LANDMARK UNIVERSITY, OMU-ARAN

LECTURE NOTE 2

COLLEGE: COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT: MECHANICAL ENGINEERING

Course code: GEC216

Course title: GENERAL ENGENEERING LABORAATORY I

Credit unit: *3 UNITS.*

Course status: *compulsory*

ENGR. ALIYU, S.J

STRESS AND STRAIN

Before we proceed to develop techniques for strain measurements, we briefly review the relationship between deflections and stress. The experimental analysis of stress is accomplished by measuring the deformation of a part under load, and inferring the existing state of stress from the measured deflections. Again, consider the rod in Figure 1. If the rod has a cross-sectional area of Ac, and the load is applied only along the axis of the rod, the normal stress is defined as

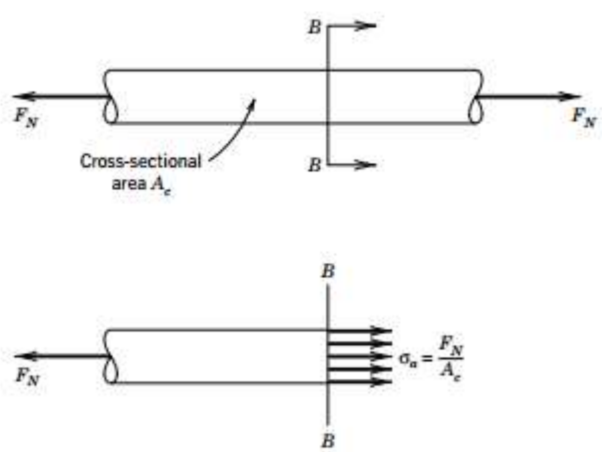

Fig. 1 Free-body diagram illustrating internal forces for a rod in uniaxial tension.

$\sigma_a = F_N/A_C$ ……………..1

Where A_c is the cross-sectional area and F_N is the tension force applied to the rod normal to the area Ac. The ratio of the change in length of the rod (which results from applying the load) to the original length is the axial strain, defined as

$\varepsilon_a = \delta L/L$ ………….. 2

Where ε_a is the average strain over the length L, δL is the change in length, and L is the original unloaded length. For most engineering materials, strain is a small quantity; strain is usually reported in units of 10^6 in./in. or 10^6 m/m. These units are equivalent to a dimensionless unit called a microstrain (me). Stress–strain diagrams are very important in understanding the behavior of a material under load. Figure 2 is such a diagram for mild steel (a ductile material). For loads less than that required to permanently deform the material, most engineering materials display a linear relationship between stress and strain. The range of stress over which this linear relationship holds is called the elastic region. The relationship between uniaxial stress and strain for this elastic behavior is expressed as

$\sigma_a = E_m \varepsilon_a$ ……………… 3

Where E_m is the modulus of elasticity, or Young's modulus, and the relationship is called Hooke's law. Hooke's law applies only over the range of applied stress where the relationship between stress and strain is linear. Different materials respond in a variety of ways to loads beyond the linear range, largely depending on whether the material is ductile or brittle. For almost all engineering components, stress levels are designed to remain well below the elastic limit of the material; thus, a direct linear relationship may be established between stress and strain. Under this assumption, Hooke's law forms the basis for experimental stress analysis through the measurement of strain.

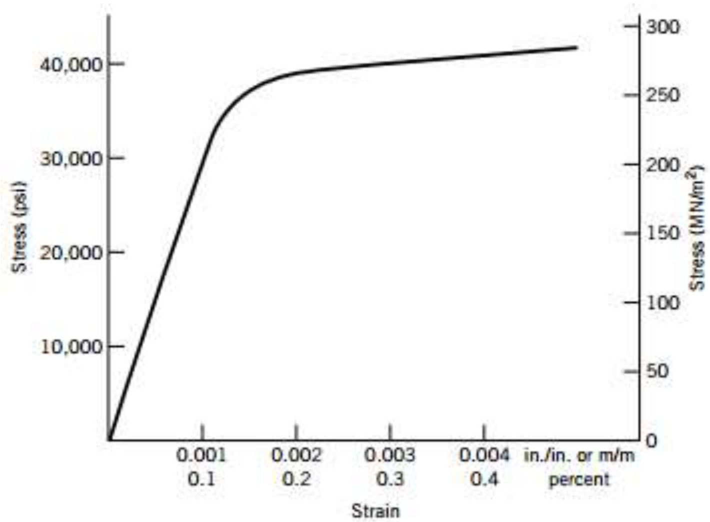

Figure 2 A typical stress– strain curve for mild steel.

Lateral Strains

Consider the elongation of the rod shown in Figure 11.1 that occurs as a result of the load FN. As the rod is stretched in the axial direction, the cross-sectional area must decrease since the total mass (or volume for constant density) must be conserved. Similarly, if the rod were compressed in the axial direction, the cross-sectional area would increase. This change in cross-sectional area is most conveniently expressed in terms of a lateral (transverse) strain. For a circular rod, the lateral strain is defined as the change in the diameter divided by the original diameter. In the elastic range, there is a constant rate of change in the lateral strain as the axial strain increases. In the same sense that the modulus of elasticity is a property of a given material, the ratio of lateral strain to axial strain is also a material property. This property is called Poisson's ratio, defined as

$$\upsilon_P = \frac{|Lateral\ strain|}{|Axial\ strain|} = \frac{\varepsilon_L}{\varepsilon_a} \quad \ldots\ldots\ldots\ldots\ldots\ 4$$

Engineering components are seldom subject to one-dimensional axial loading. The relationship between stress and strain must be generalized to a multidimensional case. Consider a two-dimensional geometry, as shown in Figure 3, subject to tensile loads in both the x and y directions, resulting in normal stresses σ_x and σ_y. In this case, for a biaxial state of stress, the stresses and strains are

$$\varepsilon_y = \frac{\sigma_y}{E_m} - \upsilon_P \frac{\sigma_x}{E_m} \qquad \varepsilon_x = \frac{\sigma_x}{E_m} - \upsilon_P \frac{\sigma_y}{E_m}$$

$$\sigma_x = \frac{E_m(\varepsilon_x + \upsilon_P \varepsilon_y)}{1 - \upsilon_P^2} \qquad \sigma_y = \frac{E_m(\varepsilon_y + \upsilon_P \varepsilon_x)}{1 - \upsilon_P^2} \quad \ldots\ldots\ldots\ldots\ldots\ 5$$

$$\tau_{xy} = G\gamma_{xy}$$

In this case, all of the stress and strain components lie in the same plane. The state of stress in the elastic condition for a material is similarly related to the strains in a complete three-dimensional situation (1, 2). Since stress and strain are related, it is possible to determine stress from measured strains under appropriate conditions. However, strain measurements are made at the surface of an engineering component. The measurement yields information about the state of stress on the surface of the part.

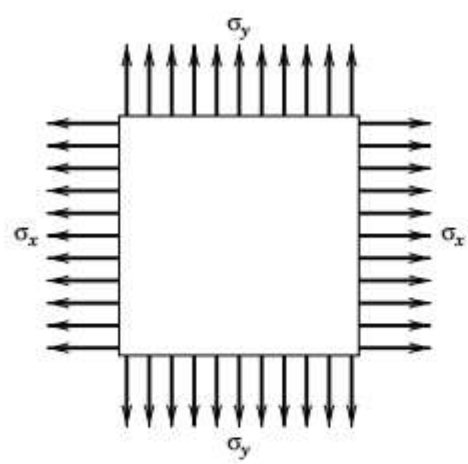

Figure 3 Biaxial state of stress.

The analysis of measured strains requires application of the relationship between stress and strain at a surface. Such analysis of strain data is described elsewhere (3), and an example provided in this chapter.

3 RESISTANCE STRAIN GAUGES

The measurement of the small displacements that occur in a material or object under mechanical load can be accomplished by methods as simple as observing the change in the distance between two scribe marks on the surface of a load-carrying member, or as advanced as optical holography. In any case, the ideal sensor for the measurement of strain would (1) have good spatial resolution, implying that the sensor would measure strain at a point; (2) be unaffected by changes in ambient conditions; and (3) have a high-frequency response for dynamic (time-resolved) strain measurements. A sensor that closely meets these characteristics is the bonded resistance strain gauge. In practical application, the bonded resistance strain gauge is secured to the surface of the test object by an adhesive so that it deforms as the test object deforms. The resistance of a strain gauge changes when it is deformed, and this is easily related to the local strain. Both metallic and semiconductor materials experience a change in electrical resistance when they are subjected to a strain. The amount that the resistance changes depends on how the gauge is deformed, the material from which it is made, and the design of the gauge. Gauges can be made quite small for good resolution and with a low mass to provide a high-frequency response. With some ingenuity, ambient effects can be minimized or eliminated. In an 1856 publication in the Philosophical Transactions of the Royal Society in England, Lord Kelvin (William Thomson) (4) laid the foundations for understanding the changes in electrical resistance that metals undergo when subjected to loads, which eventually led to the strain gauge concept. Two individuals began the modern development of strain measurement in the late 1930s—Edward Simmons at the California Institute of Technology and Arthur Ruge at the Massachusetts Institute of Technology. Their development of the bonded metallic wire strain gauge led to commercially available strain gauges. The resistance strain gauge also forms the basis for a variety of other transducers, such as load cells, pressure transducers, and torque meters.

Metallic Gauges

To understand how metallic strain gauges work, consider a conductor having a uniform cross sectional area A_c and length L made of a material having an electrical resistivity, σ_e. For this electrical conductor, the resistance, R, is given by

$$R = \sigma_e L/A_c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6$$

If the conductor is subjected to a normal stress along the axis of the wire, the cross-sectional area and the length change resulting in a change in the total electrical resistance, R. The total change in R is due to several effects, as illustrated in the total differential:

$$dR = \frac{A_C(\sigma_e dL + L d\rho_e) - \rho_e L dA_C}{A_C^2} \quad \ldots\ldots\ldots\ldots\ldots \quad 7$$

Which may be expressed in terms of Poisson's ratio as

$$\frac{dR}{R} = \frac{dL}{L}(1 + 2\upsilon_P) + \frac{d\rho_e}{\rho_e} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 8$$

Hence, the changes in resistance are caused by two basic effects: the change in geometry as the length and cross-sectional area change, and the change in the value of the resistivity, ρ_e. The dependence of resistivity on mechanical strain is called piezo-resistance, and may be expressed in terms of a piezo-resistance coefficient, π_1 defined by

$$\pi_1 = \frac{1}{E_m}\frac{d\rho_e/\rho_e}{dL/L} \ldots\ldots\ldots\ldots\ldots\ldots \quad 9$$

With this definition, the change in resistance may be expressed

$$dR/R = dL/L\left(1 + 2v_P + \pi_1 E_m\right) \ldots\ldots\ldots\ldots. \quad 10$$

Example 1

Determine the total resistance of a copper wire having a diameter of 1 mm and a length of 5 cm. The resistivity of copper is 1:7 x 10^{-8} Ω m.

KNOWN D = 1 mm

L = 5 cm

ρ_e = 1:7 x 10^{-8} Ω m

FIND: The total electrical resistance

SOLUTION

The resistance may be calculated from Equation 6 as

$$R = \sigma_e L/A_c$$

$$A_C = \frac{\pi D^2}{4} = \frac{\pi}{4}(1\ x\ 10^{-3})^2 = 7.85\ x\ 10^{-7}m^2$$

The resistance is then

$$R = \frac{(1.7\ x\ 10^{-8}\Omega\ m)(5\ x\ 10^{-2}m)}{7.85\ x\ 10^{-7}m^2} = 1.08\ x\ 10^{-3}\Omega$$

COMMENT If the material were nickel ($\rho_e = 7.8\ x\ 10^{-8}\Omega m$) instead of copper, the resistance would be 5 x 10^{-3} Ω for the same diameter and length of wire.

Example 2

A very common material for the construction of strain gauges is the alloy constantan (55% copper with 45% nickel), having a resistivity of 49 x 10^{-8} Ω m. A typical strain gauge might have a resistance of 120 Ω. What length of constantan wire of diameter 0.025 mm would yield a resistance of 120 Ω?

KNOWN The resistivity of constantan is 49 x 10^{-8} Ω m.

FIND The length of constantan wire needed to produce a total resistance of 120 Ω

SOLUTION From Equation 6, we may solve for the length, which yields in this case

$$L = \frac{RA_C}{\rho_e} = \frac{(120\Omega)(4.91 \; x \; 10^{-10} m^2)}{49 \; x \; 10^{-8} \; \Omega m} = 0.12 \; m$$

The wire would then be 12 cm in length to achieve a resistance of 120 **Ω**.

COMMENT. As shown by this example, a single straight conductor is normally not practical for a local strain measurement with meaningful resolution. Instead, a simple solution is to bend the wire conductor so that several lengths of wire are oriented along the axis of the strain gauge, as shown in Figure 4.

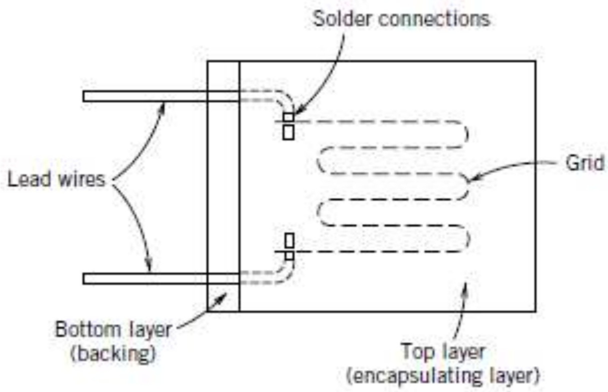

Figure 4 Detail of a basic strain gauge construction.