3.0 Thermodynamic Cycle

A system is said to operate on a thermodynamic cycle, if the processes involved in the conversion and transfer of heat and work, in and out of the system, takes place in a sequential manner, while varying temperature, pressure and other state variables within the system, and it eventually returns the system to its initial state.

Thermodynamic cycles are broadly classified into two: (i) power cycles and (ii) heat pump cycles.

3.1 Heat Engine or Power Cycle

For any cycle, the net amount of energy received through heat interactions is equal to the net energy transferred out in work interaction.

That is, $Q_{cycle} = W_{cycle}$ (3.0)

A **power cycle**, or **heat engine**, is one for which a net amount of energy is transferred out by work $W_{cycle} > 0$. Power cycles are broadly divided into two (i) vapour power cycle and (ii) gas power cycle.

In other words, a **heat engine** is a machine that transforms heat into mechanical power. Steam and gas turbines, gasoline engines, diesel engines, etc. are examples of heat engines.

A power cycle is characterized both by addition of energy by heat transfer, Q_1, and the rejection of energy by heat transfer, Q_2.

$$Q_{cycle} = Q_1 - Q_2 \quad (3.1)$$

Merging the two equations 3.0 & 3.1, we obtain,

$$W_{cycle} = Q_1 - Q_2 \quad (3.2)$$

The **thermal efficiency** of a heat engine is defined as the ratio of the net work developed to the total energy added by heat transfer. It is denoted with the symbol η_{th}.

$$\eta_{th} = \frac{W_{cycle}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad (3.3)$$

In practice, the thermal efficiency of a heat engine is less than 100%. That is, some portion of the heat supplied is rejected $Q_2 \neq 0$.

3.1.1 Heat Engines

A **heat engine** is a device operating on a cycle, that transforms heat into mechanical power. In a heat engine, heat flows from a hot cylinder to a cooler exhaust pipe and does mechanical work as the temperature drops. The net work produced by the engine is given as:

$$Q_{cycle} = Q_1 - Q_2 \quad \text{(3.1) (First law of thermodynamic for heat engines)}$$

3.2 Heat Pump Cycles

A reversed heat engine cycle is a **heat pump**. Heat pumps, refrigerators and air-conditioners operate on this cycle. The function of the system could be to supply energy to a body or to extract energy from a body. A **heat pump** is a mechanical or chemical device used for heating or cooling, while **refrigerators** and **air-conditioners** are employed to cool enclosed spaces. In the operation of heat pump or a refrigerator, heat is extracted from a low-temperature source and expelled elsewhere to a higher temperature sink. The working fluid for a heat pump and a refrigerator is a **refrigerant**.

Energy as heat Q_2, is extracted from a low temperature body and supplied as heat Q_1, to a warmer body in heat pumps and refrigerators. For this process to be achieved, mechanical work must be done on the refrigerant gas.

The heat extracted from the cold reservoir is given as:

$$Q_2 = Q_1 - W \quad (3.2)$$

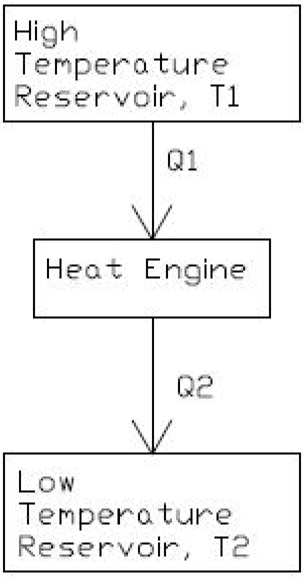

Fig (1a): Heat engine cycle

Q_1 = Heat input to engine/cycle

Q_2 = Heat rejected from engine/cycle

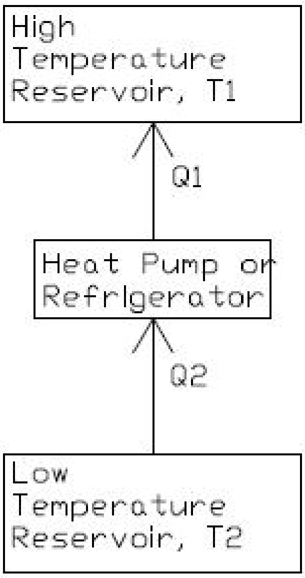

Fig (1b): Refrigerator

Q_2 = Heat rejected from condenser/cycle

Q_1 = Heat removed from evaporator/cycle

Equation (3.1) describes a mechanical work output accompanied by a drop in temperature from T_1 to T_2 while equation (3.2) describes a mechanical work input necessary to raise the temperature of gas from T_2 to T_1 in the system.

3.3 Cycle Efficiency

A Carnot engine is an ideal heat engine that operates between a high temperature reservoir and a low temperature reservoir. It has the maximum possible efficiency that cannot be attained by a **real engine**. The Carnot engine uses reversible processes to form its cycle of operation; hence it is also called a **reversible engine**.

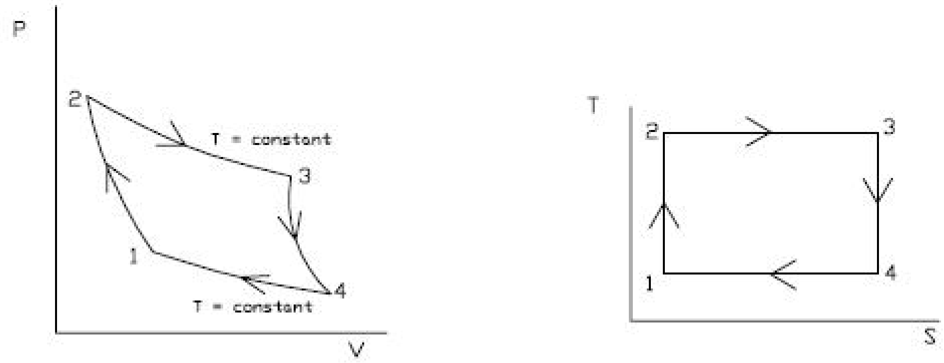

Fig 2: The <u>cycles of a Carnot Engine on a P-V and T-S Diagram respectively</u>

1 – 2 – An Isentropic Compression

2 – 3 - An Isothermal Heat addition

3 – 4 – An Isentropic expansion

4 – 1 – An Isothermal Cooling

The thermal efficiency of a heat engine is given by:

$$\eta_{th} = \frac{\text{Desired energy transfer}}{\text{Required energy input transfer}} = \frac{\text{Network output}}{\text{gross heat input}} = \frac{\oint dW}{q_1} \qquad 3.4$$

Recall from the first law of thermodynamics, in a reversible process,

$$\oint W = \oint q = q_1 - q_2 \qquad 3.4a$$

$$\eta_{th} = \frac{q_1 - q_2}{q_1} = 1 - \frac{q_2}{q_1} \qquad 3.4b$$

Where, q_2 = heat output

q_1 = heat input

Also,

$$h_{th} = 1 - \frac{T_{41}}{T_{23}} \qquad 3.4c$$

Where, the temperatures are in Kelvin K.

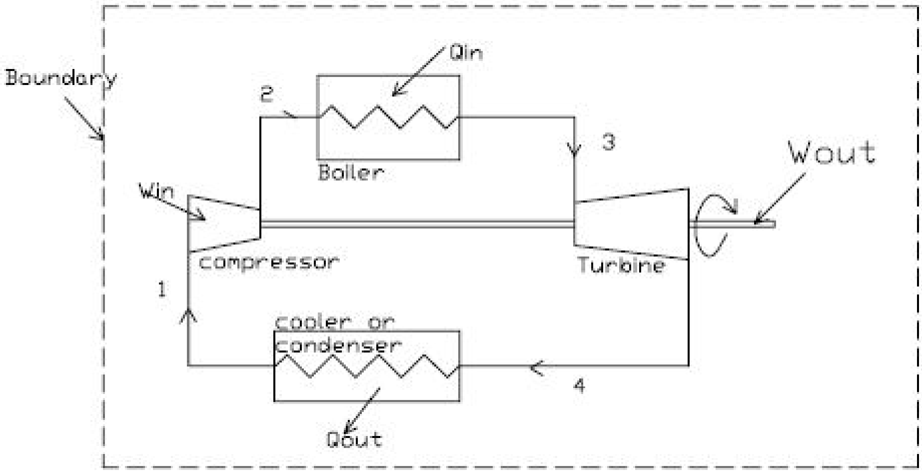

Fig 3: Schematic diagram of a steam plant operating on a Carnot cycle

Note: Carnot efficiency is dependent only on the high and low absolute temperatures of the reservoirs and is independent of the working substance used or any particular design feature of the engine.

Irreversible processes result in degradation of energy and a loss of work transfer; hence the thermal efficiency of an irreversible cycle is less than that of a **reversible cycle.**

3.4 Coefficient of Performance

Heat pumps and refrigerators are assessed by the value of their coefficient of performance (COP). Coefficient of performance is defined as the ratio of useful heating or cooling provided to the required work (net work input). In heat pumps, COP most times, exceeds 1, because work is not just converted to heat but additional heat is pumped from the source to where the heat is required. Higher COPs implies lower system operating costs.

The coefficient of performance (COP), b of a **refrigerator** is given by:

$$\beta_{REF} = \frac{\text{Heat input from low temperature source}}{\text{Net work input}} \qquad 3.5a$$

$$\beta_{REF} = \frac{q_1}{W} = \frac{q_1}{q_2 - q_1} = \frac{q_{in}}{q_{out} - q_{in}} \qquad 3.5b$$

For **heat pump**,

$$\beta_{HP} = \frac{\text{Heat output to high temperature source}}{\text{Net work input}} \qquad 3.6a$$

$$= \frac{q_2}{q_2 - q_1} = \frac{q_{out}}{q_{out} - q_{in}} \qquad 3.6b$$

Thus,

$$\beta_{HP} = \beta_{REF} + 1 \qquad 3.6c$$

Question 1

The work done on a reversed heat engine is 150KJ while the heat transfer to the engine from the low temperature reservoir is 450KJ. Determine the heat transfer to the high temperature reservoir and the COP as a refrigerator and as a heat pump.

Solution

$Q_{out} = Q_{in} + W = 450 + 150 = 600KJ$

$B_{REF} = \frac{Q_{in}}{W} = \frac{450}{150} = 3$

$B_{HP} = \frac{Q_{out}}{W} = \frac{600}{150} = 4$

$B_{HP} = B_{REF} + 1 = 3 + 1 = 4$

Question 2

A Carnot heat engine rejects 230 KJ of heat at 25 ˚C. The net cycle work is 375 KJ. Determine the thermal efficiency and the cycle high temperature.

Solution

Source heat Q_1 = ?, Temperature of the heat source T_1 = ?, Heat rejected Q_2 = 230 KJ, Temperature of the sink T_2 = 25 ˚C = 298 K, Net cycle work W = 375 KJ.

Recall,

Thermal efficiency $\eta_{th} = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$

$Q_1 = Q_2 + W = 230 + 375 = 605$ KJ

To calculate the thermal efficiency $\boldsymbol{\eta_{th}} = \frac{W}{Q_1} = \frac{375}{605} = 0.62$

To calculate the cycle's highest temperature $\mathbf{T_1}$,

$$\eta_{th} = \frac{T_1 - T_2}{T_1}, \qquad 0.62 = \frac{T_1 - 298}{T_1}$$

$$T_1 = \frac{298}{(1 - 0.62)} = 784.2 \text{ K}$$

The source temperature $\mathbf{T_1}$ = 784.2 K

Question 3

Two reversible heat engines operate in series between a source at 500 ˚C and a sink at 20 ˚C. If the engines have equal efficiencies and the first rejects 350 KJ to the second, calculate:

a) The temperature at which heat is supplied to the second engine.
b) The heat taken from the source
c) The work done by each engine, assume each engine operates on the Carnot cycle.

Solution

From the diagram,

Q1 =?, T1 = 500 ˚C = 773 K, Q2 = 350 KJ, T2 = ?, Q3 = ?, T3 = 20 ˚C = 293 K.

But, the thermal efficiencies of the engines are equal, that is $\eta_{th1} = \eta_{th2}$

Therefore,

$$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2};$$

$$\frac{773 - T_2}{773} = \frac{T_2 - 293}{T_2}$$

$$T_2 = \sqrt{(293 \times 773)}$$

$$T_2 = 476 \text{ K}$$

The temperature at which heat is supplied to the second engine T_2 = 476 K.

To calculate the thermal efficiency of the first engine, η_{th1}

$$\eta_{th1} = \frac{T_1 - T_2}{T_1} = \frac{773 - 476}{773} = 0.384 \text{ or } 38.4\ \%$$

(b) To calculate the heat taken from the source, Q1,

$$\eta_{th1} = \frac{Q_1 - Q_2}{Q_1};$$

Therefore,

$$Q_1 = \frac{350}{(1 - 0.384)} = 568.2 \text{ KJ}$$

Heat taken from the source Q_1 = 568.2 KJ.

(c) The work done by each engine assuming each operates on the Carnot cycle.

(i) For the first engine, work done is $W_1 = Q_1 - Q_2$ = 568.2 – 350 = 218.2 KJ.

(ii) For the second engine, work done is $W_2 = Q_2 - Q_3$

To calculate Q_3,

$$\eta_{th2} = \frac{Q_2 - Q_3}{Q_2}$$

Recall, $\eta_{th1} = \eta_{th2}$

Therefore,

$$0.384 = \frac{350 - Q_3}{350}$$

Heat rejected Q_3 = 215.6 KJ

The work done by the second engine W_2 = 350 – 215.6 = 134.4 KJ.