GCE214
Applied Mechanics-Statics

Lecture 01: 06/08/2017

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Class: Wednesday (3–5 pm)

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

WEEK 03 (4–8/08/ 2017)

- Introduction: Fundamental Concepts and Principles
- Representation and Resolution of Vector of Forces in 2D
- Free-body Diagram
- Equilibrium of Forces

Recommended textbook

- Vector Mechanics for Engineers: Statics and Dynamics by Beer, Johnston, Mazurek, Cornwell. 10th Edition

Applied Mechanics-Statics: Introduction

What is Mechanics?

Mechanics

Applied Mechanics
(*Mechanics applied to Engr. Problems*)

Mechanics of Rigid Bodies
(*Things that don't change shape*)

Mechanics of Deformable Bodies
(*Things that do change shape*)

Mechanics of Fluids
(*Mechanics applied to Engr. Problems*)

Statics
(*Bodies at rest*)

Dynamics
(*Bodies in motion*)

Compressible fluids

Incompressible fluids

Kinetics
(*Forces which cause motion are considered*)

Kinematics
(*Forces which cause motion aren't considered*)

Fundamental Concepts and Principles

- Basic concepts used in the study mechanics:

  - Space
  - Time
  - Mass
  - Force

- The condition of ***rest*** or motion of *particles* or *rigid bodies* is often studied in terms of the above mentioned concepts.
- A particle in this context is a very small amount of matter occupying a single point in space.
- A rigid body is a combination of large number of particles occupying fixed positions relative to each other

Fundamental Concepts and Principles

- The study of ***elementary mechanics*** rests on six fundamental principles:

  - The parallelogram law for addition of forces
  - The principle of transmissibility

  Newton's three laws of motion

  - First law of motion (inertia)
  - Second Law of motion ($F = ma$)
  - Third Law of motion (action and reaction)
  - Newton's law of gravitation

- These principles are based on experimental evidences and not on any mathematical proofs.
- Newtonian mechanics revolve round these six principles and are valid when velocities are small compared to the speed of light.
- However, in the study of the motion of atoms and some planets Newtonian mechanics will not suffice. Such study would have to be supplemented by the theory of relativity (Albert Einstein).

International System of Units (SI)

- The four concepts earlier introduced have associated kinetic units made of three *basic* and one *derived* units:

  - *Time in seconds (s)*
  - *Length in meters (m)*
  - *Mass in kilogram (kg), and*
  - *Force in Newton (N)*

- The first three units are arbitrarily defined according to some standards established by convention. For instance the second is defined as the duration of 9 192 631 770 cycles of the radiation corresponding to the transition between two levels of the fundamental state of the cesium-133 atom.
- The universal system of units, which is yet to be fully adopted by the USA, is known as the International System of units (SI). This will be used largely in our study

Numerical Accuracy

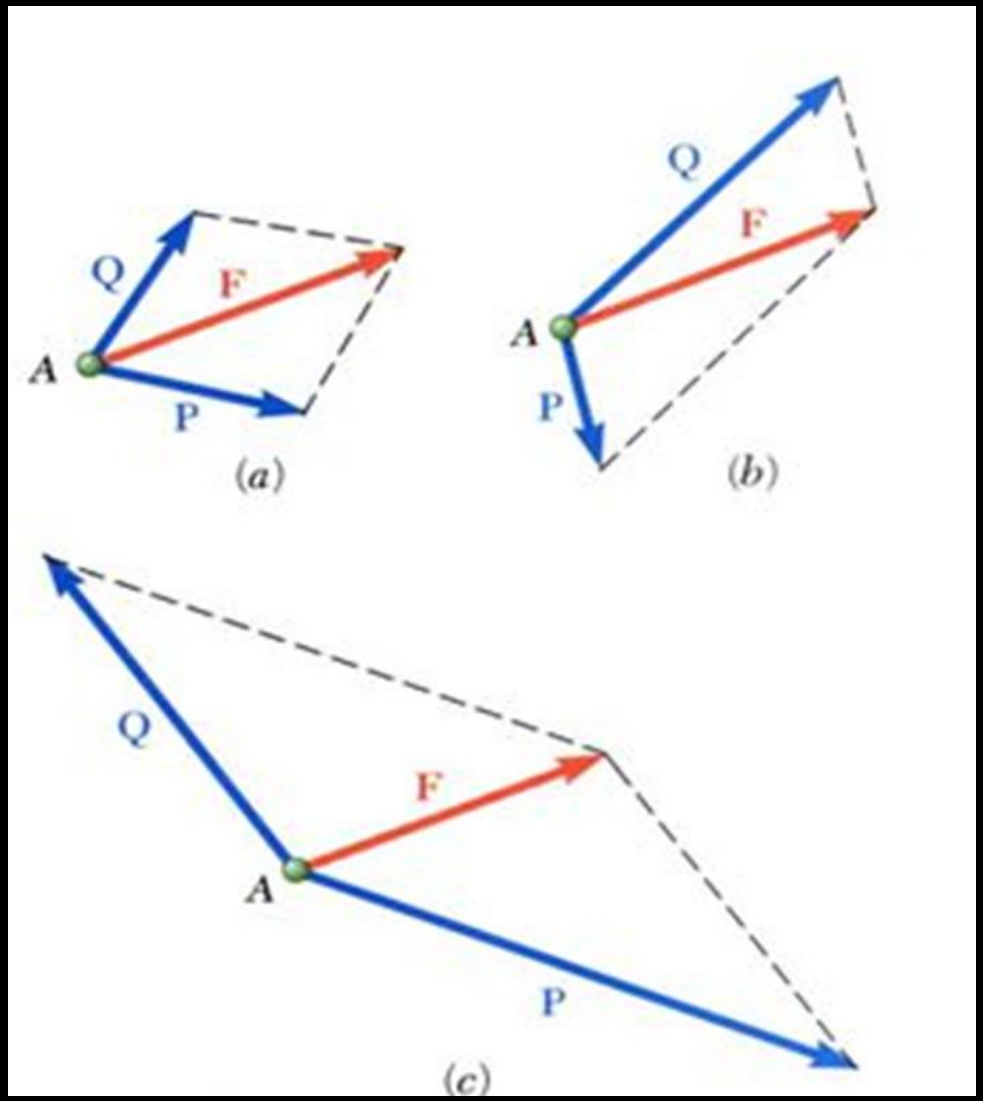
- The accuracy of the solution of a given problem depends on the accuracy of the

  - **Given data**
  - **Computations performed**

- The example of loading on a bridge say 34000 kg with a possible error of 45 kg either way.
- In most engineering problems the data are seldom known with an accuracy higher than 0.2 percent.
- A practical rule is to use 4 significant figures for values beginning with "1" and 3 for others. E.g. 55 kg mass could be read as 55.0kg and 18 kg could be read as 18.00 kg

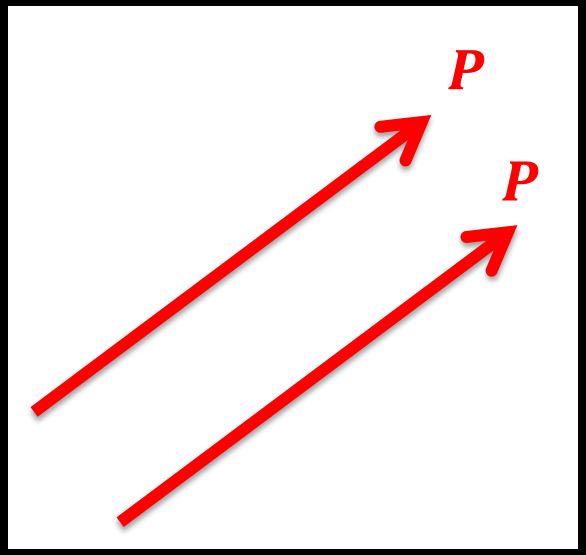
VECTORS OF FORCES IN 2D

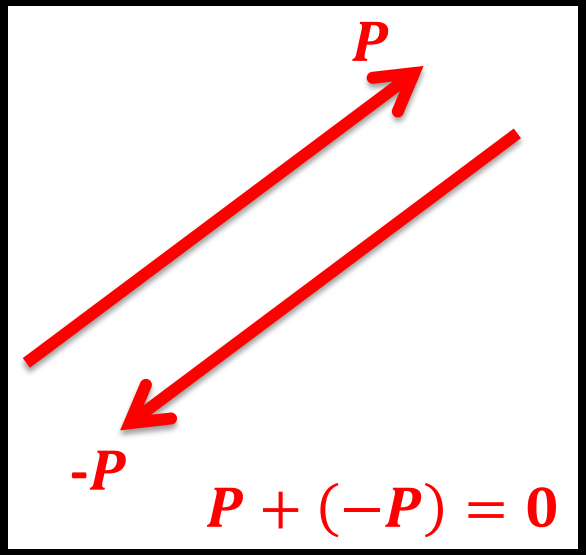
- **FORCE ON A PARTICLE. RESULTANT OF TWO FORCES**



VECTORS OF FORCES IN 2D

- VECTORS





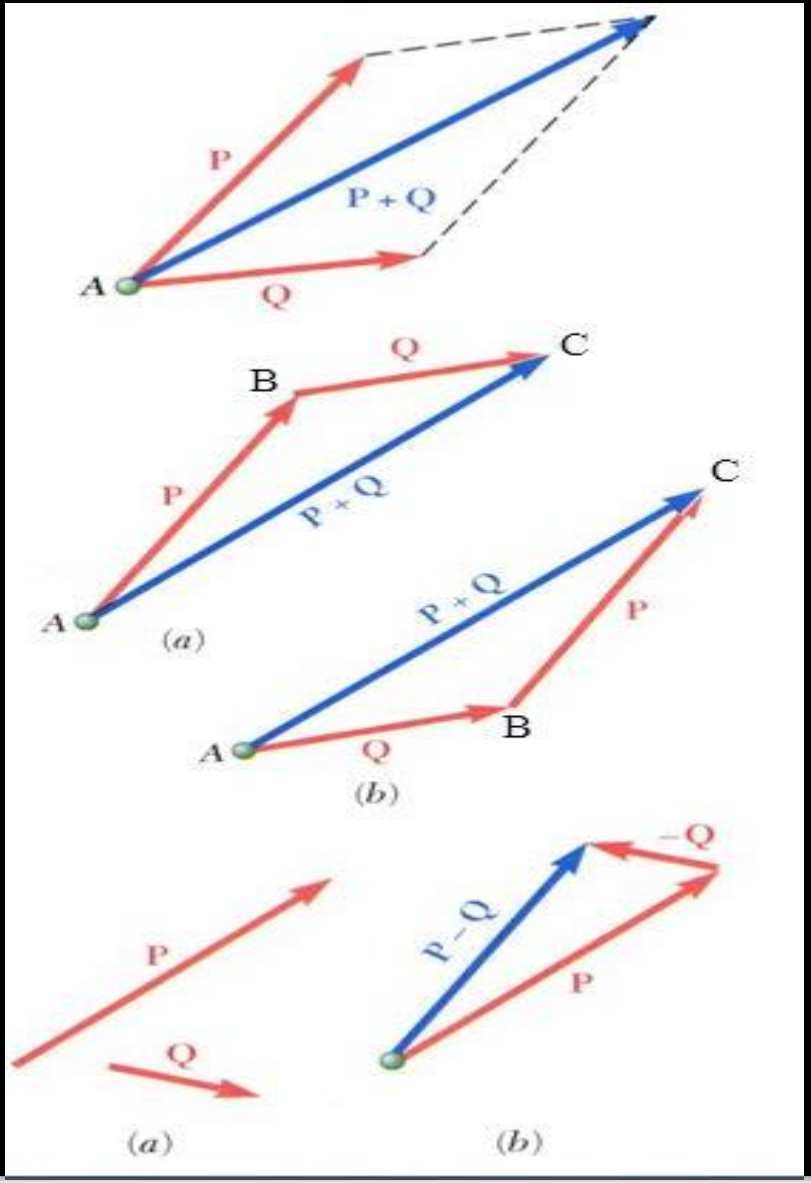
Vectors are mathematical expressions that possess magnitude and direction and add according to parallelogram law

Vector Classification

- Fixed or bound vectors (e.g force acting on a particle)
- Free vectors (e.g. couples)
- Sliding vectors (e.g force acting on a rigid body)

VECTORS OF FORCES IN 2D

- ADDITION OF VECTORS



- Parallelogram law of addition
- Triangle rule
- Law of cosines

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$\boldsymbol{R = P + Q}$$

- Law of sines

$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative

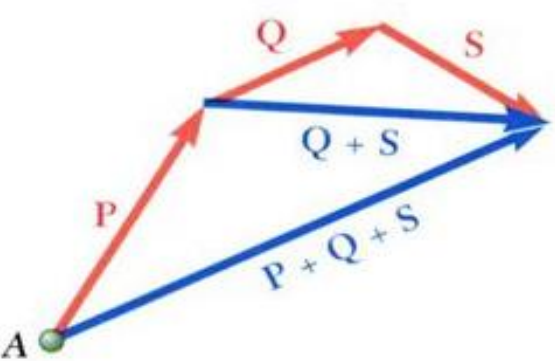
$$\boldsymbol{P + Q = Q + P}$$

- Vector subtraction

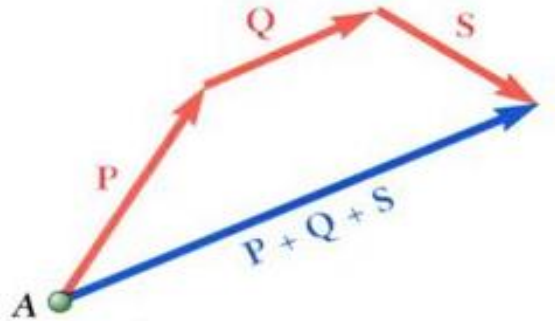
$$\boldsymbol{P + (-Q)}$$

VECTORS OF FORCES IN 2D

- ADDITION OF VECTORS

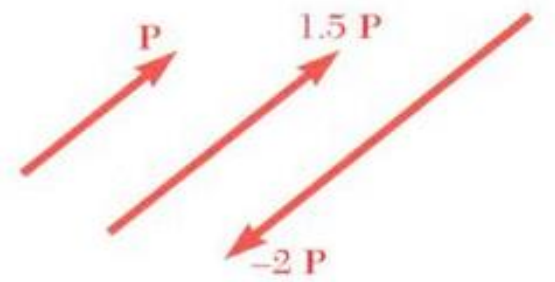


- Addition of three of more vectors by repeated application of triangle rule



- The polygon rule for the addition of three or more vectors
- Vector addition is associative

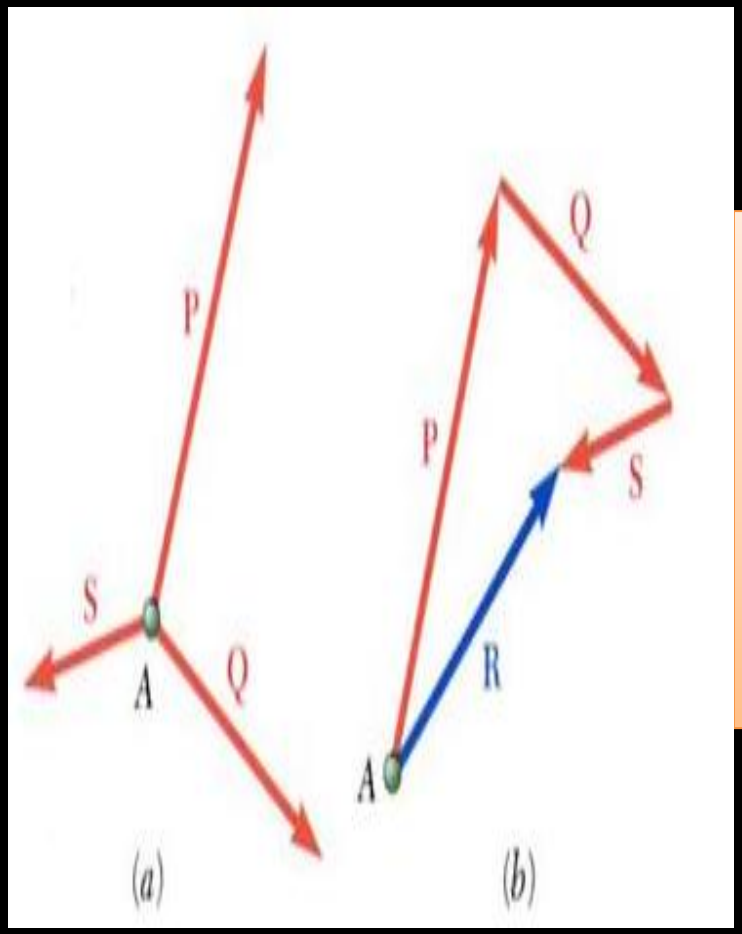
$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = \mathbf{P} + (\mathbf{Q} + \mathbf{S}) = (\mathbf{P} + \mathbf{Q}) + \mathbf{S}$$



- Multiplication of a vector by a scalar

VECTORS OF FORCES IN 2D

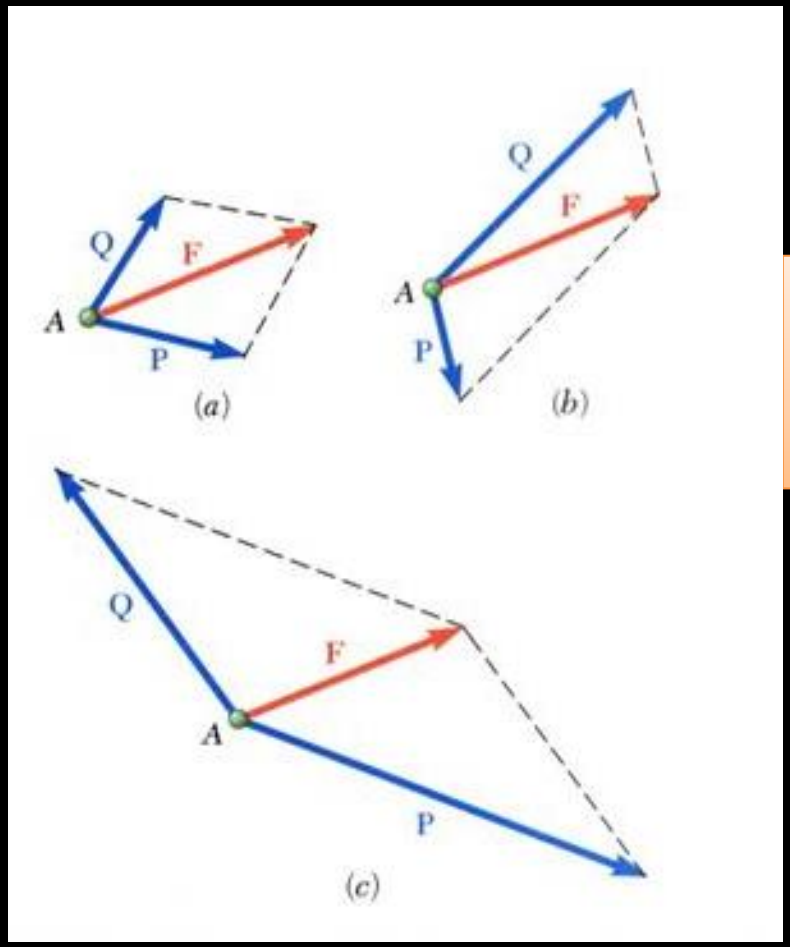
- **RESULTANT OF SEVERAL CONCURRENT FORCES**



- Concurrent forces: set of forces that pass through the same point.
- A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

VECTORS OF FORCES IN 2D

- **RESOLUTION OF A FORCE INTO COMPONENTS**

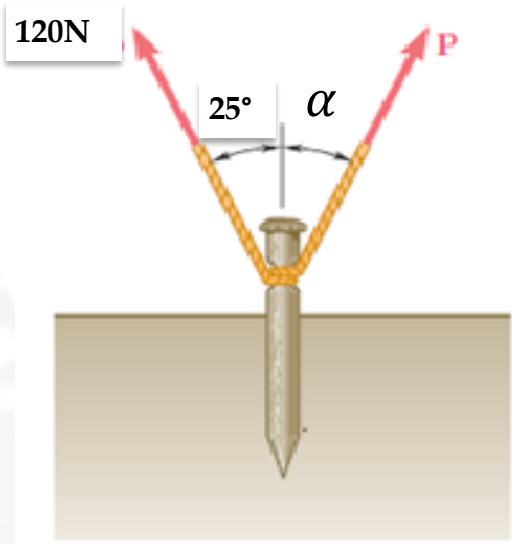


- Vector force components: two or more force vectors which, together has the same effect as a single vector force

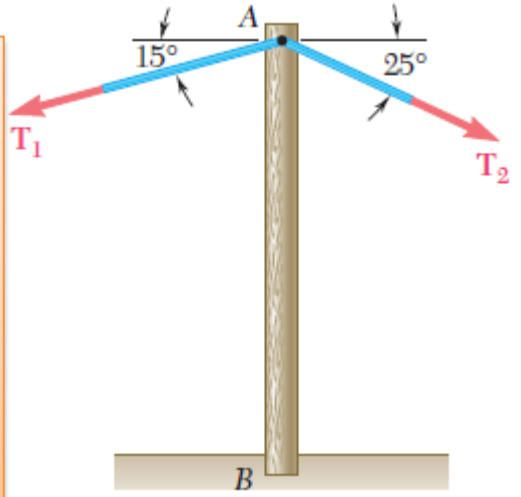
VECTORS OF FORCES IN 2D

EXAMPLES

1. A stake is being pulled out of the ground by means of two ropes as shown. Knowing that a $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

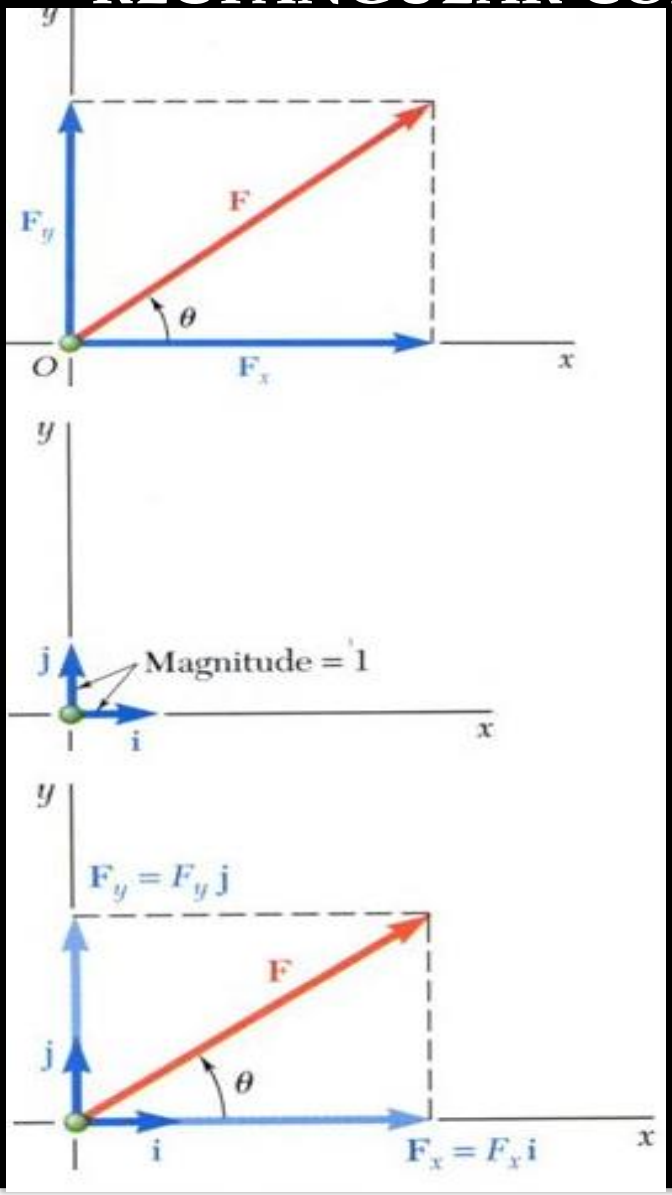


2. A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the left-hand portion of the cable is $T_1 = 362.8$ kg, determine by trigonometry (*a*) the required tension T_2 in the right-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.



VECTORS OF FORCES IN 2D

RECTANGULAR COMPONENTS OF A FORCE: UNIT VECTORS



- It is possible to resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are referred to as ***rectangular vector components*** and

$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y$$

- Define perpendicular ***unit vectors*** $\boldsymbol{i}$ and $\boldsymbol{j}$ which are parallel to the x and y axes.

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$$

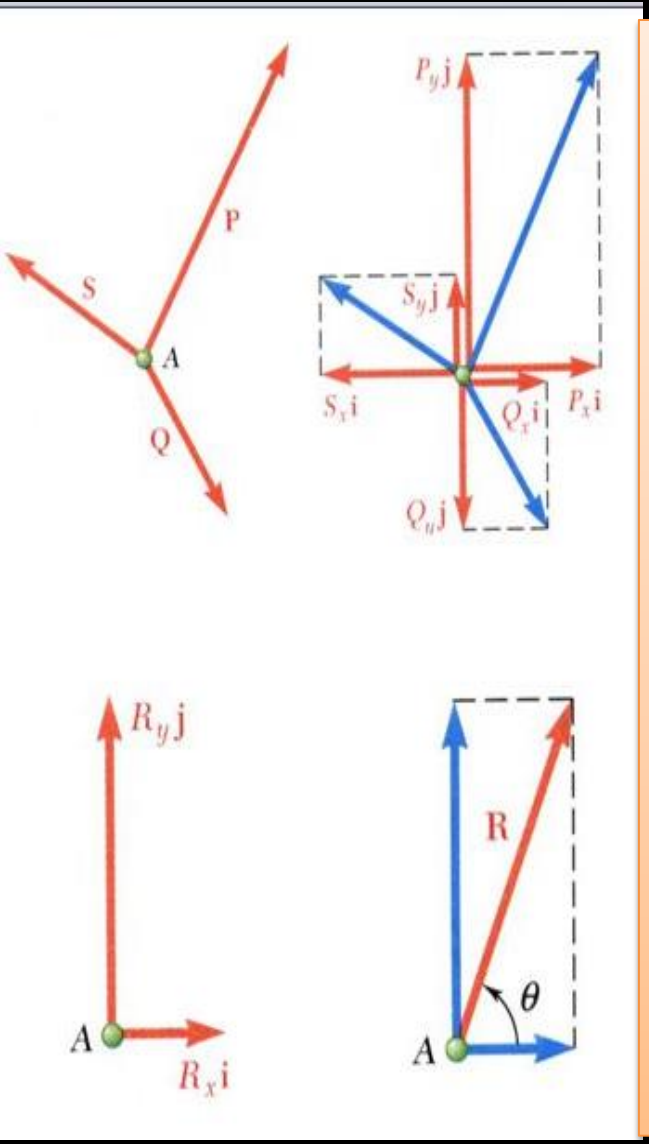
- F_x and F_y are referred to as ***scalar components*** of $\boldsymbol{F}$

VECTORS OF FORCES IN 2D

3. A force of 800 N is exerted on a bolt, A, 145° measured from the horizontal. Determine the horizontal and vertical components of the force.
4. A man pulls with a force of 300 N on a rope attached to a building. The man is positioned 8 m away horizontally and 6 m at the base of the building (vertically) from the point of attachment, A. What are the horizontal and vertical components of the force exerted by the rope at point A?
5. A force $\boldsymbol{F} = (318\,\boldsymbol{i} + 680\,\boldsymbol{j})$ N is applied to a bolt A. Determine the magnitude of the force and the angle α it forms with the horizontal.

VECTORS OF FORCES IN 2D

ADDITION OF FORCES BY SUMMING COMPONENTS



- The resultant of 3 or more concurrent forces is $\boldsymbol{R = P + Q + S}$
- Resolve each force into the rectangular components

$$R_x\boldsymbol{i} + R_x\boldsymbol{j} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + S_x\boldsymbol{i} + S_y\boldsymbol{j}$$
$$= (P_x + Q_x + S_x)\boldsymbol{i} + (P_y + Q_y + S_y)\boldsymbol{j}$$

- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces

$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y$$
$$\sum F_x \qquad \sum F_y$$

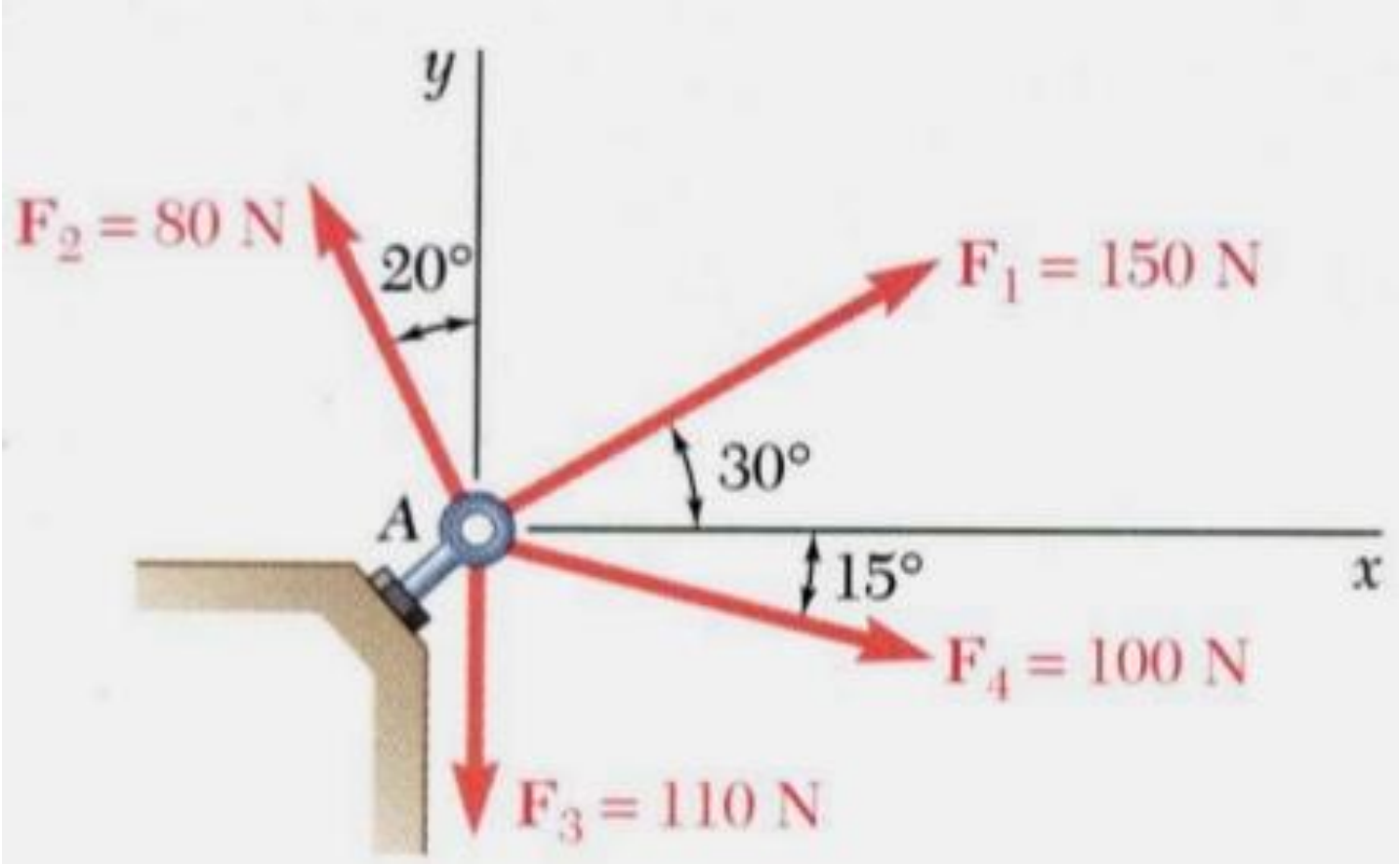
- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

VECTORS OF FORCES IN 2D

EXAMPLES

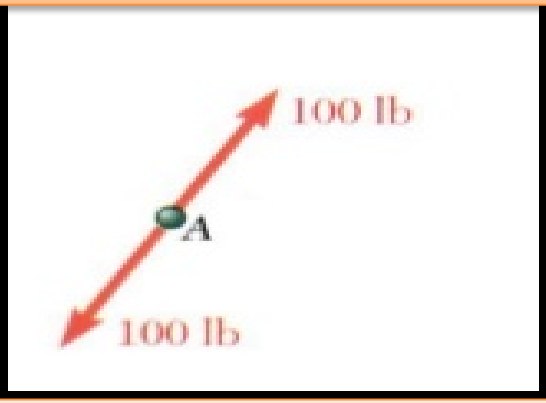
6. Four forces act on a bolt as shown in the Fig below. Determine the resultant of the forces on the bolt

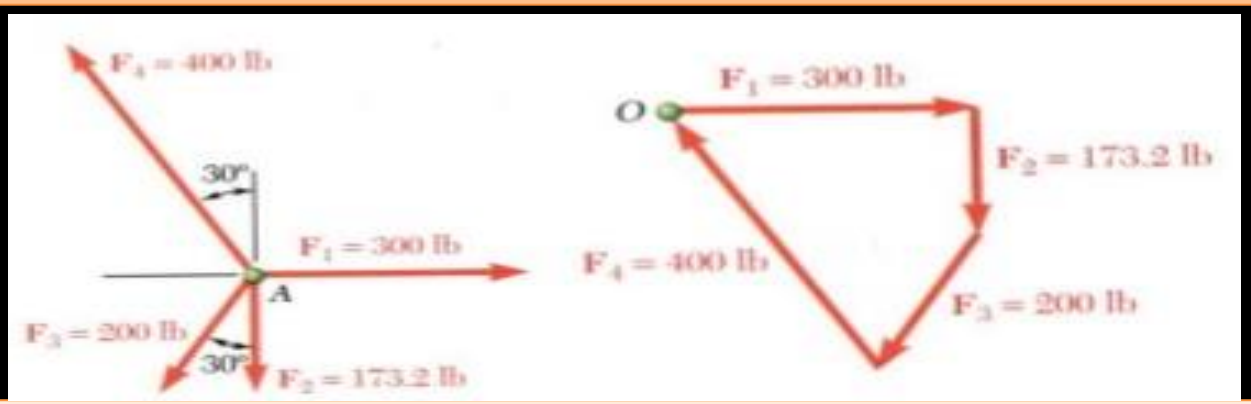


VECTORS OF FORCES IN 2D

EQUILIBRIUM OF A PARTICLE

- When the resultant of all the forces acting on a particle is zero, the particle is in ***equilibrium***
- Newton's first law: If the resultant forces acting on a particle is zero, the particle will remain at rest, or will move with constant speed in a straight line





- Particles acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite direction

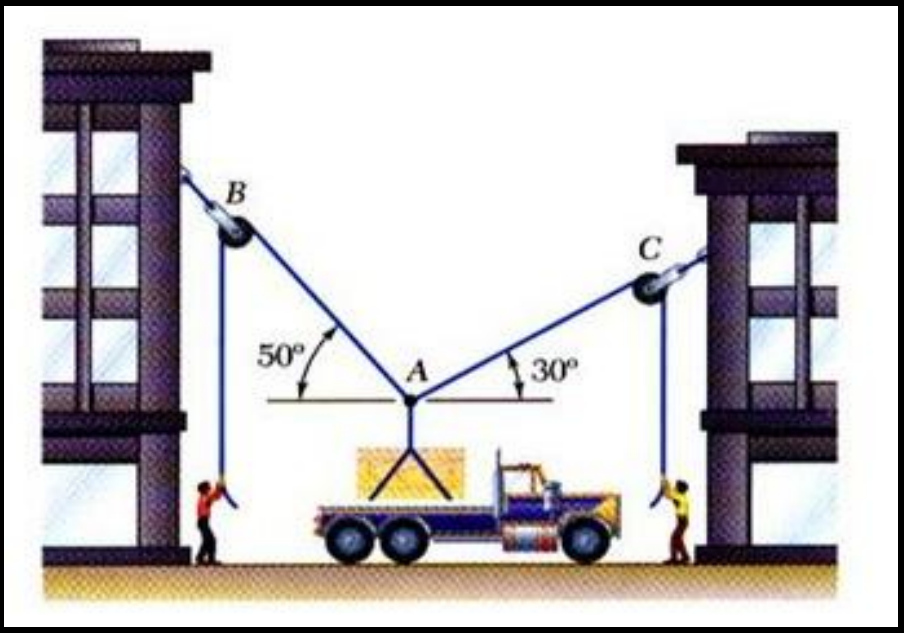
- Particle acted upon by three or more forces:
  - Graphical solution yields a closed polygon
  - algebraic solution

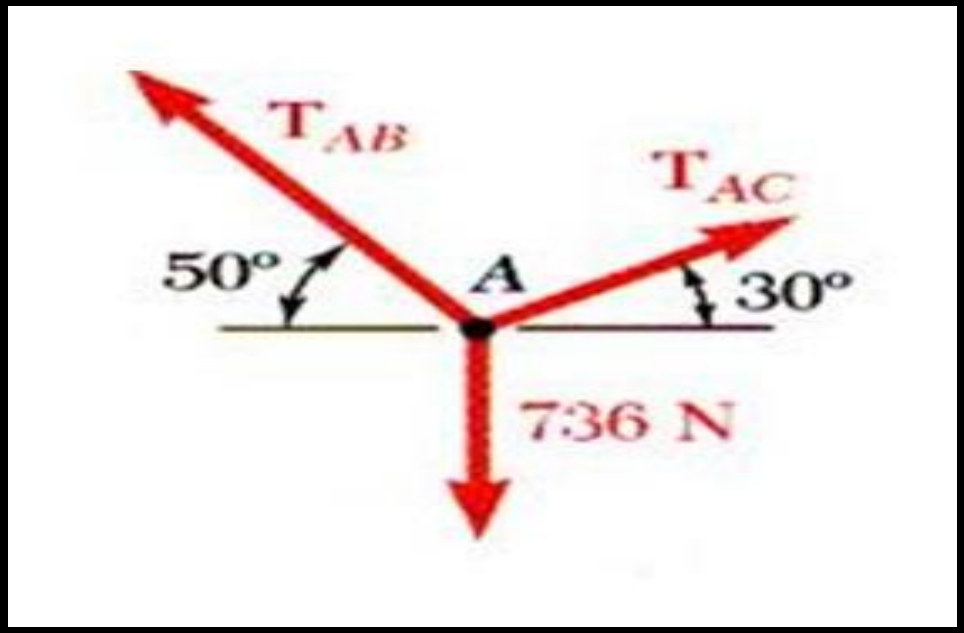
$$\boldsymbol{R} = \sum \boldsymbol{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

VECTORS OF FORCES IN 2D

- FREE-BODY DIAGRAMS





- *Space diagram:* A sketch the physical conditions of the problem, usually provided with the problem statement, or represented by the actual physical situation

- *Free Body Diagram:* A sketch showing only forces on the selected particle.
  This must be created by you

VECTORS OF FORCES IN 2D

EXAMPLES

7. In a ship-unloading operation, a 1587.57 kg automobile is supported by a cable. A rope is tied to the cable at *A* and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2°, while the angle between the rope and the horizontal is 30°. What is the tension in the rope?