

LECTURE NOTE

ON

THERMODYNAMICS (GEC 221)

Thermodynamics is the branch of science that treats the various phenomena of energy and related properties of matter especially the relationship between heat, work and properties of systems.

BASIC CONCEPTS AND DEFINITIONS

System: Any collection of matter contained within prescribed boundary, which is of interest for a particular study or analysis, is referred to as a system. A boundary may be physical or imaginary (either fixed or moving). The boundary separates the system from its surroundings or environments. There are two kinds of system, closed and open system.

Closed System: A closed system is that which allows no exchange of matter with the surroundings. A good example of a closed system is the gas contained within a cylinder that is closed at one end and a movable piston as shown in Figure 1.1. The inner surface of the piston and cylinder form the boundary of this system and the boundary is a moving one.

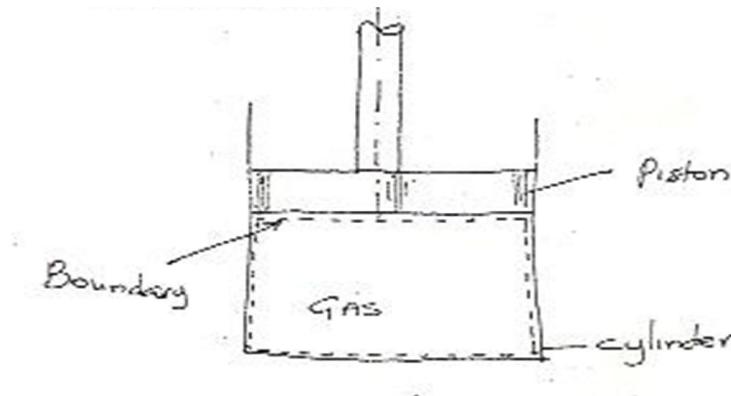


Fig. 1.1 Closed System

Open system: An open system is that which allows exchange of matter with the surroundings. i.e there is flow of mass in addition to work and heat across its boundary. With open systems the boundary is normally specified as a control surface and the volume encompassed by the surface as the control volume. The mass of matter within the control volume may be constant (though not the same molecular matter at any given instant). An example of an open system with constant mass is a water nozzle and that with varying mass is air in a rubber tube undergoing inflation.

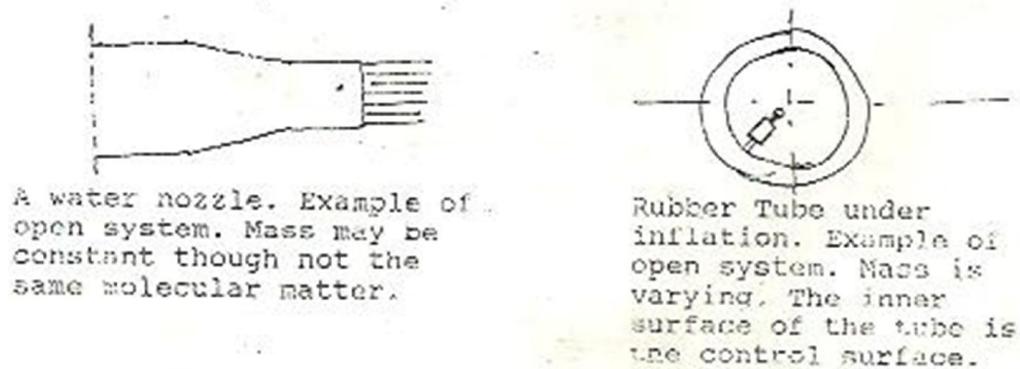


Fig. 1.2 Open System

State: At any instant or time a system is in a condition called state, which encompasses all that can be said about the result of any measurement and observation that can be performed on the system at that time.

Process: That which changes the state of a system.

Path: path of a process is the series of state points passed through during process.

Cyclic process: is one in which the final state of the system is identical with its initial state. Such a system is said to have undergone a cycle.

Property: any observable characteristics of a system is referred to as property of the system. The thermodynamic properties are pressure, volume, temperature, internal energy, enthalpy and entropy. Properties may be classified as **intensive or extensive** as they are independent or dependent of mass of the system. For example, temperature, pressure, specific volume and density are intensive properties, while total volume, total mass, total energy in a system, internal energy, enthalpy and entropy are extensive properties. If a change in the value of a property depends only on the initial and final states of the system, and is independent of the process undergone by the system during a change of state, it is called a state/ point function. e.g temperature, pressure and entropy.

State functions are written as exact differentials e.g pressure $\int_1^2 dp = P_2 - P_1$. **Path functions** have their values depend on the path followed during the process of changing their states i.e their history is important in determining their values in the final equilibrium state. Examples are heat and work written as inexact/partial differential as ∂Q and ∂w respectively. e.g heat $\int_1^2 \partial Q \neq Q_2 - Q_1$ but $\int_1^2 \partial Q = Q_{1-2}$

Specific properties: are those for a unit mass and are extensive by definition e.g. specific volume.

Energy: The general term is the capacity to produce an effect. It appears in many forms which are related to each other by the fact that the conversion can be defined with the precision.

Internal Energy: the sum of various forms of energy that a system has is the internal energy of the system. It is a property of the system. The absolute amount of internal energy that a system has is never known, but internal energy changes can be measured from any convenient datum.

Isolated System : is one that is completely impervious to its surrounding neither mass nor energy cross its boundary.

PROCESSES AND CYCLE

Thermodynamic Processes: is the path of successive states through which a system passes in changing its states. When the processes are performed on a system is such a way that the final state is identical with the initial state, the closed system is said to have undergone a thermodynamic cycle or cyclic process.

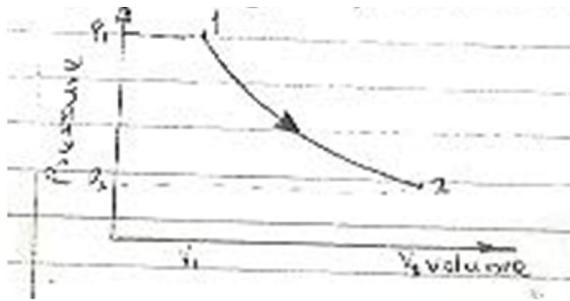


Fig. 1.3 Thermodynamic process

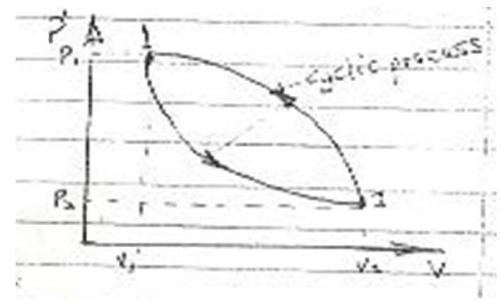


Fig. 1.4 Thermodynamic Cycle

REVERSIBILITY

Reversible Process: May be defined as a process between two states during which the system passes through a series of equilibrium states. A reversible process between two states can be drawn as a line on any diagram of properties. When a fluid undergoes a reversible process, both the fluid and its surroundings can always be restored to their original states. In practice, the fluid undergoing a process cannot be kept in its intermediate states and a continuous path cannot be traced on diagram of properties. Such a real process is called **irreversible process** and is usually represented by a dotted line joining the end states to indicate the intermediate states are indeterminate.

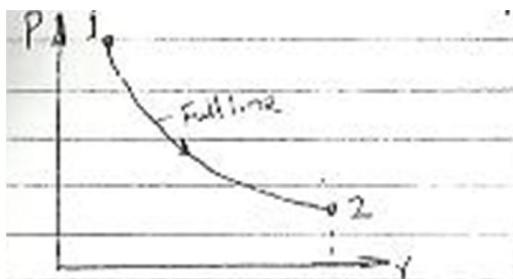


Fig. 1.5 Reversible process



Fig. 1.6 Irreversible process

The criteria for reversibility include:

- (a) Absence of friction i.e no internal or mechanical friction
- (b) No finite pressure difference between the fluid and its surroundings (i.e infinitely slow process)
- (c) No heat transfer across a finite temperature difference.

From the criteria stated above, it is apparent that no real process is reversible but a close approximation to an internal reversibility may be obtained in many practical processes.

TEMPERATURE

Temperature is an intensive property, which determines the degree of hotness or the level of heat intensity of a body. When a hot body and a cold body are brought in contact with each other, heat energy is transferred from the hot body to the cold body until the two bodies are said to have equal temperature if there is no change in any of their observable properties.

If a body A has the same temperature with body B when brought together and body B when in contact with a third body C shows no change in any observable characteristics, then bodies A and C when brought into contact will also show no change in their characteristics. That is, if two bodies are each equal in temperature to a third body, they are equal in temperature to each other. This principle of thermal equilibrium is often called the **Zerth law of Thermodynamics**.

Scale of Temperature: A temperature scale is an arbitrary thing. The Fahrenheit and Celsius (Centigrade) scales are based on melting and boiling points for water at 1 atmosphere. The respective temperatures are 32 and 212 degrees in the former and 0 and 100 degrees in the latter. The thermodynamic temperature scale which is measured from a point of absolute zero will be discussed later.

Thermometer: is a system with a readily observable characteristics termed a thermometric property. The systems are equal in temperature when there is no change in any property when they are brought into contact.

WORK AND HEAT

For a closed system to undergo change of state, this must be accompanied by the appearance of work and heat at the boundary. In mechanics, work is defined as the product of force and distance (d) moved in the direction of the force.

Mathematically, work done $W = F \times d$, where Force = pressure \times area. The basic unit of work in SI system is in Nm or Joule. Work is a transient quantity which only appears at the boundary while a change of state is taking place within a system. Work can therefore be described in thermodynamics as a form of energy transfer which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.

By convention, work done by the system on the surroundings is taken to be positive i.e. $W_{out} = +ve$ e.g when a fluid expands within a cylinder pushing a piston outwards. Conversely, work done on the system by the surroundings is taken to be negative i.e. $W_{in} = -ve$ e.g when a piston compresses a fluid within a cylinder.

Heat can also be described in an analogous manner to work as a form of energy transfer that appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings. Heat is transitory energy like work and can never be contained or possessed by a body

By sign convention, heat flowing into a system from the surroundings is taken to be positive i.e $Q_{in} = +ve$ while heat flowing from the system to the surroundings is taken to be negative. i.e $Q_{out} = -ve$. It is worth noting that both heat and work are not thermodynamic properties and therefore they are path path functions whose values depend on the particular path followed during the process. Hence heat and work are written as inexact differentials in the forms ∂Q and ∂W respectively.

i.e $\int_1^2 \partial W = W_{1-2}$ and $\int_1^2 \partial Q = Q_{1-2}$

REVERSIBLE WORK

Consider an ideal frictionless fluid contained in a cylinder behind a piston. Assume the pressure and temperature of the fluid are uniform and that there is no friction between the piston and the cylinder walls.

Let the cross-sectional area of the piston be A , and the fluid pressure be p , let the pressure of the surroundings be $(p-dp)$. The restraining force exerted by the surroundings on the piston is $(p-dp)A$. Let the piston move under the action of the force exerted by the fluid a distance dl to the right. Then, the work done by the fluid on the piston is given by the force times the distance moved. i.e. Work done = $pA \times dl = pdv$

Where dv is a small increase in volume.

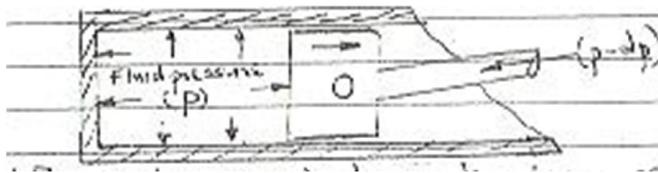


Fig. 1.7 Fluid in a cylinder undergoing expansion

Or for a mass m , $dW = mpdv$, where v is the specific volume. This is only true when criteria (a) and (b) for reversibility hold.

When a fluid undergoes a reversible process, a series state points can be joined up to form a line on a diagram of properties. The workdone by the fluid during any reversible process is therefore given by the area under the line of the process plotted on a p - v diagram as shown in Fig. 1.8.

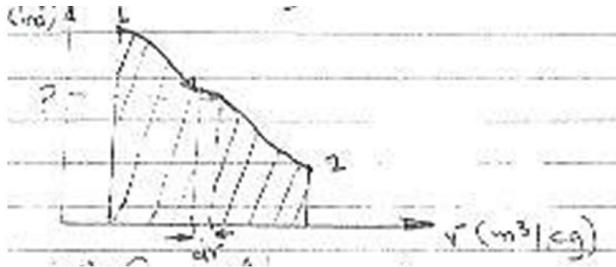
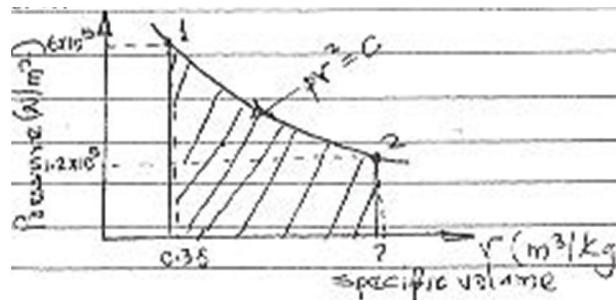


Fig. 1.8 Reversible expansion process on a p-v diagram

Example 1.1

A fluid of unit mass at an initial pressure 6 bar and specific volume of $0.36 \text{ m}^3/\text{kg}$ which is enclosed in a cylinder behind a piston undergoes a reversible expansion process according to a law $pv^2 = \text{constant}$. The final pressure is 1.2 bar. Calculate the work done during the expansion process.

Solution



Work done = shaded area = $m \int_1^2 p dv$ where $p = c/v^2$

$$W = mc \int_1^2 \frac{1}{v^2} dv = mc \left[-\frac{1}{v} \right]_{v_1}^{v_2} = mc \left[\frac{1}{v_1} - \frac{1}{v_2} \right]$$

$$\text{But } c = pv^2$$

$$\begin{aligned} c &= p_1 v_1^2 = 6 \times 10^5 \text{ N/m}^2 \times (0.36)^2 \text{ (m}^3/\text{kg)}^2 \\ &= 0.7776 \times 10^5 \text{ N/m}^2 \text{ (m}^3/\text{kg)}^2 \end{aligned}$$

$$\text{Also } p_2 v_2^2 = c \text{ and } v_2 = \sqrt{\frac{c}{p_2}}$$

$$v_2 = \sqrt{\frac{c}{p_2}} = \sqrt{\frac{0.7776 \times 10^5 \text{ N/m}^2}{1.2 \times 10^5 \text{ N/m}^2}} \text{ (m}^3/\text{kg)}^2$$

$$v_2 = 0.805 \text{ (m}^3/\text{kg)}^2$$

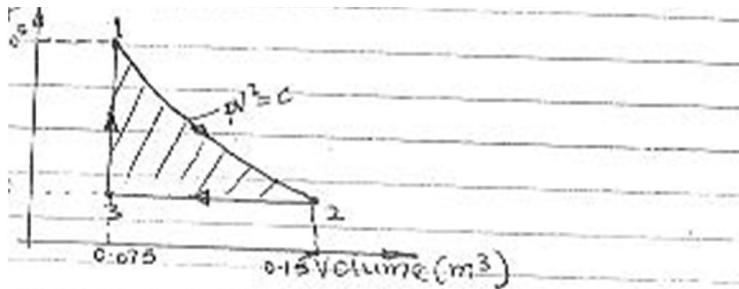
For unit mass $m = 1$

$$\begin{aligned} \therefore W &= C \left[\frac{1}{v_1} - \frac{1}{v_2} \right] = 0.7776 \times 10^3 \left[\frac{1}{0.36} - \frac{1}{0.805} \right] \frac{Nm^2(m^3/kg)^2}{m^3/kg} \\ &= 119403.73 \text{ Nm/kg} \\ &= 119.404 \text{ kJ/kg} \end{aligned}$$

Example 1.2

A fluid of unit mass which is enclosed in a cylinder at an initial pressure of 30 bar is allowed to undergo a reversible expansion process behind a piston according to a law $pv^2 = \text{constant}$ until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original position. With the piston firmly locked in position, heat is then supplied reversibly until the pressure rises to the original value of 30 bar. Calculate the net work done the fluid for an initial volume of 0.075 m^3

Solution



$$p_1 V_1^2 - p_2 V_2^2 = C$$

$$\begin{aligned} p_2 &= p_1 \left(\frac{v_1}{v_2} \right)^2 = 30 \times 10^5 \left(\frac{0.075}{0.15} \right)^2 = \frac{30 \times 10^5}{4} \\ &= 7.5 \times 10^5 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} C &= p_1 V_1^2 = 30 \times 10^5 \times 0.075^2 \text{ N/m}^2 \text{ m}^6 \\ &= 16875 \text{ N/m}^2 \text{ m}^6 \end{aligned}$$

$$\text{so, } c = pv^2, \text{ and } p = \frac{c}{v^2}$$

$$\begin{aligned} W_{12} &= \int_1^2 p dV = \int_1^2 \frac{c}{v^2} dV = \int_1^2 \frac{1}{v^2} dV = c \left[-\frac{1}{V} \right]_{v_1}^{v_2} \\ &= c \left[\frac{1}{V_1} - \frac{1}{V_2} \right] = 16875 \left[\frac{1}{0.075} - \frac{1}{0.15} \right] \text{ N/m}^2 \text{ m}^6 \frac{1}{\text{m}^3} \\ &= 112500 \text{ Nm} \end{aligned}$$

$$W_{23} = p_2 (v_3 - v_2) = 7.5 \times 10^5 (0.075 - 0.15) \text{ N/m}^2 \times \text{m}^3 = -56250 \text{ Nm}$$

$$= -56250 \text{ Nm}$$

$W_{31} = 0$ since the piston is locked in position $dV = 0$

$$\begin{aligned}\text{Net work done} &= W_{12} + W_{23} + W_{31} \\ &= 112500 - 56250 + 0 \\ &= 56250 \text{ Nm}\end{aligned}$$

Hence the net work done by the fluid is 56.25 kJ

PRACTICE QUESTIONS

1. A fluid at 10 bar is enclosed in a cylinder behind a piston, the initial volume being 0.05 m³. Calculate the work done by the fluid when it expands reversibly according to a law $pv^3 = C$ to a final volume of 0.06 m³ (7640 Nm)
2. 1 kg of a fluid is compressed reversibly according to a law $pv = 0.25$, where p is in bar and v is in m³/kg. The final volume is one-fourth of the initial volume. Calculate the work done on the fluid and sketch the process on a p - v diagram (3466 Nm)
3. 0.05 m³ of a gas at 6.9 bar expands reversibly in a cylinder behind a piston according to the law $pv^{1.2} = \text{Constant}$, until the volume is 0.08 m³. Calculate the work done by the gas and sketch the process on a p - v diagram (15480 Nm)
4. A fluid is heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of 0.1 m³/kg. It is then compressed reversibly according to a law $pv = \text{Constant}$ to pressure of 4.2 bar, then allowed to expand reversibly according to a law $pv^{1.7} = \text{Constant}$, and is finally heated at constant volume back to the initial conditions. The work done in the constant pressure process is 1515 Nm, and the mass of fluid present is 0.2 kg. Calculate the network of the cycle and sketch the cycle on a p - v diagram (781 Nm)

2.0 THE FIRST LAW OF THERMODYNAMICS

The first law is concerned with the principle of conservation of energy as applied to closed systems which undergo changes of state due to transfer of work and heat across the boundary. It is in fact, a statement of the law of conservation of energy as applied to heat energy and mechanical work that energy can neither be created nor destroyed though it can be transformed from one form to another.

The law states that, when a closed system is taken through a thermodynamic cycle, the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings.

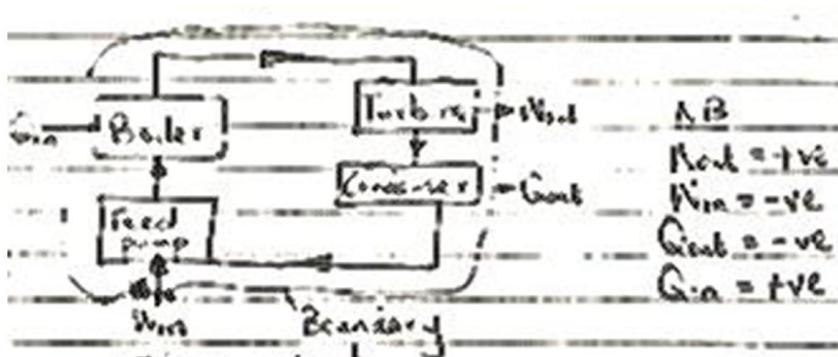
Mathematically, $\Sigma \partial Q = \Sigma \partial W$

Where Σ represents summation for a complete cycle. This implies that network done can never be greater than the heat supplied.

EXAMPLE 2.1

In a certain steam plant the turbine develops 2000 kW. The heat supplied to the steam in the boiler is 3800 kJ/kg, the heat rejected by the steam to the cooling water in the condenser is 3100 kJ/kg and the feed-pump work required to pump the condensate back into the boiler is 10 kW. Calculate the steam flow rate.

Solution



Steam Plant

$$\Sigma \partial Q = Q_{in} - Q_{out} = 3800 - 3100 = 700 \text{ kJ/kg}$$

Let the steam flow rate be \dot{m} kg/s

Then, $\Sigma \partial Q = \dot{m} \times 700 = 700 \dot{m} \text{ kW}$

$$\Sigma \partial W = W_{\text{out}} - W_{\text{in}} = 2000 - 10 = 1990 \text{ kW}$$

But $\Sigma \partial Q = \Sigma \partial W$

$$700 \dot{m} = 1990$$

$$\dot{m} = 1990/700 = 2.84 \text{ kg/s}$$

i.e. mass flow rate of steam required = 2.84 kg/s

COROLLARIES OF THE FIRST LAW

Corollaries are basic propositions or inferences about the behaviour of thermodynamic systems deduced from the laws of thermodynamics. Corollaries of the first law of thermodynamics are stated as follow:

COROLLARY 1

There exists a property of a closed system such that a change in its value is equal to the difference between the heat supplied and the work done during any change of state.

Mathematically, $\Sigma (\partial Q - \partial W) = du$

Where U denotes the property so discovered called internal energy of the system.

i.e. $Q - W = (U_2 - U_1)$ for a non-flow process.

This equation is called **non-flow energy equation (NFEE)**

Internal energy, which is the intrinsic energy of a body not in motion, being a property, can be said to reside in the system and it can be increased or decreased by a change of state.

COROLLARY 2

The internal energy of a closed system remains unchanged if the system is isolated from its surroundings.

COROLLARY 3

A perpetual motion machine of the first kind is impossible. That is, a system cannot produce work continuously without absorbing heat-energy from the surroundings.

NON FLOW ENERGY EQUATION

From the first corollary of the first law, the following can be rightly deduced.

Gain in internal energy = Net heat supplied - Network done

$$\text{Mathematically, } dU = \sum_1^2 \partial Q - \sum_1^2 \partial W$$

For a single process between state 1 and state 2, we have, $U_2 - U_1 = Q - W$ for a non-flow process or in differential form, $dU = \partial Q - \partial W$, where U is the specific internal energy in kJ/kg

This equation is true for both reversible and irreversible processes. For reversible non-flow process $dw = pdv = mpdv$

But $\partial Q = dU + \partial W = du + pdv$ for unit mass

$$Q = (u_2 - u_1) + \int_1^2 pdv$$

The above equation is true for ideal reversible non-flow processes.

EXAMPLE 2.2

In the compression stroke of an internal combustion engine, the heat rejected to the cooling water is 145 kJ/kg and the work input is 190 kJ/kg. Calculate the change in specific internal energy of the working fluid stating whether it is a gain or loss.

SOLUTION

$$Q_{\text{out}} = 145 \text{ kJ/kg}, W_{\text{in}} = 190 \text{ kJ/kg}$$

$$U_2 - U_1 = Q - W$$

$$= -145 - (-190)$$

$$= -145 + 190$$

$$= 45 \text{ kJ/kg}$$

$$\text{gain in internal energy} = 45 \text{ kJ/kg}$$

Example 2.3

In the cylinder of an air motor, the compressed air has a specific internal energy of 4200 kJ/kg at the beginning of the expansion and a specific internal energy of 2050 kJ/kg after expansion. Calculate the heat flow to or from the cylinder when the work done by the air during the expansion is 1050 kJ/kg.

SOLUTION

$$U_1 = 4200 \text{ kJ/kg}, U_2 = 2050 \text{ kJ/kg}, W_{\text{out}} = +1050 \text{ kJ/kg}$$

$$U_2 - U_1 = Q - W$$

$$2050 - 4200 = Q - 1050$$

$$-2150 = Q - 1050$$

$$Q = -2150 + 1050$$

$$Q = -1100 \text{ kJ/kg}$$

i.e. heat rejected by the air = 1100kJ/kg

PRACTICE QUESTIONS

1. An air compressor which compresses air at constant internal energy rejects 80 kJ of heat to the cycling water for every kilogram of air. Calculate the work done during compression stroke per kilogram of air (80 kJ/kg)
2. In the compression stroke of a gas engine the work done on the gas by the piston is 90 kJ/kg and the heat rejected to the cooling water is 52 kJ/kg. Calculate the change in specific internal energy (38 kJ/kg)
3. The gases in the cylinder of an internal-combustion engine has a specific internal energy of 950 kJ/kg and a specific volume of 0.08 m³/kg at the beginning of expansion. The expansion of the gases may be assumed to take place according to a reversible law $pv^{1.4} = \text{constant}$, from 50 bar to 1.6 bar. The specific internal energy after expansion is 180 kJ/kg. Calculate the heat rejected to the cylinder cooling water per kilogram of gases during the expansion stroke (-144 kJ/kg).

THE STEADY FLOW ENERGY EQUATION

The governing equation for an open system with mass transfer across its boundary is referred to as **steady flow energy equation (SFEE)**. This is analogous to non-flow energy equation for a closed system.

For a flow to be regarded as steady, the mass flow must be constant and the same at inlet and outlet and the fluid properties at any point in the open system must not vary with time.

Consider a unit mass of a fluid with specific internal energy u flowing steadily and moving with velocity C through a steam power plant shown in Fig. 2.1 below

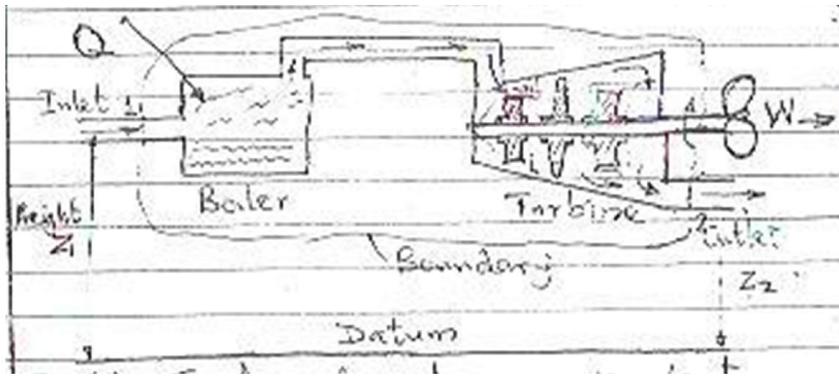


Fig. 2.1 Section of a steam power plant

The system above constitutes an open system with its boundary shown cutting the inlet pipe at section 1 and outlet pipe at section 2.

This boundary is often referred to as a control surface and the system as a control volume. Assuming a steady flow of heat Q per kg of fluid is supplied at the boiler and each kg of fluid does work W at the turbine.

Also considering an element of fluid at the entry of the boiler shown below.

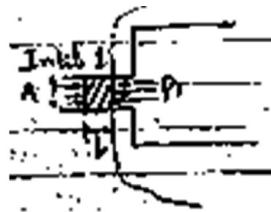


Fig. 2.2 section at inlet of boiler

Energy required to push fluid element across the boundary

$$= p_1 A_1 \times l = p_1 A_1 l = p_1 \times \text{volume of fluid element}$$

\therefore Energy required to move fluid element across the boundary at the inlet = $p_1 v_1$; v_1 = specific volume at inlet.

Similarly,

Energy required to move fluid element across the boundary at the exit = $p_2 v_2$; v_2 = specific volume at exit.

Total energy entering the system consists of the energy of the flowing fluid at inlet, energy term $p_1 v_1$ and the heat supplied Q i.e.

$$E_{in} = u_1 + \frac{1}{2} c_1^2 + g z_1 + p_1 v_1 + Q$$

Total energy leaving the system consists of energy of the flowing fluid at the exit, energy term $p_2 v_2$ and the output work W

$$E_{out} = u_2 + \frac{1}{2} c_2^2 + g z_2 + p_2 v_2 + W$$

Since the flow of fluid into and out of the system with heat and work crossing the boundary are steady, then $E_{in} = E_{out}$. Thus

$$u_1 + \frac{1}{2} c_1^2 + g z_1 + p_1 v_1 + Q = u_2 + \frac{1}{2} c_2^2 + g z_2 + p_2 v_2 + W$$

The sum of specific internal energy and $p v$ term is given the symbol h and is called specific enthalpy i.e. $h = u + p v$

By writing h for $(u + p v)$, the equation reduces to

$$h_2 + \frac{1}{2} c_1^2 + g z_1 + Q = h_2 + \frac{1}{2} c_2^2 + g z_2 + W$$

$$Q - W = (h_2 - h_1) + \frac{1}{2} (c_2^2 - c_1^2) + g (z_2 - z_1)$$

where Q and W are the heat and work transfers per unit mass flow through the system.

The above equation is called the **steady flow energy equation (SFEE)** and provides the basic means for studying most open systems of importance in engineering. In nearly all problems in practice, the potential energy term $g(z_2 - z_1)$ is either zero or negligible compared with other terms. It has been included in the equation for the sake of completeness and hence can be omitted in analysis.

For steady flow, the mass flow rate over a cross section of the flow at inlet is constant and equal to the mass flow rate at exit.

$$\text{i.e. } \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

and mass flow rate is the ratio of volume flow rate (AC) and specific volume (v) hence, mass

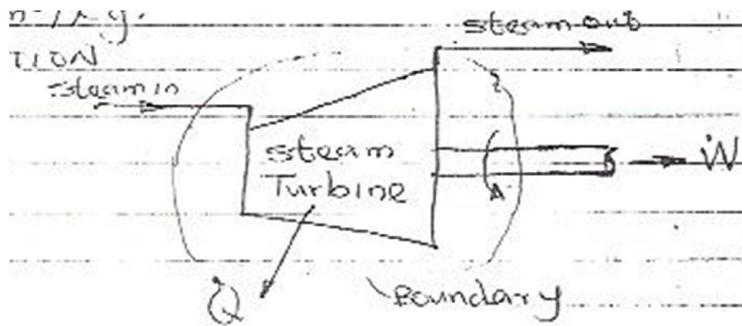
$$\text{flow rate } \dot{m} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} = \rho A C$$

This equation which expresses the principle of conservation of mass in steady flow is known as **continuity equation**.

EXAMPLE 2.4

In a steam turbine unit, the mass flow rate of steam through the turbine is 17 kg/s and the power developed by the turbine is 14000 kW. The specific enthalpies of the steam at inlet and exit of the turbine are 1200 kJ/kg and 360 kJ/kg while the velocities are 60 m/s and 150 m/s respectively. Calculate the rate at which heat is rejected from the turbine. Also find the area of the inlet pipe given that the specific volume of the steam at inlet is 0.5 m³/kg.

SOLUTION



Given

$\dot{m} = 17 \text{ kg/s}$, $W = 14000 \text{ kW}$, $h_1 = 1200 \text{ kJ/kg}$, $h_2 = 360 \text{ kJ/kg}$

$C_1 = 60 \text{ m/s}$, $C_2 = 150 \text{ m/s}$, $v_1 = 0.5 \text{ m}^3/\text{kg}$

To determine Q

From SFEE, neglecting the potential energy term

$$Q - W = \dot{m} (h_2 - h_1) + \frac{1}{2} \dot{m} (c_2^2 - c_1^2)$$

$$Q = 14,000 = 17(360 - 1200) + \frac{1}{2}(17)(150^2 - 60^2)$$
$$= -14280 + 160.65 = -14119.35 \text{ kW}$$

$$Q = -14119.35 + 1400 = -119.35 \text{ kW}$$

i.e. heat rejected = 119.35 kW

To determine inlet area A_1

From the continuity equation,

$$m = \frac{A_1 C_1}{v_1} \text{ and } A_1 = \frac{m v_1}{C_1}$$

$$A_1 = \frac{m v_1}{C_1} = \frac{17 \times 0.5}{60}$$

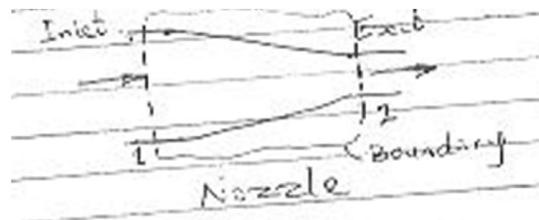
$$= 0.142 \text{ m}^2$$

i.e. inlet area $A_1 = 0.142 \text{ m}^2$

EXAMPLE 2.5

At the inlet to a nozzle employed for increasing the velocity of a steadily flowing fluid, the specific enthalpy and the velocity of the fluid are 3025 kJ/kg and 60 m/s respectively. The specific enthalpy of the fluid at the exit is 760 kJ/kg. The nozzle is horizontal and there is a negligible heat loss from it. Calculate the fluid velocity at the exit, and also find the mass flow rate of the fluid when the inlet area is 0.1 m^2 and inlet specific volume is $0.19 \text{ m}^3/\text{kg}$. If the specific volume at the nozzle exit is $0.5 \text{ m}^3/\text{kg}$. Determine the exit area of the nozzle.

SOLUTION



Solution

Given

$$h_1 = 3025 \text{ kJ/kg}, C_1 = 60 \text{ m/s}, h_2 = 2790 \text{ kJ/kg}$$

$$A_1 = 0.1 \text{ m}^2, v_1 = 0.19 \text{ m}^3/\text{kg}, v_2 = 0.5 \text{ m}^3/\text{kg}$$

Determine C_2

From SFEE, neglecting the potential energy term

Once the nozzle is horizontal i.e. $z_1 = z_2$

$$Q - W = (h_2 - h_1) + 1/2 (C_2^2 - C_1^2)$$

But no heat and work cross the boundary

i.e. $Q = 0$ and $W = 0$

$$\begin{aligned}
h_1 + 1/2 C_1^2 &= h_2 + 1/2 C_2^2 \\
3025 \times 10^3 + 1/2 (60)^2 &= 2790 \times 10^3 + 1/2 C_2^2 \\
3026.8 \times 10^3 - 2790 \times 10^3 &= 1/2 C_2^2 \\
C_2^2 &= 2(236.8 \times 10^3) \\
&= 473.6 \times 10^3 \\
C_2 &= \sqrt{4736 \times 10^3} \\
C_2 &= 688.19 \text{ m/s}
\end{aligned}$$

i.e. fluid velocity at exit = 688.19 m/s

From continuity equation,

$$\dot{m} = \frac{A_1 C_1}{v_1} = \frac{0.1 \times 60}{0.19}$$

Inlet mass flow rate = 31.59 kg/s

$$\text{Also, } \dot{m} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

$$\begin{aligned}
A_2 &= \frac{m v_2}{C_2} = \frac{31.59 \times 0.5}{688.19} \\
&= 0.02295 \text{ m}^2
\end{aligned}$$

i.e. the exit area $A_2 = 0.02295 \text{ m}^2$

PRACTICE QUESTIONS

1. A steady flow of steam enters a condenser with a specific enthalpy of 2300 kJ/kg and a velocity of 350 m/s. The condensate leaves the condenser with a specific enthalpy of 160 kJ/kg and a velocity of 70 m/s. Calculate the heat transfer to the cooling fluid per kilogram of steam condensed (02199 kJ/kg)
2. A steam turbine receives a steam flow of 4.35 kg/s and the power output is 500 kW. The heat loss from the casing is negligible. Calculate the change in specific enthalpy across the turbine when the velocity at entrance is 60 m/s, the velocity at exit is 360 m/s, and the inlet pipe is 3m above the exhaust pipe. (433 kJ/kg)
3. A turbine operating under steady-flow conditions receives steam at the following state: pressure 13.8 bar, specific volume 0.143 m³/kg, specific internal energy 2590 kJ/kg, velocity 30 m/s. The state of the steam leaving the turbine is as follows: pressure 0.35 bar,

specific volume $4.37 \text{ m}^3/\text{kg}$, specific internal energy 2360 kJ/kg , velocity 90 m/s Heat is rejected to the surroundings at the rate of 0.25 kW and the rate of steam flow through the turbine is 0.38 kg/s .

Calculate the power developed by the turbine (102.7 kW)