



LANDMARK UNIVERSITY, OMU-ARAN

LECTURE NOTE 2

COLLEGE: COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT: MECHANICAL ENGINEERING

Course code: GEC 223

Course title: **FLUID MECHANICS I**

Credit unit: 2 UNITS.

Course status: *compulsory*

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GEC 223- INTRODUCTION TO FLUID MECH.

Boundary layer

- A consequence of this no-slip condition is the formation of velocity gradients and a boundary layer near a solid interface.

Flow in a pipe



Initial flat
Velocity profile

Fully developed
Velocity profile

- The existence of a boundary layer helps explain why dust and scale can build up on pipes, because of the low velocity region near the walls

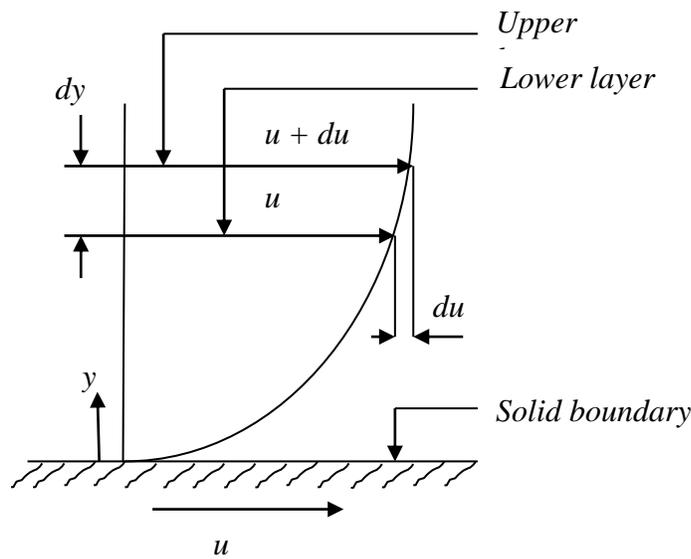
- The Boundary layer is a consequence of the stickiness of the fluid, so it is always a region where viscous effects dominate the flow.
- The thickness of the boundary layer depends on how strong the viscous effects are relative to the inertial effects working on the flow.
- Think of a fluid as being composed of layers like the individual sheets of paper. When one layer moves relative to another, there is a resisting force.
- This frictional resistance to a shear force and to flow is called viscosity. It is greater for oil, for example, than water.

Viscosity.

Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow. It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

“An ideal fluid has no viscosity”. There is no fluid which can be classified as a perfectly ideal fluid. However, the fluids with very little viscosity are sometimes considered as ideal fluid.

Viscosity of fluids is due to cohesion and interaction between particles. In the figure below, when two layers of fluid, at a distance ‘dy’ apart, move one over the other at different velocities, say ‘u’ and ‘u + du’, the velocity together with relative velocity causes a shear stress acting between the fluid layers. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to ‘y’. It is denoted by τ (called Tau).



Velocity variation near a solid boundary

Mathematically $\tau \propto \frac{du}{dy}$ or $\tau = \mu \cdot \frac{du}{dy}$, where, μ = constant of proportionality and is known as coefficient of dynamic velocity or only viscosity. $\frac{du}{dy}$ = Rate of shear stress or rate of shear deformation or velocity gradient. From the figure above, we have $\mu = \frac{\tau}{\left[\frac{du}{dy}\right]}$, Thus viscosity may also be defined as the shear stress required to produce unit rate of shear strain.

Units of viscosity.

$$\therefore \mu = \frac{\text{force/area}}{\text{length/time} \times 1/\text{length}} = \frac{\text{force/length}^2}{\frac{1}{\text{time}}} = \frac{\text{forcetime}}{\text{length}^2}$$

Kinematic viscosity:

Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by ν (called nu)

$$\text{Mathematically, } \nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}, \text{ Unit} = \text{m}^2/\text{s}$$

Shear Stress

The shear stress is part of the pressure tensor. However, here, and many parts of the book, it will be treated as a separate issue. In solid mechanics, the shear stress is considered as the ratio of the force acting on area in the direction of the forces perpendicular to area. Different from solid, fluid cannot pull directly but through a solid surface. Consider liquid that undergoes a shear stress between a short distance of two plates as shown in Figure (1.3).

The upper plate velocity generally will be: $U = f(A, F, h)$

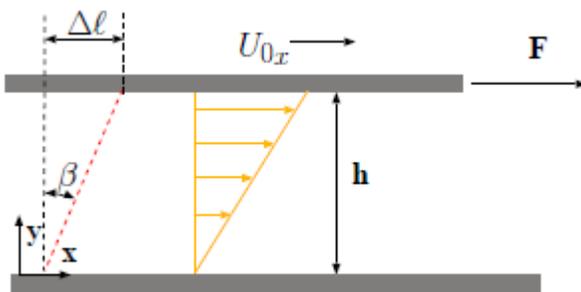
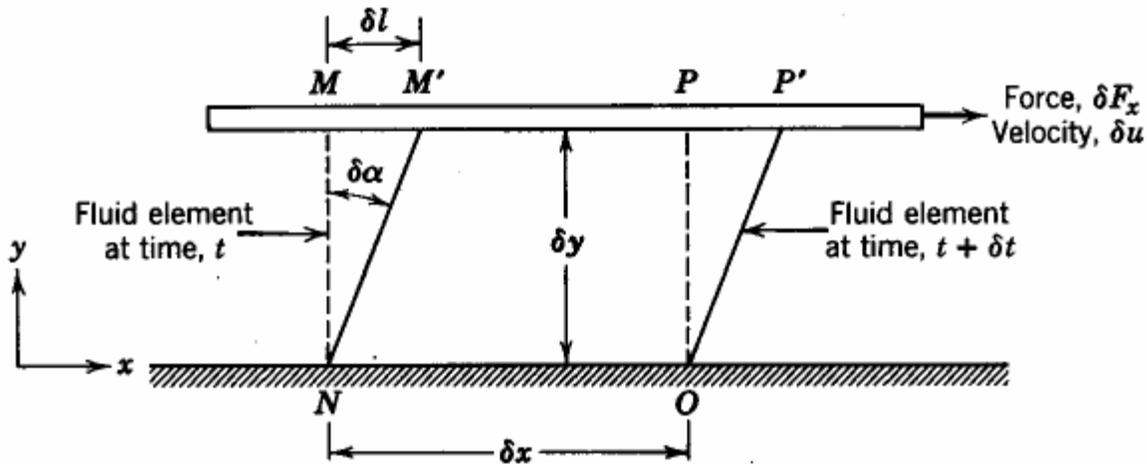


Fig. -1.3. Schematics to describe the shearstress in fluid mechanics.

Where A is the area, the F denotes the force, h is the distance between the plates. From solid mechanics study, it was shown that when the force per area increases, the velocity of the plate increases also. Experiments show that the increase of height will increase the velocity up to a certain range. Consider moving the plate with a zero lubricant ($h \sim 0$) (results in large force) or a large amount of lubricant (smaller force).



Deformation of a fluid element.

The crosshatching represents (a) solid plates or planes bonded to the solid being sheared and (b) two parallel plates bounding the fluid in (b). The fluid might be a thick oil or glycerin, for example.

- Within the elastic limit of the solid, the shear stress $\tau = F/A$ where A is the area of the surface in contact with the solid plate.
- However, for the fluid, the top plate does not stop. It continues to move as time t goes on and the fluid continues to deform.
- Consider a block or plane sliding at constant velocity δu over a well-oiled surface under the influence of a constant force δF_x .
- The oil next to the block sticks to the block and moves at velocity δu . The surface beneath the oil is stationary and the oil there sticks to that surface and has velocity zero.
- **No-slip boundary condition**—The condition of zero velocity at a boundary is known in fluid mechanics as the “no-slip” boundary condition.

- It can be shown that the shear stress τ is given by

$$\tau = \mu \frac{\delta u}{\delta y}$$

- The term du/dy is known as the velocity gradient and as the rate of shear strain.
- The coefficient is the coefficient of **dynamic** viscosity, μ . ($\text{kg/m}\cdot\text{s}$)
- And we see that for the simple case of two plates separated by distance d , one plate stationary, and the other moving at constant speed V : $\tau = \mu \frac{\delta u}{\delta y} = \mu \frac{V}{h}$.

coefficient of **dynamic** viscosity μ . is:-

- Intensive property of the fluid.
- Dependent upon both temperature and pressure for a single phase of a pure substance.
- Pressure dependence is usually weak and temperature dependence is important.

Effect of temperature on viscosity.

Viscosity is effected by temperature. The viscosity of liquids decreases but that of gases increases with increase in temperature. This is due to the reason that in liquids the shear stress is

due to the inter-molecular cohesion which decreases with increase of temperature. In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules, normal to the direction of motion. The molecular activity increases with rise in temperature and so does the velocity of gas.

For liquids, $\mu_T = Ae^{\beta/T}$

For gases: $\mu_T = \frac{bT^{1/2}}{1+a/T}$

Where,

μ_T = Dynamic viscosity at absolute temperature T.

A and β = Constants for given liquid, and

a and b = Constants for given gas.

EX. 1

A plate 0.05mm distant from a fixed plate moves at 1.2 m/s and required a force of 2.2N/m² to maintain this speed. Find the viscosity of the fluid between the plates.

Solution,

Velocity of the moving plate, $u = 1.2$ m/s

Distance between the plates, $dy = 0.05 = 0.05 \times 10^{-3}$.

Force on the moving plate $F = 2.2\text{N/m}^2$.

Viscosity of the fluid, μ :

We know that $\tau = \mu \frac{\delta u}{\delta y}$, where,

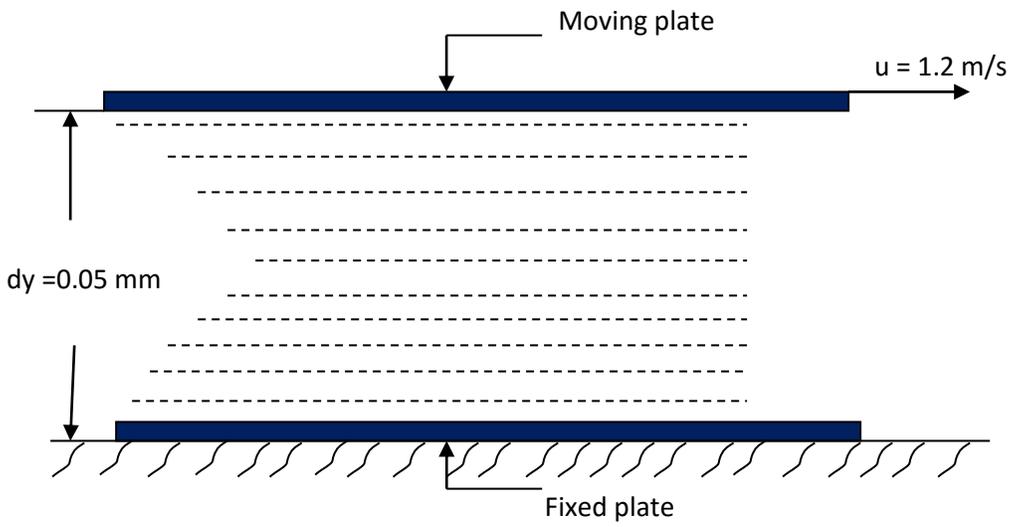
τ = shear stress or force per unit area = 2.2 N/m².

du = change of velocity = $u - 0 = 1.2$ m/s.

dy = change of distance = 0.05×10^{-3} m

$\therefore 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$ or,

$\mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} \text{N-s} / \text{m}^2$.



Energy and Heads of flowing Liquids

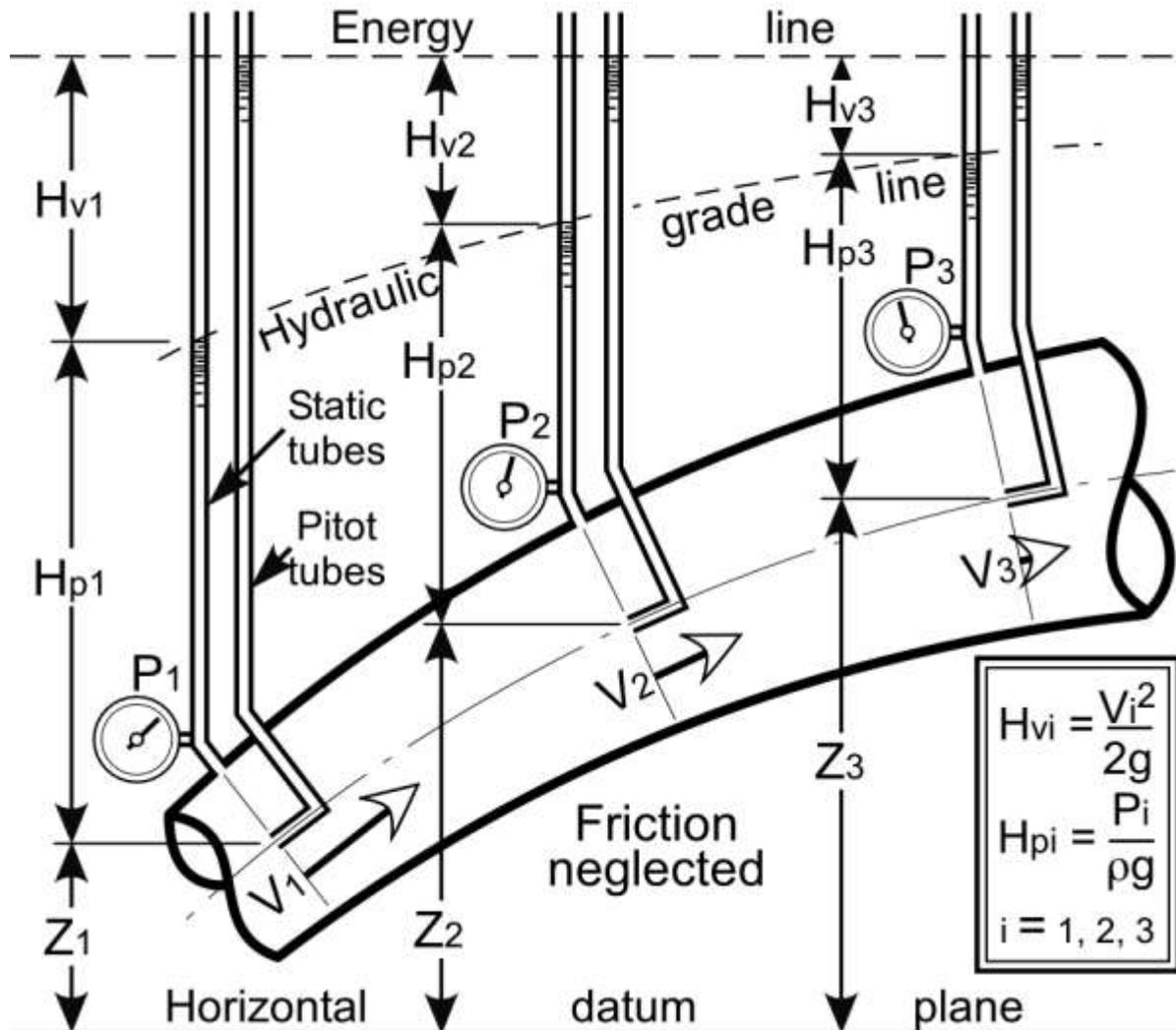


Fig.1 – Total Heads in a pipe (Professor Allan C. Wightley, 2002, Weir Slurry Group Technology) – Omega-s file.

Water and other liquids in a steady state find their own levels. Water, in a dam on a high mountain has potential energy or, said in another way; it has the potential for doing work. On the way down to lower grounds the potential energy converts to kinetic energy, which does the actual work, like driving a turbine-generator set or a water mill or float a barge down a river. Alternatively all this energy can be wasted, like in a waterfall. Water is stored in dams mostly for the following reasons: (1) in hydroelectric schemes, for electric power generation, (2) in irrigation schemes, for crop cultivation, (3) in town water circulation systems, as one of the essential ingredients of life and (4) in river flood mitigation schemes. Only in hydroelectric schemes must water be stored at high altitudes, where it has lots of convertible potential energy. In irrigation and water circulation schemes, pumps are used to impart kinetic energy to the water to circulate it through the pipes and/or to elevate it to higher grounds. Fluid flow is always from the point of highest energy to the lowest. Specific Energy is the capacity to do work per unit weight [N.m/N], which reduces to head in metres [m]. It is important to note and remember that specific energy of a fluid at a given point is numerically equal to its elevation or Potential Head Z at that point. If an object falls from a building of height H , it will reach a velocity

$V = \sqrt{2gH}$ (1) - just before it hits the ground. Conversely, if the same object is propelled upwards, with an initial velocity V , it will reach a height

$$H_v = V^2/2g \text{ (2)}$$

Which is another form of energy and it is called Velocity Head.

When dealing with liquids and heads we must invariably deal also with pressures. If we have a vertical cylinder of height H and diameter D filled with some liquid, the weight of the liquid is $W = \rho \cdot g \cdot H \cdot D \cdot \pi/4$. Dividing this by the cross-sectional area of the cylinder $D^2\pi/4$ we get the Static Pressure acting on the bottom of the cylinder:

$$P = \rho \cdot g \cdot H \text{ (3)}$$

As we can see, this pressure is independent of the cylinder's diameter, its shape or volume. Pressure is a function purely of the height and density of the liquid. So, if the liquid is water, a pressure gauge at the bottom of a 10 metres high cylinder shows a pressure

$P = 1000 \times 10 \times 9.81 \times 1 = 98,100 \text{ Pa}$ or 98.1 kPa i.e. close to normal atmospheric pressure. If the liquid is mercury (SG= 13.6), the pressure is 13.6 times greater or 1334 kPa. In either case, if we move the gauge up the cylinder, the pressure decreases linearly to 0 at the top. From the pressure equation (3) we can extract the corresponding Pressure Head:

$$H_p = P/(\rho \cdot g) \text{ (4)}$$

A liquid passing through a pipeline at a fixed flow rate follows all directional changes and its local velocities grow and fall inversely with the respective pipe cross-sections. Total head H (or energy) of the fluid along the pipeline, referred to a horizontal datum plane, is made up of:

- (1) the Static head Z , equal to the vertical position of the pipe section,
- (2) the Pressure head H_p , obtained from a pressure gauge P in the pipe or from a liquid column and
- (3) the Velocity head H_v , which can be either calculated or obtained by means of a Pilot-Static tube set.

If we neglect internal friction, the sum:

$$H = Z + H_p + H_v \text{ 5}$$

remains constant along the pipeline. This summation is shown in Fig.1 and is known as the Bernoulli equation.

It is important to note that:

- (1) the sums of Z and H_p along the pipeline define the Hydraulic Grade Line (HGL) and
- (2) the sums of Z , H_p and H_v along the pipeline define the Energy Line (EL).

If we considered internal friction head losses H_f as well, then the EL would show a continuous drop from left to right in Fig.1.

If the pipeline had a constant cross-section, the amount of drop per unit length of pipe would be constant i.e. the EL would be a straight line with a constant falling slope.

On the other hand, if the pipeline had variable cross-sections along the way as shown in Fig.1, then the EL would be a curved line with a falling but continuously changing slope.

The Head Equation

Figure 2 shows part of continuous fluid flow in a duct. Between the two observation sections 1 and 2 no energy is transferred to or from the fluid and the flow is assumed to be frictionless.

Thus the total energy of the fluid relative to a horizontal reference plane T at the two sections must be equal. The total energy comprises components for potential energy, pressure energy and kinetic energy, and for a fluid particle with a mass m the energy at the observation sections is as follows:

Section	1	2
Potential Energy	mgh_1	mgh_2
Pressure Energy	$mg \frac{p_1}{\rho g}$	$mg \frac{p_2}{\rho g}$
Kinetic Energy	$\frac{1}{2}mv_1^2$	$\frac{1}{2}mv_2^2$

Where ρ is the fluid density and g the acceleration of gravity. For a flow without losses the total energy in section 1 and 2 will be equal, thus

$$mgh_1 + mg \frac{p_1}{\rho g} + \frac{1}{2}mv_1^2 = mgh_2 + mg \frac{p_2}{\rho g} + \frac{1}{2}mv_2^2 \dots\dots\dots 6$$

Dividing both sides of the equation with the term mg it is obtained

$$h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \dots\dots\dots 7$$

This equation is called Bernoulli's equation after the engineer who first derived it. The terms of the equation are expressed as **heads**, and the terms are consequently called static head, pressure head and kinetic head, respectively. The equation is essential for fluid mechanics and can be used to account for many hydrodynamic phenomena, such as the decrease in pressure that accompanies a reduction in a flow cross section area. In this case the fluid velocity increases, and for the total head to remain constant and assuming the potential head remains unchanged, the pressure term or static head, must decrease.

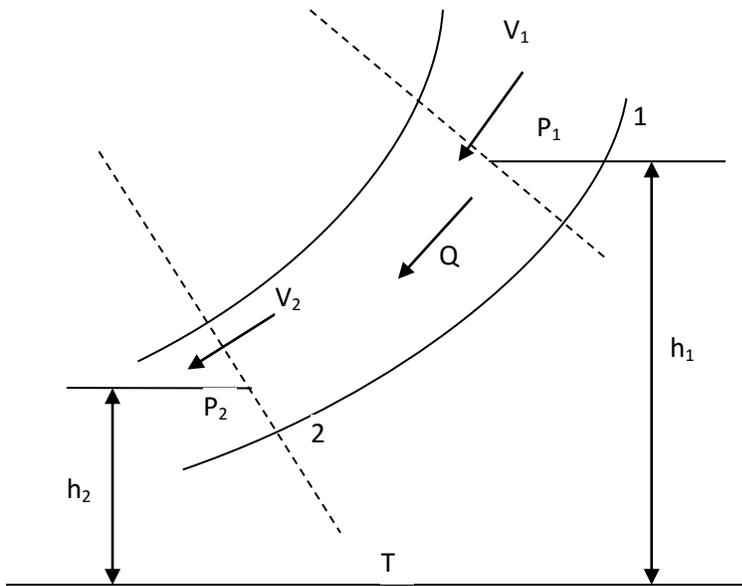


Fig.2.

Section showing flow of liquid through two observation cross sections. T is a reference plane for the potential heads h_1 and h_2 , p_1 and p_2 are the prevailing pressures and v_1 and v_2 the fluid velocities at sections 1 and 2.

Flow with Losses or Addition of Energy

If there are losses in the flow between section 1 and section 2 in Figure 2, the head equation 1 can be written

$$h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + H_r \dots\dots\dots 8$$

Where H_r is the head loss.

If energy is added to the flow by placing a pump between section 1 and section 2 in Figure 2, the equation 8 can be written

$$h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + H = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + H_r \dots\dots\dots 9$$

Where H is the pump total head.

Fluid Flowing from a Container

An example of the application of the Bernoulli equation is the calculation of the flow rate of a fluid flowing freely from an open container. Figure 3 shows an open container with an outlet

orifice near the bottom. For practical purposes the area A_1 is assumed much larger than the orifice area A_2 , and the atmospheric pressure p_1 in the container is equal to that outside the orifice, p_2 .

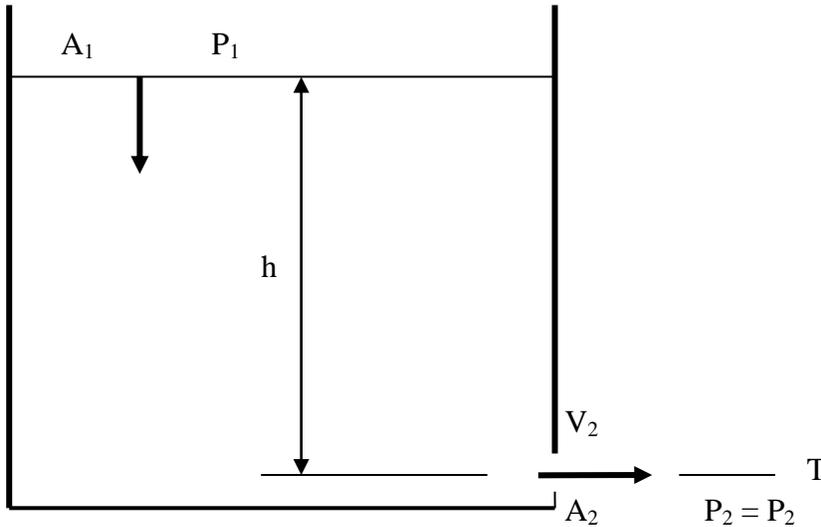


Fig.3

Section of a fluid container with an outlet orifice near the bottom. A_1 and A_2 are the cross section areas of the surface and the outlet orifice, h the height difference between surface and orifice centre line, v_1 surface recession velocity and v_2 liquid outlet velocity through the orifice. Ambient pressure is constant.

Choosing the centre line of the orifice as the reference plane T , the term h_2 is equal to zero and h_1 equal to h . Because A_1 is much larger than A_2 , the kinetic head $\frac{v_1^2}{2g}$ can be assumed as zero. Thus the head equation 7 can be written

$$h = \frac{v_1^2}{2g} \dots\dots 10$$

Whence,

$$V_2 = \sqrt{2gh} \dots\dots 11$$

For volume flow without losses is obtained

$$q_1 = A_2 \sqrt{2gh} \dots\dots 12$$

To accommodate for losses present, a flow coefficient μ is added to equation 12, whence

$$q_1 = \mu A_2 \sqrt{2gh} \dots\dots 13$$

The flow coefficient μ is dependent on the shape of the orifice, and can be obtained from text books on the subject. If the fluid level in the container is allowed to recede, the level height h will change, which will have to be accommodated for in calculations.