

REVERSIBLE NON-FLOW PROCESS

CONSTANT VOLUME PROCESS (ISOCHORIC PROCESS)

In a constant volume process, the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable, so work cannot be done on or by the system other than parallel which work input which is negative and this can be carried out in practice by some method of during the fluid. It will be assumed that constant volume implies zero work unless otherwise stated.

From the non-flow energy equation for unit mass, since no work is done, i.e. $W = 0$, we therefore have $Q = u_2 - u_1$

Or for mass m , of the working substance

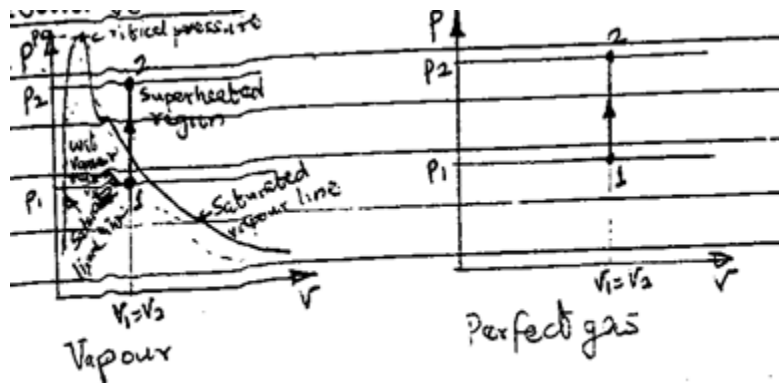
$$Q = U_2 - U_1$$

That is, all the heat supplied in a constant volume process goes to increasing the internal energy.

This equation can be written in differential form as $\delta Q = dU$

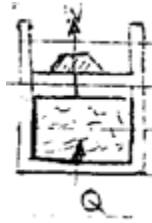
For a perfect gas, $Q = mc_v (T_2 - T_1)$

The figures below show the p-v diagrams of constant volume process for a vapour and a perfect gas



CONSTANT PRESSURE PROCESS (ISOBARIC PROCESS)

A closed system undergoing a process at constant pressure is illustrated in the figure below



The fluid is enclosed in a cylinder by a piston on which rests a constant weight. If heat is supplied, the fluid expands and work is done by the system in overcoming the constant force. Heat is extracted, the fluid contracts and work is done on the system by the constant force .

From the non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

Work might also be done in the system simultaneously by turning the fluid with a paddle, and this negative quantity of work must be included in the term W if no paddle work is done on the system and the process is reversible i.e. $W = \int p dv = p dv$, we have

$$\partial Q = du + p dv$$

$$\text{Or } \partial Q - p dv = du$$

Since p is constant, this can be integrated to give

$$Q - p(v_2 - v_1) = (u_2 - u_1)$$

A further implication of the energy equation is possible if a new property is introduced. Since p is constant, $p dv$ is identical with $d(pv)$

Thus the energy equation can be written as

$$\partial Q - d(pv) = du$$

$$\partial Q = d(u + pv)$$

But specific enthalpy $h = u + pv$

$$\text{i.e. } \partial Q = dh$$

or in the integrated form

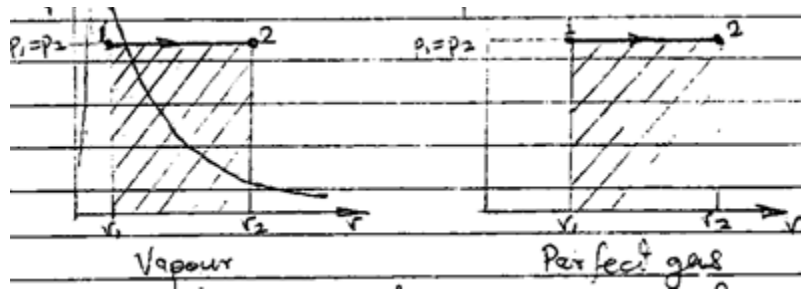
$$Q = h_2 - h_1$$

or for mass m , of a fluid

$$Q = H_2 - H_1 \text{ (reversible process only)}$$

That is the heat added in a reversible constant pressure process is equal to the increase in enthalpy. The above equation does not apply if the process is irreversible.

For a perfect gas, $Q = mc_p (T_2 - T_1)$



The figures above show the p-v diagrams of constant pressure process for a vapour and a perfect gas

NB: The areas shaded represent the work done by the fluid i.e. $(v_2 - v_1)$

CONSTANT TEMPERATURE PROCESS (ISOTHERMAL PROCESS)

When the quantities of heat and work are so proportioned during an expansion or compression that the temperature of the fluid remains constant the process is said to be isothermal when a fluid in a cylinder behind a piston expands from a high pressure to a low pressure, there is tendency for the temperature to fall. In isothermal expansion heat must be added continuously in order to keep the temperature at the initial value.

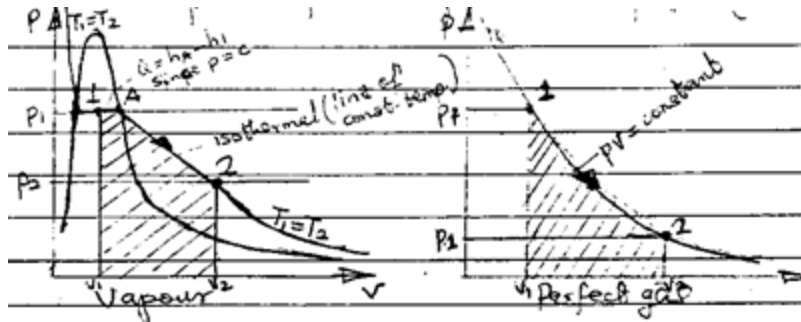
Similarly in an isothermal compression heat must be removed from the fluid continuously during the process. It is possible to show that for a reversible isothermal process, a certain definite relationship must exist between p and v, and consequently, the work done has a definite value that can be predicted.

For any reversible process the energy equation is $\delta Q - pdv = du$

Or

$$Q - \int_1^2 pdv = (u_2 - u_1)$$

The figures below show the p-v diagrams of isothermal process for a vapour and a perfect gas



For isothermal process of a perfect gas which is assumed to have an equation of state.

$$pv = RT = \text{constant, since } T = \text{constant}$$

$$\text{i.e. } pv = \text{constant}$$

or

$$p_1 v_1 = p_2 v_2$$

The work done by a perfect gas which undergoes a reversible isothermal expansion from state 1 to state 2 as shown in the figure above is given by the shaded area and can be evaluated as:

$$W = \int_1^2 p dv$$

In this case $pv = c$ or $p = c/v$

$$\therefore W = \int_1^2 \frac{c}{v} dv = c \int_1^2 \frac{1}{v} dv$$

$$= c [\ln v]_{v_1}^{v_2} = c \ln \left(\frac{v_2}{v_1} \right)$$

Since $c = p_1 v_1 = p_2 v_2$

$$W = p_1 v_1 \ln \left(\frac{v_2}{v_1} \right) = p_2 v_2 \ln \left(\frac{v_2}{v_1} \right) \text{ per unit mass of gas}$$

For mass, m , of the gas

$$W = p_1 v_1 \ln \left(\frac{v_2}{v_1} \right) = p_2 v_2 \ln \left(\frac{v_2}{v_1} \right)$$

From $p_1 v_1 = p_2 v_2$, we have

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$\therefore W = p_1 v_1 \ln \left(\frac{p_1}{p_2} \right) = p_2 v_2 \ln \left(\frac{p_1}{p_2} \right)$$

Also $p_1 v_1 = RT$

$$\text{Hence } W = RT \ln \left(\frac{p_1}{p_2} \right)$$

And for mass, m , of the gas

$$W = mRT \ln\left(\frac{p_1}{p_2}\right)$$

From Joule's law for perfect gas,

$$du = cvdT$$

For isothermal process $dT = 0$

$$\text{i.e } du = 0$$

That is the internal energy remains constant in an isothermal process for a perfect gas

From non-flow energy equation, we have

$$\partial Q = dU + \partial W$$

$$\text{Since } dU = 0$$

$$\therefore \partial Q = \partial W$$

or

$$Q = W$$

That is, the quantity of heat supplied is equal to the work done in an isothermal process for a perfect gas only.

ADIABATIC PROCESS

An adiabatic process is the one in which heat is prevented from crossing the boundary of the system. i.e. no heat is transferred to or from the fluid during the process. That is, an adiabatic process is one undergone by a system which is thermally insulated from its surroundings.

From the non-flow energy equation

$$Q = u_2 - u_1 + W$$

For an adiabatic process $Q = 0$, we have

$$-W = (u_2 - u_1)$$

or

$$W = (u_1 - u_2)$$

This equation is true for an adiabatic non-flow process whether or not the process is reversible.

In an adiabatic compression process all the work done on the fluid goes to increasing the internal energy of the fluid. Similarly in an adiabatic expansion process, the work done by the fluid is at the expense of a reduction in the internal energy of the fluid.

A reversible adiabatic process is called isentropic (constant entropy) process.

For a vapour undergoing a reversible adiabatic process the work done can be found from equation given above by evaluating u_1 and u_2 from tables. For a perfect gas, a law relating p and v has been obtained for a reversible adiabatic process as $pv^\gamma = \text{constant}$

Each perfect gas undergoing a reversible adiabatic process have its own value of γ

POLYTROPIC PROCESS

The constant volume and constant pressure processes can be regarded as limiting cases of a more general type of process in which both the volume and pressure change, but in a certain specified manner.

In many real processes it is found that the states during an expansion or compression can be described approximately by a relation of the form $pv^n = \text{constant}$ where n is a constant called the index of expansion or compression, and p and v are average values of pressure and specific volume for the system.

Compressions and expansion of the form $pv^n = \text{constant}$ are called polytropic processes. When $n = 0$, the relation reduces to $p = \text{constant}$ (isobaric process) and when $n = \infty$, it can be seen to reduce to $v = \text{constant}$ by writing it in the form $p^{\frac{1}{n}} v = \text{constant}$

For any reversible process, $W = \int p dv$

For a process in which $pv^n = \text{constant}$, we have $p = \frac{c}{v^n}$

Where c is a constant

$$\begin{aligned} \text{Thus, } W &= \int_{v_1}^{v_2} \frac{c}{v^n} dv = c \int_{v_1}^{v_2} \frac{dv}{v^n} = c \left(\frac{v^{-n+1}}{-n+1} \right)_{v_1}^{v_2} \\ &= c \left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1} \right) = c \left(\frac{v_2^{1-n} - v_1^{1-n}}{-n+1} \right) \\ &= c \left(\frac{-(v_1^{1-n} - v_2^{1-n})}{-(n-1)} \right) = c \left(\frac{v_1^{1-n} - v_2^{1-n}}{n-1} \right) \\ &= \left[\frac{p_1 v_1^n v_1^{1-n} - p_2 v_2^n v_2^{1-n}}{n-1} \right] \end{aligned}$$

Since constant $c = p_1 v_1^n = p_2 v_2^n$

$$W = \frac{p_1 v_1 - p_2 v_2}{n-1}$$

This equation is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^n$$

The integrated form of the energy equation for a reversible polytropic process can therefore be written as

$$Q - \left(\frac{p_1 v_1 - p_2 v_1}{n-1} \right) = (u_2 - u_1)$$

In a polytropic process the index 'n' depends on in the heat and work quantities during the process. The various processes considered previously are special cases of the polytropic process.

When $n = \infty$, $p v^\infty = c$ or $p^{\frac{1}{\infty}} v = \text{constant}$ (isobaric process)

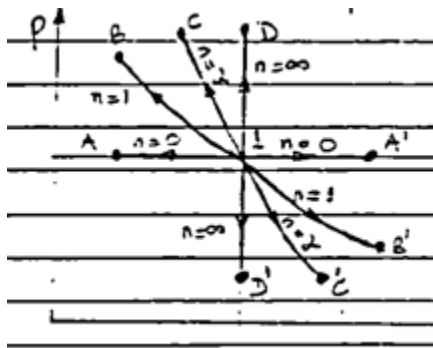
When $n = \infty$, $p v = c$ or $p v^{1/\infty} = C$ i.e. $v = \text{constant}$ (isochoric)

When $n = 1$, $p v = c$ i.e. $T = \text{constant}$ (isothermal process)

Since $p v/T = \text{constant}$ for a perfect gas

When $n = \gamma$, $p v^\gamma = \text{constant}$ i.e. reversible adiabatic process

This is illustrated on a p-v diagram shown below



State 1 to A is constant pressure cooling ($n = 0$)

State 1 to B is isothermal compression ($n = 1$)

State 1 to C is reversible adiabatic compression ($n = \gamma$)

State 1 to D is constant volume heating ($n = \infty$)

Similarly,

State 1 to A^1 is isobaric heating

State 1 to B^1 is isothermal expansion

State 1 to C^1 is reversible adiabatic expansion

State 1 to D^1 is isochoric cooling