4.0 IDEAL CYCLES IN ENGINES (AIR STANDARD CYCLES) CONTD

4.1.2 Diesel Cycle

The following processes take place in an air-standard Diesel cycle:

Process 1 -2: Isentropic compression of air takes place from state 1 to state 2.

Process 2-3: constant pressure heat addition takes place from state 2 to state3.

Process 3–4: Isentropic expansion occurs from state 3 to state 4.

Process 4-1: heat rejection at constant volume occurs from state 4 to state 1.

To calculate the thermal efficiency of the diesel engine, the heat supplied and the heat rejected are required.

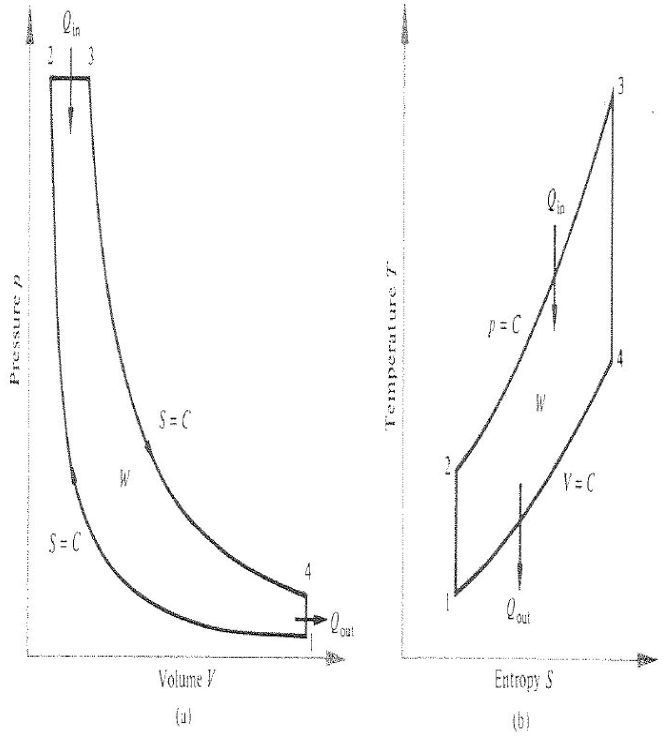

Figure 2: (a) The P-V diagram for the air-standard Diesel cycle (b) The T-S diagram for the air-standard Diesel cycle.

(i) Show that the thermal efficiency (η_{th}) of an engine operating on a diesel cycle is:

$$\eta_{th} = 1 - \frac{1}{\gamma_v^{\gamma-1}} \left(\frac{\gamma_C^{\gamma} - 1}{\gamma(\gamma_C - 1)} \right)$$

Where,

γ_v is the engine compression ratio

γ_C is the cut-off ratio

γ is the ratio of specific heats

$$\eta_{th} = 1 - \frac{Q_2}{Q_1}$$

The heat supplied at constant pressure is given as:

$$Q_1 = mc_p\left(T_3 - T_2\right)$$

The heat rejected at constant volume Q_2 is given as:

$$Q_2 = mc_v\left(T_4 - T_1\right)$$

Substituting (2) and (3) into (1) we have:

$$\eta_{th} = 1 - \frac{c_v\left(T_4 - T_1\right)}{c_p\left(T_3 - T_2\right)} = 1 - \frac{T_1\left(\left(\frac{T_4}{T_1}\right) - 1\right)}{\gamma \cdot T_2\left(\left(\frac{T_3}{T_2}\right) - 1\right)}$$

At the isentropic compression stage,

$$\frac{T_2}{T_1} = \gamma_v{}^{\gamma - 1}$$

Therefore,

$$T_1 = T_2\left(\frac{1}{\gamma_v}\right)^{\gamma - 1}$$

Cut off ratio $\gamma_c = \frac{T_3}{T_2}$

$$T_3 = T_2\gamma_c$$

For the isentropic expansion stage:

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = \left(\frac{v_3}{v_2} \cdot \frac{v_2}{v_4}\right)^{\gamma - 1} = \left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma - 1}$$

Since $\frac{v_3}{v_2} = \gamma_c$ and $\frac{v_4}{v_2} = \gamma_v$

Hence $T_4 = T_3\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma - 1} = T_2\gamma_c\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma - 1}$

Recall,

$$\frac{T_4}{T_1}=\frac{T_2\gamma_c\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma-1}}{T_2\left(\frac{1}{\gamma_v}\right)^{\gamma-1}}=\gamma_c{}^{\gamma}$$

$$\frac{T_3}{T_2}=\frac{T_2\gamma_c}{T_2}=\gamma_c$$

$$\eta_{th}=1-\frac{1}{\gamma_v{}^{\gamma-1}}\left(\frac{\gamma_C{}^{\gamma}-1}{\gamma(\gamma_C-1)}\right)$$

QUESTION 2

An engine operates on the air standard diesel cycle. The inlet temperature and pressure are 27˚C and 100KPa respectively. The compression ratio is 12:1 and the heat addition is 1800KJ/kg. Calculate the maximum temperature and pressure of the cycle, the thermal efficiency and the mean effective pressure.

Solution

For the isentropic compression process from state 1 to state 2,

$Pv^{\gamma}=C$ and $Tv^{\gamma-1}=C$

$\gamma=1.4$

T_1 = 300 K; P_1 = 100kPa; γ_c = 12 and heat supplied Q_{23} = 1800KJ/kg.

To calculate the air temperature at the end of compression,

$T_2=T_1\gamma_v{}^{\gamma-1}=300(12)^{1.4-1}$ = 810.58 K.

The pressure at the end of the compression stroke P_2 is given as:

$P_2=P_1\gamma_v{}^{\gamma}=100(12)^{1.4}=3242.30KPa$

The constant pressure heat addition process:

$Q_{23}=c_p(T_3-T_2)=1800\text{ kJ/kg}$

$$T_3=T_2+\frac{1800}{1.005}=810.58+1791.04$$

T_3 = 2601.62 K.

The cycles maximum temperature T_3 = 2601.62 K.

The cycles maximum pressure P_3 = P_2 = 3242.30kPa.

The specific volume at the end of injection $v_3=\frac{RT_3}{P_3}=\frac{(0.287)(2601.62)}{3242.30}=0.2303m^3/kg$

For the isentropic expansion process from state 3 to state 4,

$$v_4 = v_1$$

R is the specific gas constant for dry air = 0.287kJkg^{-1}K^{-1}

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(300)}{100} = 0.861 m^3/kg$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1}$$

$$T_4 = 2601.62\left(\frac{0.2303}{0.861}\right)^{1.4-1} = 1535.18K$$

$$P_4 = P_3\left(\frac{v_3}{v_4}\right)^{\gamma} = 3242.30\left(\frac{0.2303}{0.861}\right)^{1.4} = 511.75 \text{ kPa}$$

Heat rejected at constant volume between state 4 and state 1

c_v is the specific heat capacity at constant volume = 0.718kJ/kgK

$$Q_{41} = c_v(T_4 - T_1) = 0.718(1535.18 - 300) = 886.86 kJ/kg$$

The Thermal efficiency of the air standard diesel cycle η_{th}

$$\eta_{th} = \frac{Q_{23} - Q_{41}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{886.86}{1800} = 0.5073 \text{ or } 50.73\%$$

Calculation of the Mean Effective Pressure (MEP)

$$\text{MEP} = \frac{W_{net}}{v_1 - v_2} = \frac{Q_{23} - Q_{41}}{v_1 - v_2}$$

$$v_2 = \frac{v_1}{12} = \frac{0.861}{12} = 0.07175 m^3/kg$$

$$\text{MEP} = \frac{1800 - 886.86}{0.861 - 0.07175} = 1226.93 kPa$$

The Mean Effective Pressure (MEP) = 1226.93kPa.