from u to v, then,

$$f = ma = \frac{m(v-u)}{t}$$

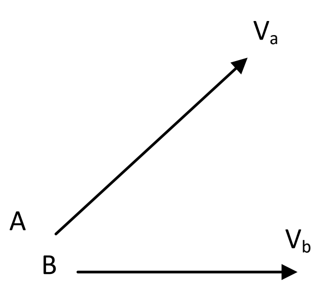
or

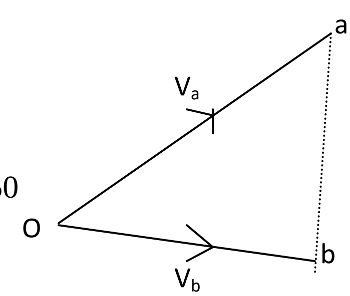
$$ft = m(v-u)$$

$\therefore$ impulse of force = change of momentum

Relative Velocity

If two bodies A and B are moving with velocities V_a and V_b respectively, fig 1.2 then the relative velocity of one to the other is the vector difference of v_a and v_b,



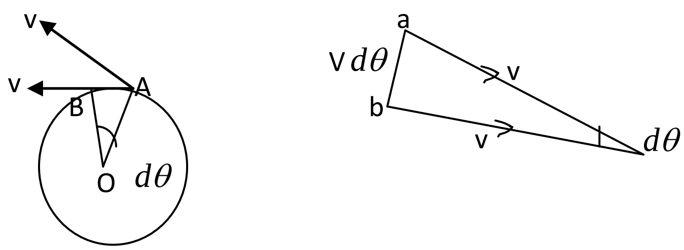


i.e. if vectors Oa and Ob, representing Va and Vb in magnitude, direction and sense, are drawn from the same point O, then ab represents the velocity of B relative to A and ba the velocity of A relative to B.

If Oa and Ob represent the velocities of the same body at different times, then ab represents the change in velocity.

Centripetal acceleration and centrifugal force

Consider a body of mass in moving in a circular path of radius r with constant speed V, fig. 1.3. If it moves from A to B in time dt and the angle AOB is dθ



then, from the relative velocity diagram, the change of velocity is represented by ab

Thus, change of velocity $= Vd\theta$

$$\therefore acceleration = V\frac{d\theta}{dt}$$

i.e. $a = v\omega$

where ω is the angular velocity of OA

But $V = \omega r$ ....recall

$$\therefore \ a = w^2 r \ \ or \ v^2/r$$

This acceleration is directed towards the centre of rotation, o, and is called **Centripetal acceleration**. The radially inward, or centripetal force required to produce this acceleration is given by

$$F = ma = mw^2 r$$

Or $= mv^2/r$

If a body rotates at the end of an arm, this force is provided by the tension in the arm. The reaction to this force acts at the centre of rotation and is called the centripetal force.

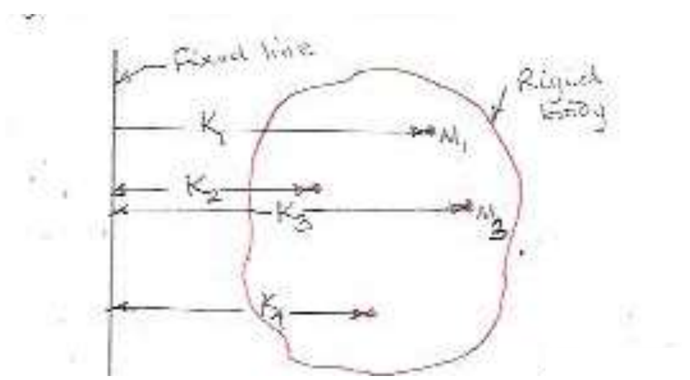
A common concept of centrifugal force in engineering problems is to regard it as the radially outward force which must be applied to a body to convert the dynamical condition to the equivalent static condition; this is known as d'Alembert's principle.

Mass Moment of Inertia

If the mass of every particle of a body is multiplied by the square of its perpendicular distance from a fixed line, then the sum of these quantities (for the whole body) is known as mass moment of inertia of the body. It is denoted by I.

Consider a body of total mass compose of small particles of masses m_1, m_2, m_3, m_4 etc. If k_1, k_2, k_3, k_4 are the distances of these masses from a fixed line, as shown in fig 1.4, then the mass moment of Inertia of the whole body is given by

$$I = m_1(k1)^2 + m_2(k2)^2 + m_3(k_3)^3 + m_4(k_4)^2 + ...$$



If the total mass of body may be assumed to concentrate at one point (known as centre of mass or centre of gravity), at a distance k from the given axis, such that

$$m.k^2 = m_1(k_1)^2 + m_2(k_2)^2 + m_3(k_3)^2 + m_4(k_4)^2 + ...$$

$$then \quad I = m.k^2$$

The distance K is called the **radius of Gyration.** This is defined as the distance from a given reference where the whole mass of body is assumed to be concentrated to give the same value of I.

Angular Momentum or Moment of Momentum

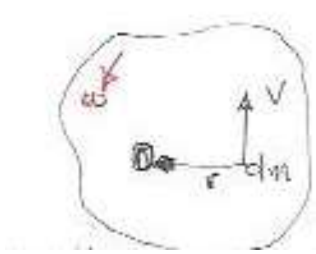
Consider a body of total mass m rotating with an angular velocity of ω rad/s, about the fixed axis O as shown below. Since the body is composed of numerous small particles, therefore let us take one of these small particles having a mass dm and at a distance r from the axis of rotation. Let v be its linear velocity acting tangentially at any instant.

Recall, momentum is the product of mass and velocity, therefore momentum of mass $dm = dm\,x\ v = dm\ x\ \omega \times r.$ ....$(\because V = \omega.r)$

and

moment of momentum of mass dm about O

$$= dm\,x\,\omega \times r \times r = dm\,x\,r^2 \times \omega$$



Where I_m = Mass moment of Inertia of mass dm about O=dmxr^2

$\therefore$ Moment of momentum or angular momentum of the whole body about 0

$$= \int I_m.\omega = I.\omega$$

Where $\int I_m$ = Mass moment of Inertia of the whole body about O

Thus, the angular momentum or the moment of momentum is the product of mass moment of inertia (I) and the angular velocity (ω) of the body.

Torque

It may be defined as the product of force and the perpendicular distance of its line of action from the given point or axis.

The Newton's second Law of motion, when applied to rotating bodies, state that the torque is directly proportional to the rate of angular momentum. Mathematically

$$Torque,\ T\ \alpha\ \frac{d(I.\omega)}{dt}$$

$$T = I\ \alpha\ \frac{d\omega}{dt} = I.\alpha\ldots\ldots\ldots\ldots\ldots\ldots\left(\therefore \frac{d\omega}{dt} = \alpha\right)$$

Example 1

The flywheel of a steam engine has a radius of gyration of 1m and mass 2500kg. The starting torque of the steam engine is 1500 Nm and may be assumed constant. Determine:

(i) angular acceleration of the flywheel

(ii) kinetic energy of the flywheel after 10 seconds from the start

Solution:

Given: K = 1m; m = 2500kg, T=1500Nm

Let α = Angular acceleration of the flywheel

From mass moment of inertia I = m.k^2

= 2500 x 1^2

= 2500kgm^2

We know that torque (T) = I. α

1500 = 2500 x α

Or

$$\alpha = \frac{1500}{2500} = 0.6 rad/s^2$$

(ii) We know that $\omega_2 = \omega_1 + \alpha .t$

= 0+0.6 x 10

= 6 rad/s

$\therefore$ Kinetic energy of the flywheel,

$$E = \frac{1}{2} I(\omega_2)^2$$

$$= \frac{1}{2} \times 2500 \times 6^2$$

$$= 4500J = 45kJ$$