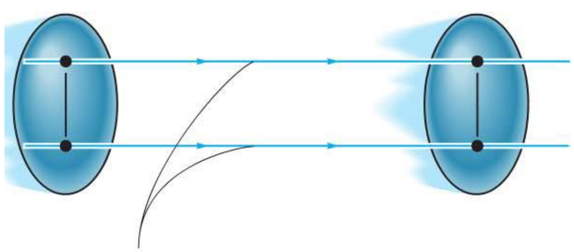
GEC 241-APPLIED MECHANICS II-DYNAMICS

PLANAR KINEMATICS OF RIGID BODIES

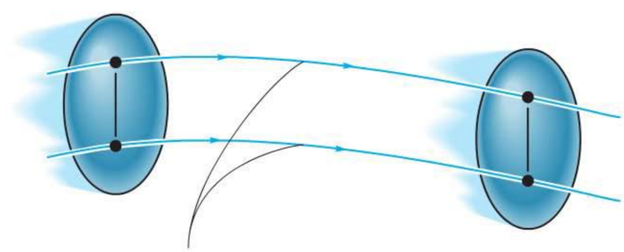
This study is important for the design of gears, cams and mechanisms used for many mechanical operations. The application of the equations of motion is normally used once the kinematics has been understood which will relate the forces on the body to the body's motion.

The planar motion of a body occurs when all the particles of a rigid body move along paths which are equidistant from a fixed plane. There are three types of rigid body planar motion in order of increasing complexity, they are:

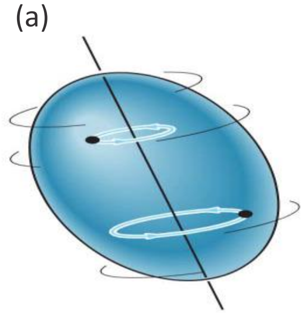
1. Translation: It occurs when a line in the body remains parallel to its original orientation throughout the motion. When the paths of motion for any two points on the body are parallel lines, he motion is called rectilinear translation. If the paths of motion are along curved lines which are equidistant, the motion is called curvilinear translation.
2. Rotation about a fixed axis: When a rigid body rotates about a fixed axis, all the particles of the body except those which lie on the axis of rotation move along circular paths.
3. General plane motion: When a body is subjected to general plane motion, it undergoes a combination of translation and rotation. The translation occurs within a reference plane and the rotation occurs about an axis perpendicular to the reference plane.



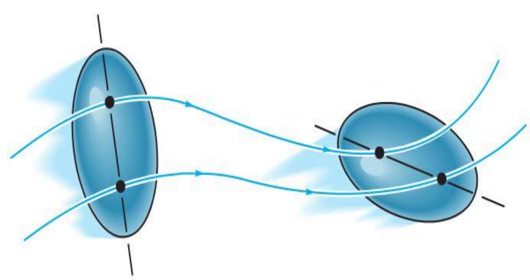
Path of rectilinear translation
(a)



Path of curvilinear translation
(b)



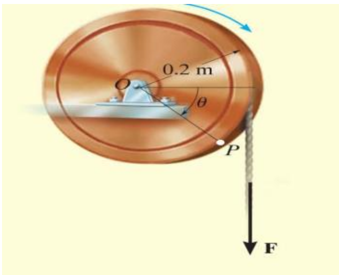
Rotation about a fixed axis
(c)



General plane motion
(d)

NB: Review first week lecture on rectilinear motion

Example1: A cord is wrapped around a wheel which is initially at rest when $\theta = 0$. If a force is applied to the cord and gives it an acceleration $a = (4t)\, m/s^2$, where t is in seconds, determine, as a function of time, (a) the angular velocity of the wheel and (b) the angular position of line OP in radians.



Solution

The point P on the wheel has motion about a circular path and the acceleration of this point has both tangential and normal components. The tangential component is $(a_p)_t = (4t)\, m/s^2$, since the cord is wrapped around the wheel and moves tangent to it. Hence the angular acceleration of the wheel is

$$(a_p)_t = \alpha r$$

$$(4t)\, m/s^2 = \alpha(0.2m)$$

$$\alpha = (20t)\, rad/s^2$$

Using the result above, the wheel's angular velocity ω can now be determined

$$\alpha = \frac{d\omega}{dt} = (20t)\, rad/s^2$$

$$\int_0^{\omega} d\omega = \int_0^{t} 20t\, dt$$

$$\omega = 10t^2\, rad/s$$

$$\omega = \frac{d\theta}{dt} = (10t^2)\, rad/s$$

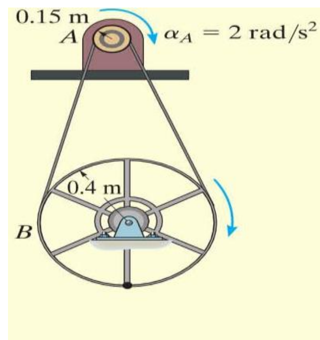
$$\int_0^{\theta} d\theta = \int_0^{t} 10t^2\, dt$$

$$\theta = 3.33t^3\, rad$$

Example 2: The motor shown in the photo is used to turn a wheel and attached blower contained within the housing. The details of the design are shown in the figure (a). If the pulley A connected to the motor begins to rotate from rest with a constant angular acceleration of $\alpha_A = 2\, rad/s^2$, determine the magnitudes of the velocity and acceleration of point P on the wheel, after the pulley has turned two revolutions. Assume the transmission belt does not slip on the pulley and wheel.

Solution

Firstly, convert the two revolutions to radians. Note that there are 2π rad in one revolution, then



$\theta_A = 2\,rev\left(\frac{2\pi\,rad}{1\,rev}\right) = 12.57rad$

Since α_A is constant, the angular velocity of pulley A is therefore

$$\omega^2 = \omega_0^2 + 2a_c(\theta - \theta_0)$$

$$\omega_A^2 = 0 + 2(2\,rad/s^2)(12.57\,rad - 0)$$

$$\omega_A = 7.090\,rad/s$$

The belt has the same speed and tangential component of acceleration as it passes over the pulley and wheel. Thus,

$$v = \omega_A r_A = \omega_B r_B$$

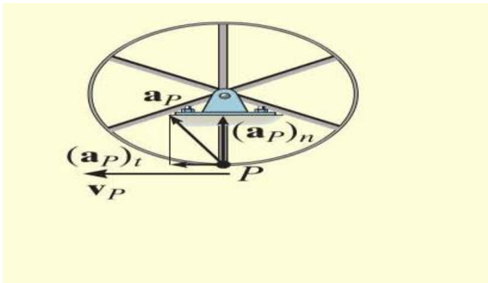
$$7.090\,rad/s\ (0.15m) = \omega_B(0.4m)$$

$$\omega_B = 2.659\,rad/s$$

$$a_t = \alpha_A r_A = \alpha_B r_B;\ 2\,rad/s^2\ (0.15m) = \alpha_B(0.4m)$$

$$\alpha_B = 0.75\,rad/s^2$$

Motion of P. As shown on the kinematics diagram, we have



$$v_p = \omega_B r_B = 2.659\, rad/s\, (0.4m) = 1.06\, m/s$$

$$(a_p)_t = \alpha_B r_B = 0.75\, rad/s^2\, (0.4m) = 0.3\, m/s^2$$

$$(a_p)_n = \omega_B^2 r_B = (2.659\, rad/s)^2 (0.4m) = 2.827\, m/s^2$$

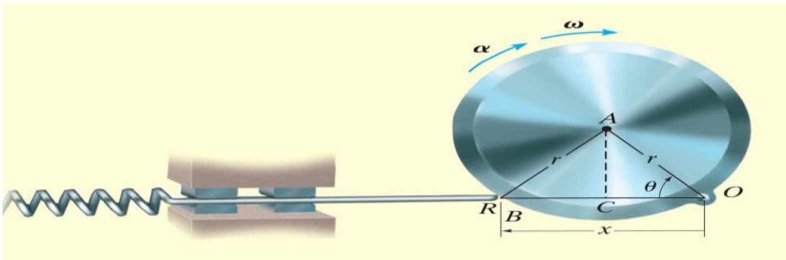
Thus,

$$a_p = \sqrt{0.3^2 + 2.827^2} = 2.84\, m/s^2$$

ABSOLUTE MOTION ANALYSIS

A body subjected to general plane motion undergoes a simultaneous translation and rotation. If the body is represented by a thin slab, the slab translates in the plane of the slab and rotates about an axis perpendicular to this plane. The motion can be completely specified by knowing both the angular rotation of a line fixed in the body and the motion of a point on the body. One way to relate this is to use a rectilinear position coordinate s to locate the point along its path and an angular position coordinate θ to specify the orientation of the line. The two coordinates are then related using the geometry of the problem. By direct application of the time-differential equations $v = ds/dt,\ a = dv/dt\,, \omega = d\theta/dt\ and\ \alpha = d\omega/dt$, the motion of the point and the angular motion of the line can then be related. However, this same procedure may be used to relate the motion of one body undergoing either rotation about a fixed axis or translation to that of a connected body undergoing general plane motion.

Example 3: The end of rod R shown in the figure below maintains contact with the cam by means of a spring. If the cam rotates about an axis passing through point O with an angular acceleration α and angular velocity ω , determine the velocity and acceleration of the rod when the cam is in the arbitrary position .



Position Coordinate Equation. Coordinates θ and x are chosen in order to relate the rotational motion of the line segment OA on the cam to the rectilinear translation of the rod. These coordinates are measured from the fixed point 0 and can be related to each other using trigonometry. Since OC = CB = r cos θ, then

$$x = 2r\ cos\theta$$

Time Derivatives.

Using the chain rule of calculus, we have

$$\frac{dx}{dt} = -2r(sin\theta)\frac{d\theta}{dt}$$

$$v = -2r\omega sin\theta$$

$$\frac{dv}{dt} = -2r\left(\frac{d\omega}{dt}\right)sin\theta - 2r\omega(cos\theta)\frac{d\theta}{dt}$$

$$a = -2r(\alpha sin\theta + \omega^2 cos\theta)$$

NOTE: The negative signs indicate that v and a are opposite to the direction of positive x. This seems reasonable when you visualize the motion.